Uncertainty analysis in National Greenhouse Gas Inventories

Support to the implementation of the Enhanced Transparency Framework under the Paris Agreement by developing countries

28-30 October 2025

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Typical problems in developing countries

1. Limited or no technical capacity. Priority is always on GHG accounting
2. Limited staff with statistical background and experience in uncertainty assessment
3. Lack of data collection on uncertainty (institutional arrangements for data collection)
4. Data reported without associated uncertainty
5. Use of default uncertainty data from the 2006 IPCC GLs may not represent national circumstances or level of aggregation
6. Limited/lack of knowledge of 2006 IPCC GLs, tools/software

TUESDAY

2. Uncertainty associated with input data

- Overview of uncertainty analysis in national GHG inventories
- Key concepts and causes of uncertainties associated with input data

WEDNESDAY

3. Approach 1 - Propagation of errors for uncertainty analysis

- Key concepts of the method to combine uncertainties
- Application to different cases

THURSDAY

4. Approach 2 - Monte Carlo simulation for uncertainty analysis

- Key concepts of the method to combine uncertainties
- Selection of the probabilistic functions
- Application to different cases

Practical exercises and typical challenges

Energy, IPPU, Agriculture, LULUCF and Waste

2

Session 2. Uncertainty associated with input data

By the end of this session, you will:

1. Know where to find more details
2. Understand the terminology
3. Learn strategies to deal with uncertainty
4. Quantify uncertainty based on available data

Learning objectives

Uncertainty overview [1]

2006 IPCC Guidelines for

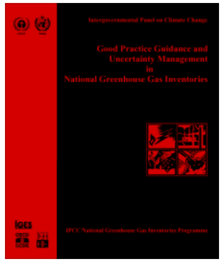
National Greenhouse Gas Inventories

Guidelines | Energy | IPPU | AFOLU | Waste

Vol. 2-5 – uncertainty sections for each category

Vol. 1 - Ch. 3: uncertainty

Good Practice Guidance and Uncertainty Management in National Greenhouse Gas Inventories



Chapter 1 Introduction
Chapter 2 Energy
Chapter 3 Industrial Processes
Chapter 4 Agriculture
Chapter 5 Waste
Chapter 6 Quantifying Uncertainties in Practice
Chapter 7 Methodological Choice and Recalculation
Chapter 8 Quality Assurance and Quality Control

Uncertainty overview [2]

General approach

Uncertainty

Lack of knowledge of the true value of a variable that can be described as a probability density function (PDF). Uncertainty depends on the analyst's state of knowledge, which in turn depends on the quality and quantity of applicable data as well as knowledge of underlying processes and inference methods.

Uncertainty analysis

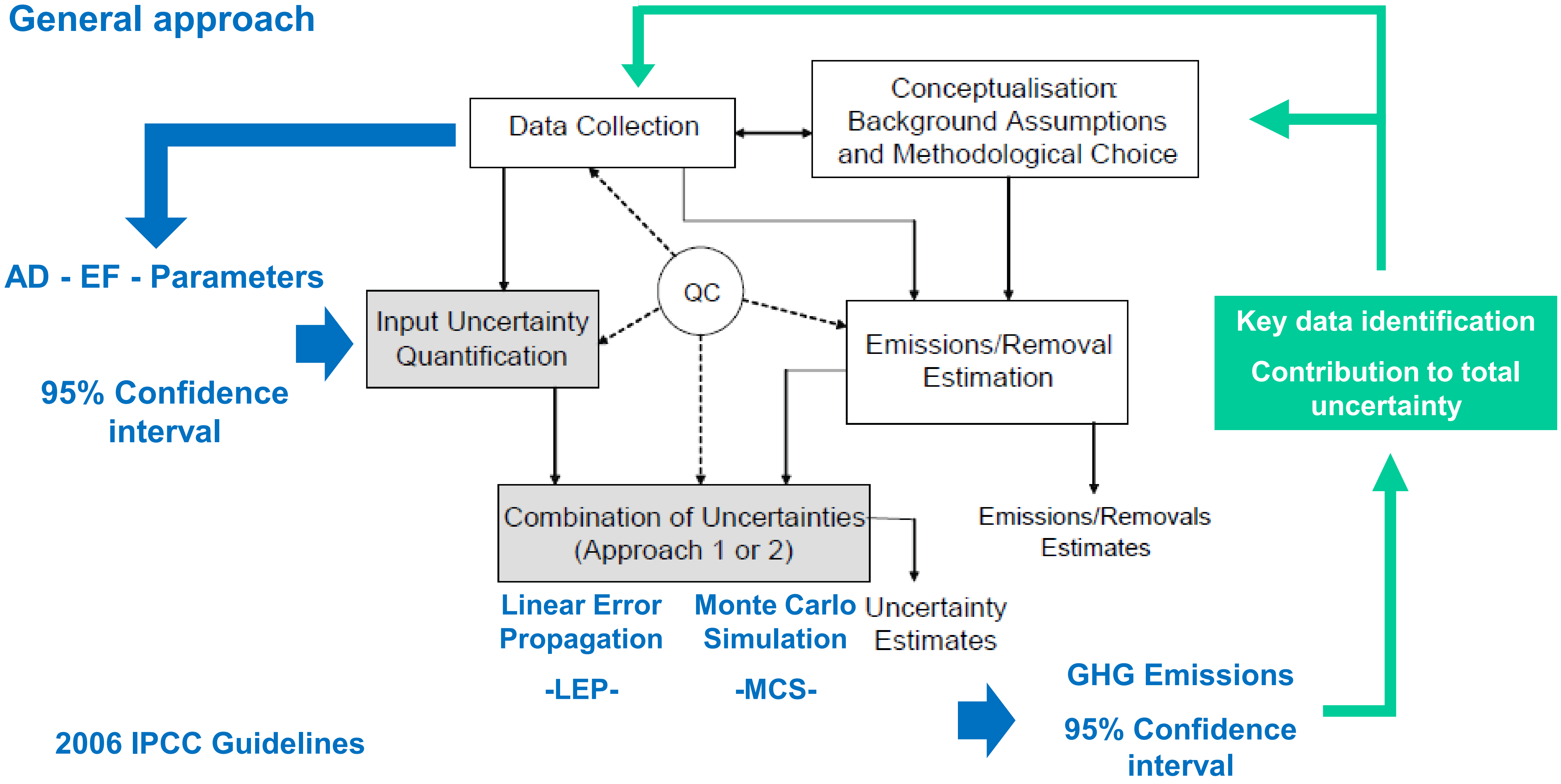
An uncertainty analysis should be seen, first and foremost, as a means to help prioritise national efforts to reduce the uncertainty of inventories in the future, and guide decisions on methodological choice.
Quantitative uncertainty analysis is performed by estimating the 95 percent confidence interval of the emissions and removals estimates for individual categories and for the total inventory

Uncertainty assessment

The term "ASSESSMENT" is intended to convey an exercise that includes the investigation of quantitative and qualitative aspects. In the glossary to the Guidelines, "uncertainty analysis" is defined as only a quantitative exercise.

Uncertainty overview [3]

General approach



2006 IPCC Guidelines

Uncertainty overview [4]

Key concepts

Confidence interval: range that encloses the true, but unknown value, with a determined confidence (probability). Typically, a 95 percent confidence interval is used in greenhouse gas inventories.
Alternative interpretation: Range that may safely be declared to be consistent with observed data or information

Probability Density Function (PDF): describes the range and relative likelihood of possible values.
For emission inventory, it is used to describe uncertainty in the estimate of a quantity that is a fixed constant whose value is not exactly known.

Sensitivity analysis: method to determine which of the input uncertainties to an inventory contributes most substantially to the overall uncertainty.

Uncertainty overview [5]

Key concepts

Accuracy: *Agreement between the true value and the average of repeated measured observations or estimates of* a variable.
An accurate measurement or prediction lacks bias or, equivalently, systematic error.

Bias / Systematic error: *Lack of accuracy. Bias (systematic error), can occur because of failure to capture all relevant processes involved or because the available data are not representative of all real-world situations, or because of instrument error.*

Precision: *Agreement among repeated measurements of the same variable.*
Better precision means less random error. Precision is independent of accuracy.

Random errors: *Random variation above or below a mean value. Random error is inversely proportional to* precision. Usually, the random error is quantified with respect to a mean value, but the mean could be biased or unbiased. Thus, random error is a distinct concept compared to systematic error.

Uncertainty overview [6]

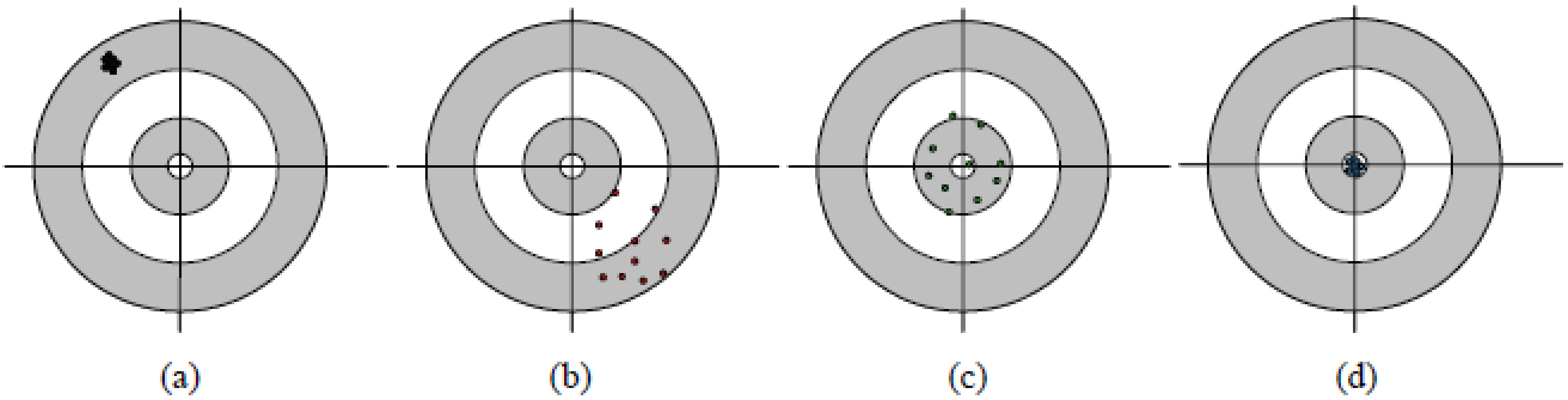
Key concepts

Lack of knowledge of the true value

How far is the true value from the value used?

Accuracy (systematic errors or bias) vs. Precision (random errors)

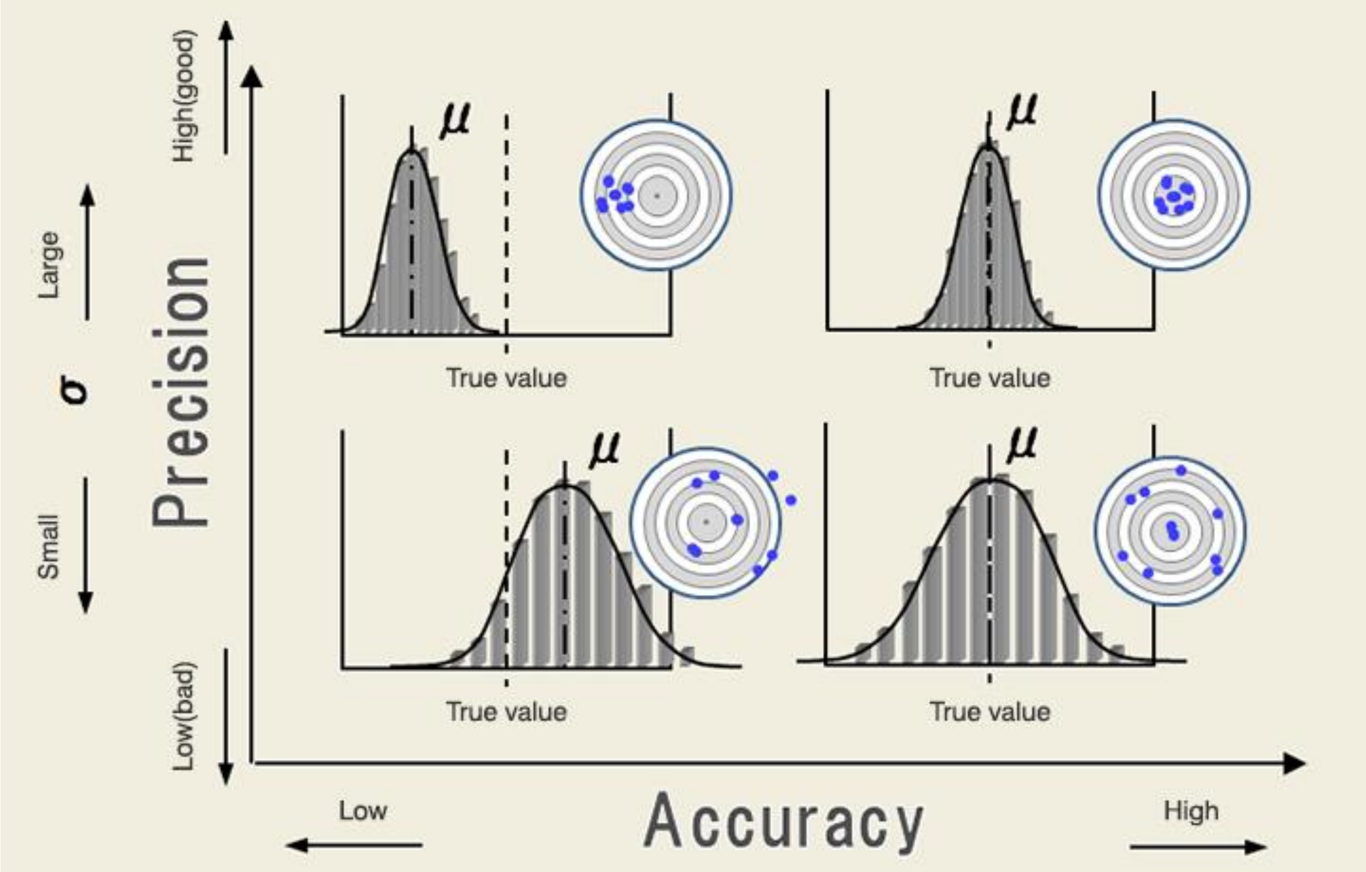
(a) inaccurate but precise; (b) inaccurate and imprecise; (c) accurate but imprecise; and (d) precise and accurate



Uncertainty overview [7]

Key concepts

Accuracy (systematic errors or bias) vs. Precision (random errors)



Source: Hitachi, 3. Semiconductor - Accuracy and Precision, Fig.3-5

Uncertainty overview [8]

Key concepts

The **quantitative uncertainty analysis** tends to deal primarily with **random errors** based on the **inherent variability** of a system and the **finite sample size** of available data, **random components of measurement error**, or **inferences** regarding the random component of uncertainty obtained from **expert judgment**

→ **PRECISION**

It is important to recognize that **some uncertainties that are not addressed by statistical means may exist**, including those arising from **omissions or double counting**, or other **conceptual errors**, or from **incomplete understanding** of the processes that may lead to inaccuracies in estimates developed from models.
Bias or systematic errors

→ **ACCURACY**

Uncertainty overview [9]

Terminology

i) Confidence interval: range that encloses the true value with a determined confidence (probability)

95 % CI → 2.5 th [……………………..] 97.5 th

	Absolute	Relative
Symmetric: 80 (64-96)	**80 ± 16**	**80 ± 20%**
Asymmetric: 80 (72-104)	**80 -8/+24**	**80 -10%/+30%**

64 [--------80--------] 96

16 | 16

16/80 = 0.2

72 [-----80--------------] 104

Uncertainty

U_i

Large uncertainty:

100 (50-200) **100 -50/+100** **100 -50%/+100%** **100 ± factor of 2** (half or double)

Factor of "n" **(mean/n) and (mean x n)**

Uncertainty overview [10]

Terminology

TABLE 2.2
DEFAULT EMISSION FACTORS FOR STATIONARY COMBUSTION IN THE ENERGY INDUSTRIES
(kg of greenhouse gas per TJ on a Net Calorific Basis)

Fuel		CO_2			CH_4			N_2O		
		Default Emission Factor	Lower	Upper	Default Emission Factor	Lower	Upper	Default Emission Factor	Lower	Upper
Crude Oil		73 300	71 100	75 500	r 3	1	10	0.6	0.2	2
Orimulsion		r 77 000	69 300	85 400	r 3	1	10	0.6	0.2	2
Natural Gas Liquids		r 64 200	58 300	70 400	r 3	1	10	0.6	0.2	2
Gasoline	Motor Gasoline	r 69 300	67 500	73 000	r 3	1	10	0.6	0.2	2
Gasoline	Aviation Gasoline	r 70 000	67 500	73 000	r 3	1	10	0.6	0.2	2
Gasoline	Jet Gasoline	r 70 000	67 500	73 000	r 3	1	10	0.6	0.2	2
Jet Kerosene		r 71 500	69 700	74 400	r 3	1	10	0.6	0.2	2
Other Kerosene		71 900	70 800	73 700	r 3	1	10	0.6	0.2	2
Shale Oil		73 300	67 800	79 200	r 3	1	10	0.6	0.2	2
Gas/Diesel Oil		74 100	72 600	74 800	r 3	1	10	0.6	0.2	2
Residual Fuel Oil		77 400	75 500	78 800	r 3	1	10	0.6	0.2	2

75.5 [---77.4---] 78.8 77.4 (-2.5% ; +1.8%) 1 (---3-----------) 10 3 -67% / +233%

Approx. 77.4 ± 2% **Approx. 3 ± factor of 3**

Uncertainty overview [11]

Terminology

Emission factor uncertainties

These will be the same as the uncertainties associated with estimation of the litter and dead organic matter stocks per unit area on the previous land use. Uncertainties need not be estimated where zero carbon density in litter and dead organic matter pools is assumed for Cropland. Where this is not the case, uncertainties should be assessed by analysis of local data and should both exceed a factor of about 2.

Uncertainties associated with carbon stocks and other parameter values are likely to be at least a factor of three unless country-specific data are available from well designed surveys.

Solid storage[b]	The storage of manure, typically for a period of several months, in unconfined piles or stacks. Manure is able to be stacked due to the presence of a sufficient amount of bedding material or loss of moisture by evaporation.	0.005	Factor of 2	Judgement of IPCC Expert Group in combination with Amon *et al.* (2001), which shows emissions ranging from 0.0027 to 0.01 kg N_2O-N (kg N)$^{-1}$.

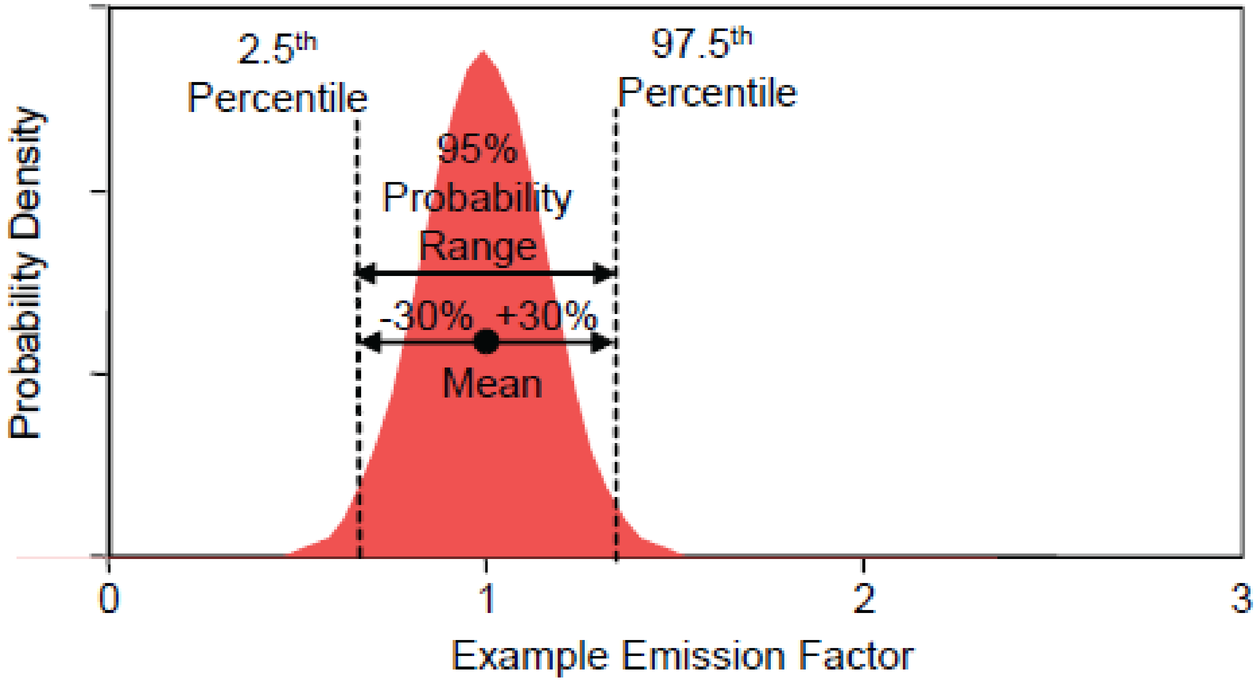
(see Annex 10A.1). Table 10.11 presents the enteric fermentation emission factors for cattle. A range of emission factors is shown for typical regional conditions. As shown in the table, the emission factors vary by over a factor of four on a per head basis.

Uncertainty overview [12]

Terminology

ii) Probability Density Function: range and relative likelihood of possible values

Symmetric (normal distribution)

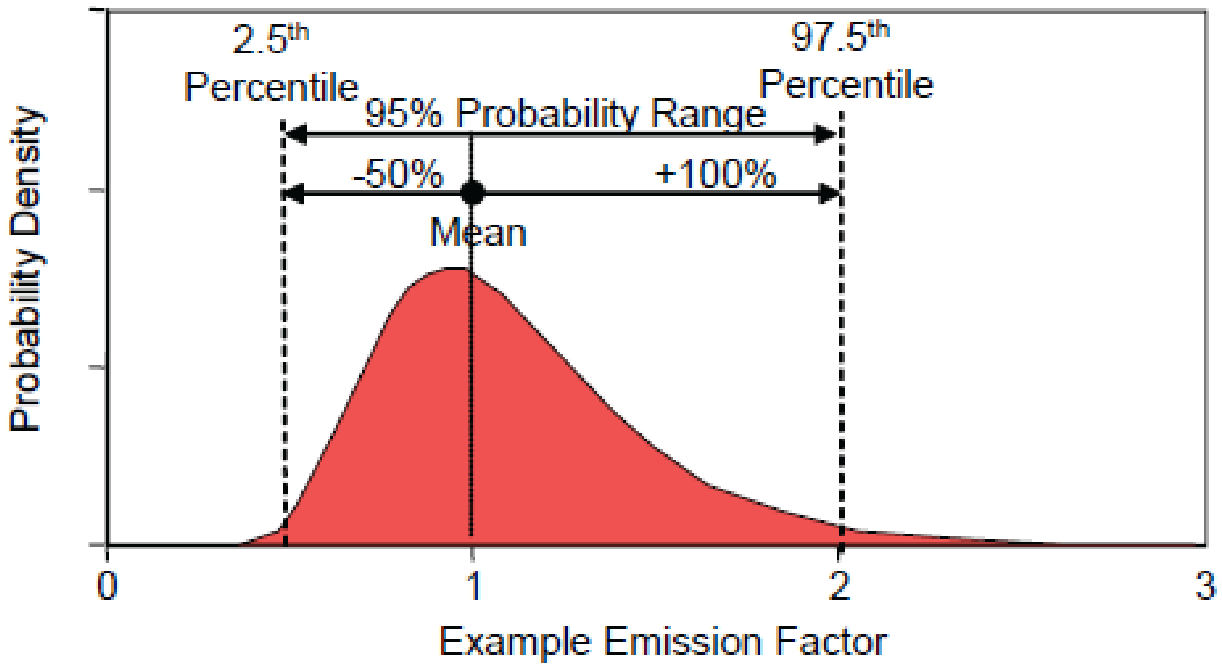


Uncertainty overview [13]

Terminology

ii) **Probability Density Function**: range and relative likelihood of possible values

Asymmetric (Skewed)



Causes of uncertainty [1]

Causes of uncertainty: Animal population? Age? Livestock characterization? Diet?

Causes of uncertainty [2]

Causes of uncertainty: Land representation? Stand volume? Carbon stock? Below ground biomass?

Causes of uncertainty [3]

Causes of uncertainty: Pipe length? No. of fittings? Gas composition? Maintenance? Venting?

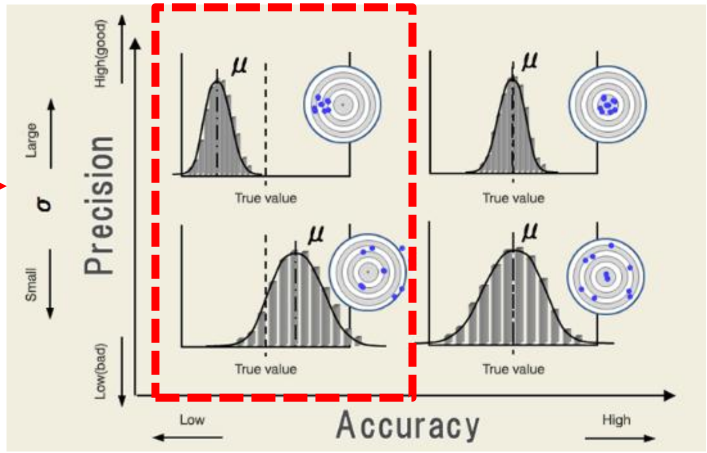
Causes of uncertainty [4]

Causes of uncertainty: Waste generation? Composition? Climate? Treatment? Management type?

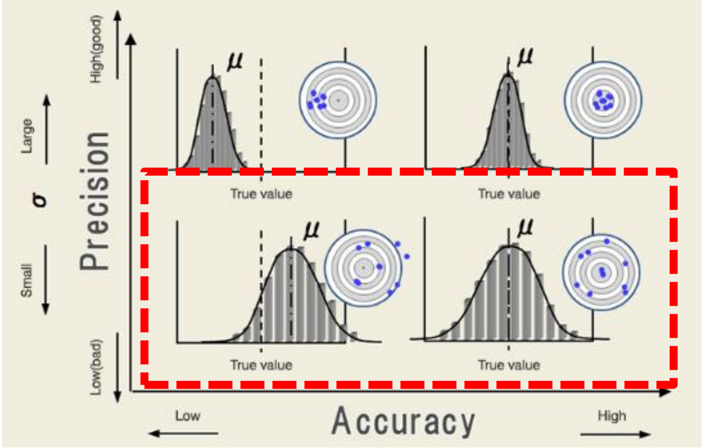
Causes of uncertainty [5]

Causes of uncertainty

1. **Lack of completeness** — **-Bias**



2. **Model** — **-Bias and Random errors**
3. **Lack of data** — **-Bias and Random errors**
4. **Representativeness of data** — **-Bias**
5. **Random sampling error** — **-Random errors**



6. **Measurement errors** — **-Bias and Random errors**
7. **Misreporting** — **-Bias**
8. **Data gaps** — **-Bias and Random errors**

Uncertainty associated with activity data [1]

National statistics: census, survey

- Activity data are closely linked to economic activity
- well established price incentives and fiscal requirements for accurate accounting

National statistics

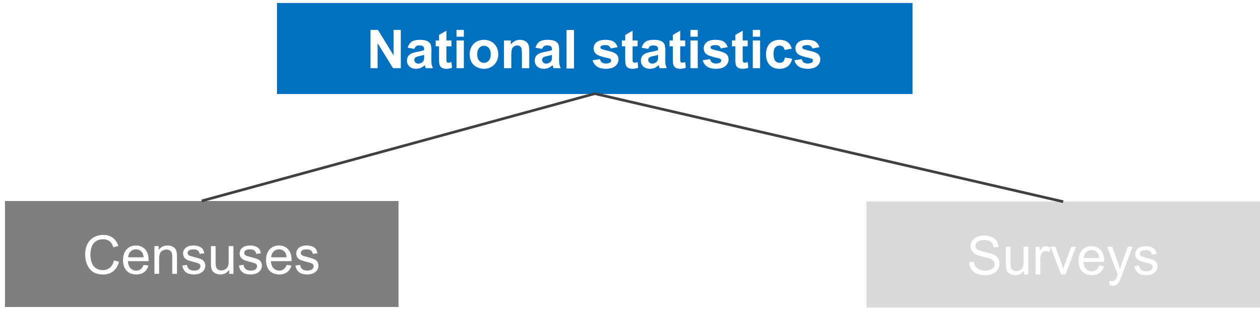
V1_Ch2. - Annex 2A.2
2006 IPCC Guidelines
General guidance on performing surveys

Censuses	Surveys
Counting every instance of a particular activity. Typically includes both systematic and random errors.	Counting a portion of a particular activity. Typically includes both systematic and random errors.
Random errors are typically normally distributed and uncorrelated	**Random errors are typically normally distributed and uncorrelated**

Contact the statistical agencies directly

Uncertainty associated with activity data [2]

National statistics: census

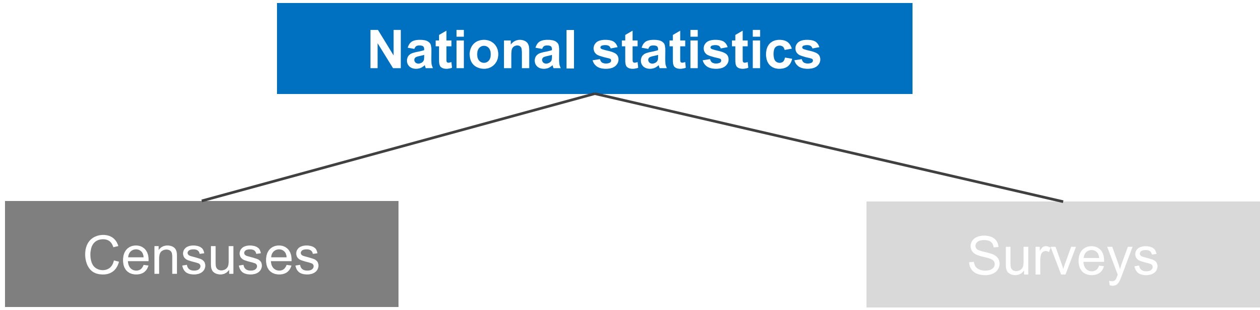


- Check for the size of random errors, look for fluctuations over time, and differential fluctuations in series that ought to be highly correlated with the data of interest
- To check for bias errors, cross-check the data of interest with other, related information. (look up and down the supply chain for fuels, or highly correlated activities with the data of interest, for instance reported fuel input vs. electricity output).
- Interpretation of statistical differences, within, for instance, national energy data are an example of cross checking (e.g. reference approach)..

Often ‘precise but inaccurate’

Uncertainty associated with activity data [3]

National statistics: census



Practical exercises

Uncertainty associated with activity data [4]

National statistics: census

Periodic publications

ENERGY: liquid fuels commercialization. Fuel consumption statistics are published by the Ministry of Energy every year for gasoil and gasoline. The statistics contains the most updated information for the current and previous years.

Evaluate consistency and identify fluctuations over time in series to derive the uncertainty of the data

Year	Gasoil	Gasoline
2016	**659.034**	**479.291**
2017	**662.157**	**504.563**
2018	**666.065**	**533.358**
2019	**770.377**	**599.242**
2020	**866.303**	**554.953**
2021		
2022		

Source: commercialization tables, 12 July 2021

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Source: commercialization tables, 18 June 2022

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2022	**884.250**	**426.849**

Source: commercialization tables, 23 June 2023

Uncertainty associated with activity data [4]

National statistics: census

4,5% U ≈ 5% 3,7%

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Uncertainty associated with activity data [5]

National statistics: census

Highly correlated activities
ENERGY: fuel consumption vs electricity generation. Fuel consumption and electricity generation are reported every year by the electricity grid administrator.
Evaluate the reported data, calculate the efficiency to identify any inconsistency and assess uncertainty.

	Total
Gas Turbine Generation (MWh)	**59.757.516**
Natural Gas Consumption (10^3 m^3)	**14.975.637**

42%

NCV: 48 TJ/Gg

Density: 0,714 kg/m^3

efficiency ?

Uncertainty associated with activity data [6]

National statistics: census

Highly correlated activities
ENERGY: fuel consumption vs electricity generation. Fuel consumption and electricity generation are reported every year by the electricity grid administrator.
Evaluate the reported data, calculate the efficiency to identify any inconsistency and assess uncertainty

	January	February	March	April	May	June	July	August	September	October	November	December	Total
GT (MWh)	5.155.358	4.554.661	5.010.912	4.657.051	4.714.604	5.288.447	5.432.010	5.592.661	5.316.355	4.290.471	4.512.174	5.232.812	59.757.516
NG (10^3 m^3)	1.711.896	1.375.323	1.480.325	1.331.261	1.345.921	1.572.943	157.041	1.674.304	1.632.617	1.376.352	1.323.954	1.622.381	14.975.637
	0,33	0,30	0,30	0,29	0,29	0,30	0,03	0,30	0,31	0,32	0,29	0,31	0,25

NCV: 48 TJ/Gg

Density: 0,714 kg/m^3

Monthly efficiency ?

	January	February	March	April	May	June	July	August	September	October	November	December	Total
Generation (TJ)	18.559	16.397	18.039	16.765	16.973	19.038	19.555	20.134	19.139	15.446	16.244	18.838	**215.127**
Consumption (TJ)	58.670	47.135	50.734	45.625	46.127	53.908	5.382	57.382	55.953	47.170	45.375	55.602	**513.245**
Efficiency	32%	35%	36%	37%	37%	35%	363%	35%	34%	33%	36%	34%	**42%**

Uncertainty associated with activity data [7]

National statistics: census

≈ 20%

Highly correlated activities
ENERGY: fuel consumption vs electricity generation. Fuel consumption and electricity generation are reported every year by the electricity grid administrator.
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NG (10^3 m^3)	1.711.896	1.375.323	1.480.325	1.331.261	1.345.921	1.572.943	157.041	1.674.304	1.632.617	1.376.352	1.323.954	1.622.381	14.975.637
							1.570.410						18.017.687

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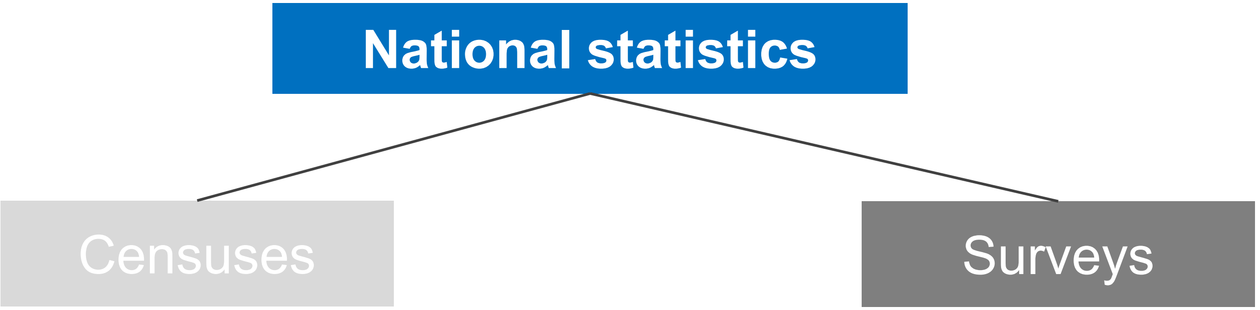
NG (10^3 m^3)	1.711.896	1.375.323	1.480.325	1.331.261	1.345.921	1.572.943	1.570.410	1.674.304	1.632.617	1.376.352	1.323.954	1.622.381	**18.017.687**
	32%	35%	36%	37%	37%	35%	36%	35%	34%	33%	36%	34%	35%

Uncertainty associated with activity data [8]

National statistics: survey



- Sample size and inter-individual variability
- Typical cases: consumer surveys, home expenses survey, land use surveys or forest cover surveys
- The agency conducting the sample will normally be able to advise on sampling error.
- If there is no information available, it may be possible to identify, or infer, the sample and population sizes and calculate sampling error directly.

precision depends on sample size, accuracy depends on sampling design

Uncertainty associated with activity data [9]

National statistics: survey

Heterogeneity (standard deviation) vs. Uncertainty in sample mean (standard error)

$$Uncertainty = \pm\left(\frac{1.96 \cdot \sigma}{\mu}\right) \cdot 100\%$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$$

$$Uncertainty = \pm\left(\frac{1.96 \cdot SE}{\mu}\right) \cdot 100\%$$

$$SE = \frac{\sigma}{\sqrt{n}}$$

Heterogeneity (standard deviation)	Uncertainty in sample mean (standard error)
Variability <u>within</u> the sample	Variability <u>of the mean</u> of the sample
Applicable for individual value	Applicable for country average
Standard deviation tends to remain constant	Standard error falls as sample size grows

Uncertainty associated with activity data [10]

National statistics: survey

National statistics

- Censuses
- Survey

Practical exercises

Uncertainty associated with activity data [11]

National statistics: survey

LULUCF: Carbon stock in forest from surveys

a) Given a sample with 80 individual values, calculate the mean, standard deviation and standard error.

b) If the sampling is repeated ten times, calculate the mean for each sample and the standard deviation of the sampling distribution of the mean and compare with a).

	Samp. 1	Samp. 2	Samp. 3	Samp. 4	Samp. 5	Samp. 6	Samp. 7	Samp. 8	Samp. 9	Samp. 10
Mean	102	99	101	100	97	100	100	101	102	96
STD	18	21	18	22	20	19	19	20	20	21
SE	2,0	2,4	2,0	2,5	2,3	2,2	2,1	2,2	2,2	2,4

Mean	STD
101	2,1

Uncertainty associated with activity data [12]

National statistics: survey

LULUCF: Carbon stock in forest from surveys

Calculate the uncertainty of the carbon stock obtained from the sampling to be used in the GHG inventory. The emissions from land use change will be calculated for the entire country in the year in which the survey was carried out.

Sample size: 30

Average C stock: 93.7 tC/ha

Standard deviation: 10.2 tC/ha

σ or SE ?

$$Uncertainty = \pm\left(\frac{1.96 \cdot SE}{\mu}\right) \cdot 100\%$$

$$SE = \frac{\sigma}{\sqrt{n}}$$

SE: 1.9 tC/ha

U: 4 %

What if the carbon stock is applied to account for emissions in one deforested area?

Uncertainty associated with activity data [13]

National statistics: survey

WASTE: Municipal solid waste amount and composition

1) The amount per capita is obtained from a sample that covers vehicles collecting in a wide range of areas: urban and rural, wealthy and poor, with and without gardens, etc. and covering several periods throughout the year.

Uncertainty for the entire MSW category? σ or SE ?

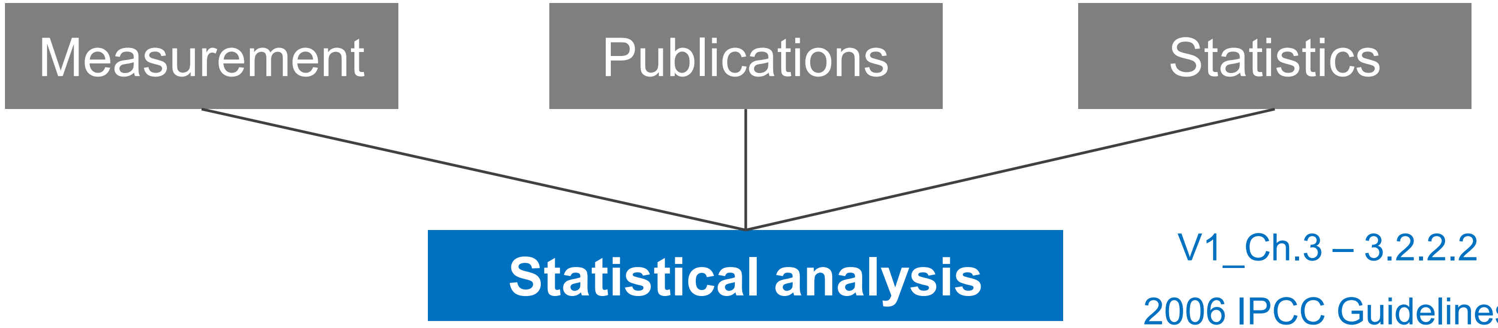
Uncertainty for the emissions from managed landfills? σ or SE ?

2) The composition of the MSW was done through a survey at different landfills. A national waste composition was calculated based on the results and is used to calculate the methane emissions in each individual landfill.

Uncertainty for the composition based on amount from each landfill? σ or SE ?

Uncertainty associated with empirical data [1]

Techniques for quantifying uncertainties



V1_Ch.3 – 3.2.2.2
2006 IPCC Guidelines

- Approach to quantify uncertainty in inventories, mainly associated to emission factors and other estimation parameters

- 6 steps approach to apply systematically

Uncertainty associated with empirical data [2]

Techniques for quantifying uncertainties

Statistical analysis

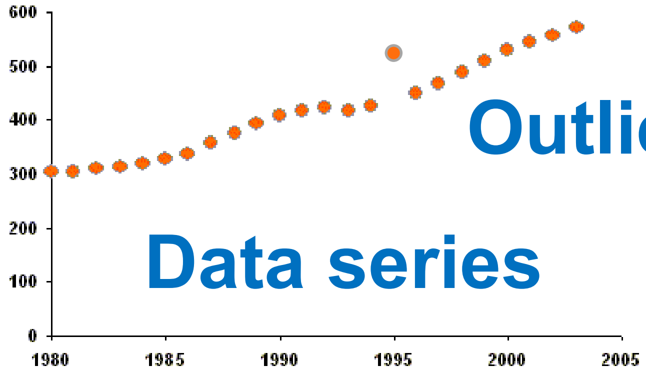
1. Compilation and evaluation of a database
2. Visualisation of data by developing empirical distribution functions
3. Fitting, evaluation, and selection of alternative PDF
4. Characterisation of uncertainty in the mean of the distributions for variability
5. Input to a probabilistic analysis to estimate uncertainty in total emissions
6. Sensitivity analysis

Uncertainty associated with empirical data [3]

Techniques for quantifying uncertainties

Statistical analysis

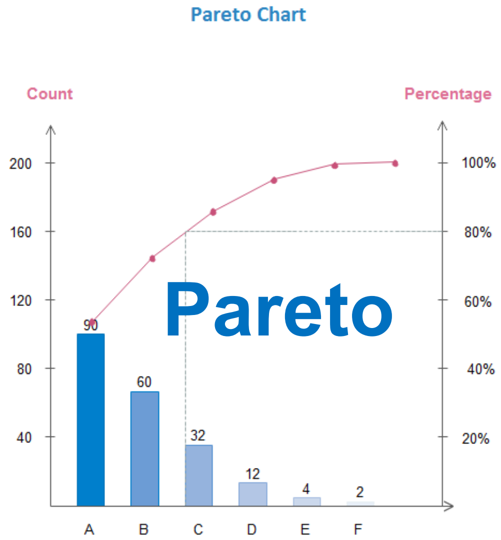
1. Compilation and evaluation of a database
2. Visualisation of data by developing empirical distribution functions



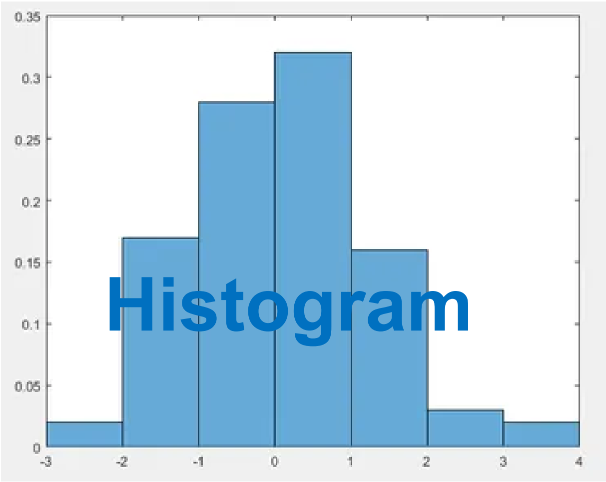
Outliers

Data series

Consistency



Pareto



Histogram

Uncertainty associated with empirical data [4]

Techniques for quantifying uncertainties

Statistical analysis

3. Fitting, evaluation, and selection of alternative PDF

Possible PDF

Physical process

Expert judgement

Variability

Goodness of fit test

Uncertainty associated with empirical data [5]

Techniques for quantifying uncertainties

Statistical analysis

4. Characterisation of uncertainty in the mean of the distributions for variability
5. Input to a probabilistic analysis to estimate uncertainty in total emissions
6. Sensitivity analysis

σ or SE ? **95% CI** → U_i

$U_i \rightarrow U_{Total}$

$U_{Total} \rightarrow X_i$

Uncertainty associated with empirical data [6]

Statistical analysis

Attention!

Measurements taken for another purpose may not be representative.

For example, methane measurements made for safety reasons at coal mines and landfills may not necessarily reflect total emissions because they may have been made only when methane emissions were suspected of being high, as a compliance check.

In such cases, the ratio between the measured data and total emissions should be estimated for the uncertainty analysis.

Conclusions

Uncertainty assessment

- It is a means to help prioritise national efforts to reduce the uncertainty of inventories in the future
- It guides decisions on methodological choice
- It helps understand the quality of the information use
- It is a requirement of GHG Inventories

Assessment of uncertainty in the input parameters should be part of the data collection !

End of day 1
Thank you!

Diego M. Ezcurra

TUESDAY

2. Uncertainty associated with input data

- Overview of uncertainty analysis in national GHG inventories
- Key concepts and causes of uncertainties associated with input data

WEDNESDAY

3. Approach 1 - Propagation of errors for uncertainty analysis

- Key concepts of the method to combine uncertainties
- Application to different cases

THURSDAY

4. Approach 2 - Monte Carlo simulation for uncertainty analysis

- Key concepts of the method to combine uncertainties
- Selection of the probabilistic functions
- Application to different cases

Practical exercises and typical challenges

Energy, IPPU, Agriculture, LULUCF and Waste

3

Session 3. Methods to combine uncertainties: Approach 1 Propagation of errors

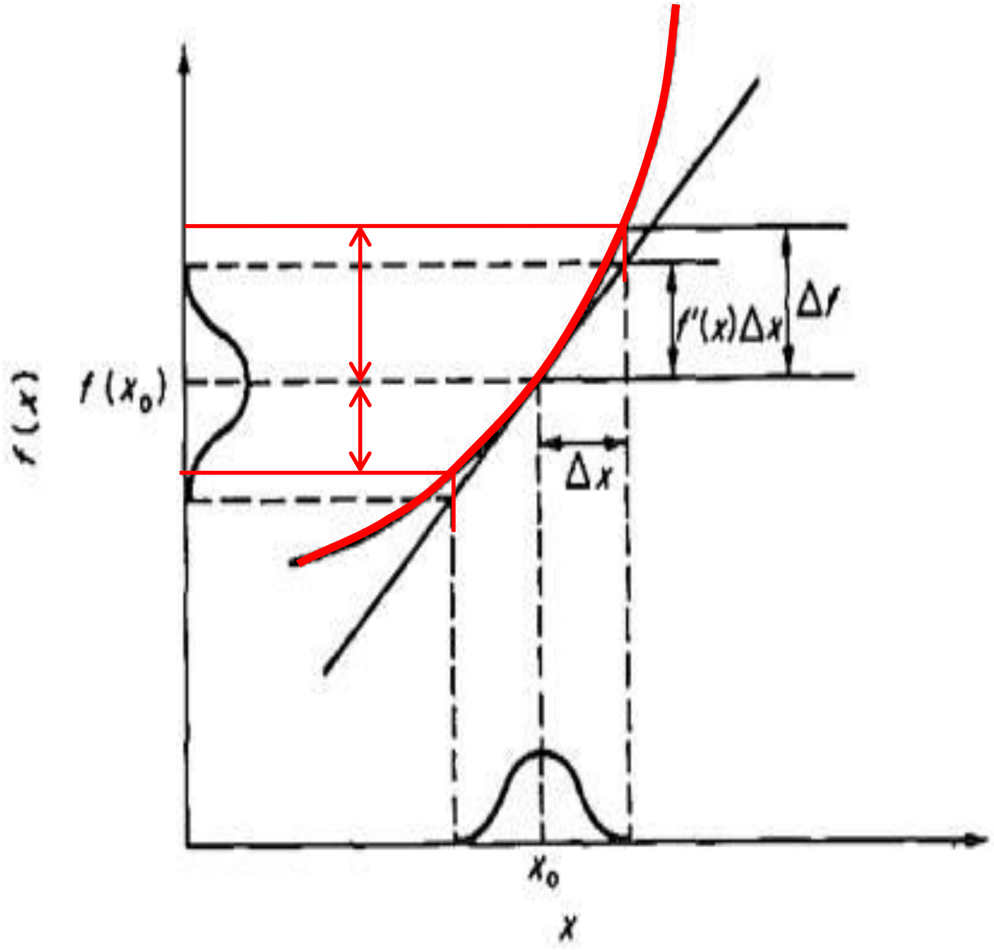
By the end of this session, you will:

1. Understand the basics and assumptions
2. Learn how to apply it to several cases
3. Identify key variables and avoid pitfalls
4. Learn how to deal with asymmetric cases

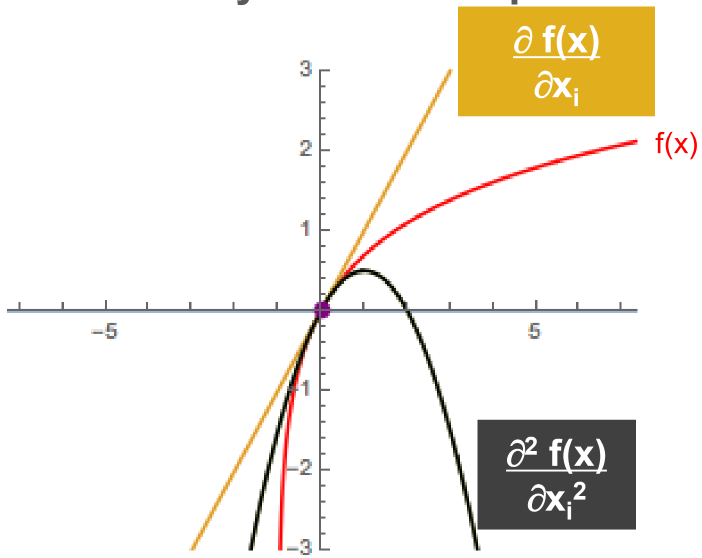
Learning objectives

Combining uncertainty: Approach 1 [1]

Linear Error Propagation (LEP)



First order Taylor series expansion



Assumptions:

- **Small std. Deviation (~30% from the mean)**
- **Symmetric (not skewed)**
- **Normal distribution**
- Uncorrelated variables
- Multiplication or addition

Equations 3.1 and 3.2

Combining uncertainty: Approach 1 [2]

Linear Error Propagation (LEP) **Theory behind it – Don’t panic! Bear with me :)**

Taylor 1st order: $f = f(a;b) + \frac{\partial f}{\partial a}\delta a + \frac{\partial f}{\partial b}\delta b + \cdots$ ➡ $f - f(a;b) = \frac{\partial f}{\partial a}\delta a + \frac{\partial f}{\partial b}\delta b$ ➡ $\delta f = \frac{\partial f}{\partial a}\delta a + \frac{\partial f}{\partial b}\delta b$

Variance of a summation:

$$\sigma_f^2 = \left|\frac{\partial f}{\partial a}\right|^2 \sigma_a^2 + \left|\frac{\partial f}{\partial b}\right|^2 \sigma_b^2 + 2\,\frac{\partial f}{\partial a}\,\frac{\partial f}{\partial b}\,\sigma_{ab}$$

Case f = a x b:

$$\sigma_f^2 = b^2\,\sigma_a^2 + a^2\,\sigma_b^2 + 2\,a\,b\,\sigma_{ab}$$

Or, in relative terms

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{b\,\sigma_a}{a\,b}\right)^2 + \left(\frac{a\,\sigma_b}{a\,b}\right)^2 + \frac{2\,a\,b\,\sigma_{ab}}{(a\,b)^2}$$

$u_f^2 = u_a^2 + u_b^2$ ➡ $u_f = \sqrt{u_a^2 + u_b^2}$ ➡ $U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$

Equations 3.1

Assumptions:

1) Small std. Deviation (~30%)

2) Symmetric (not skewed)

Normal distribution

3) Uncorrelated variables

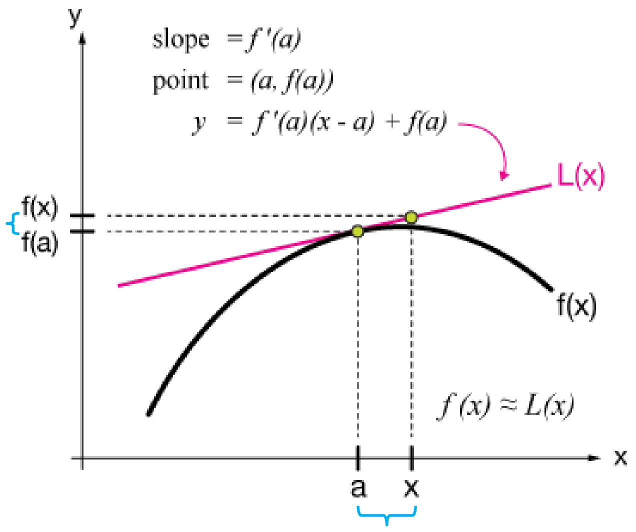
4) Multiplication or addition

Combining uncertainty: Approach 1 [3]

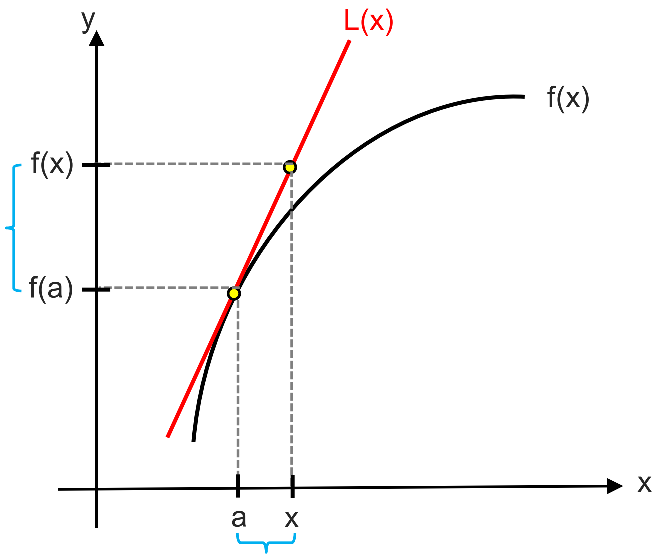
Linear Error Propagation (LEP)

$$\sigma_f^2 = \left|\frac{\partial f}{\partial a}\right|^2 \sigma_a^2 + \left|\frac{\partial f}{\partial b}\right|^2 \sigma_b^2$$

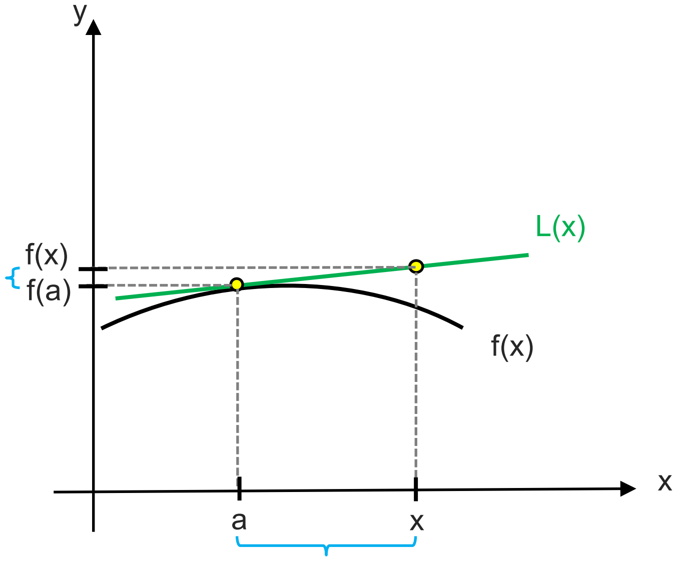
Small impact from the input uncertainty



Large impact from the input uncertainty



Small impact from large input uncertainty



The most important is how the uncertainty of the data affects the result. **!**

Combining uncertainty: Approach 1 [4]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1

COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

U_i: relative

E1 + E2 + … + En

EQUATION 3.2

COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

x_i: CO_2e

Combining uncertainty: Approach 1 [4]

Linear Error Propagation (LEP)

Practical exercises

Combining uncertainty: Approach 1 [5]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1

COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

U_i: relative

Example: CO_2 emissions due to fuel consumption

$E = C \times EF$

Gasoil consumption = 18 710 GJ

$E = 18\,710\,[GJ] \times 0.0741\,[\frac{t}{GJ}] = 1\,386\,tCO_2$

Uncertainty in activity data: 10%

Uncertainty in emission factor: 2%

(kg of greenhouse gas per TJ on a Net Calorific Basis)

Fuel	CO_2		
	Default Emission Factor	Lower	Upper
Gas/Diesel Oil	74 100	72 600	74 800

[72 600 - 74 100] / 74 100	[74 800 - 74 100] / 74 100
2%	**1%**

Uncertainty in emissions = $\sqrt{10^2 + 2^2} = 10.2\% \sim 10\%$

U = $\sqrt{0.1^2 + 0.02^2} = 0.102 = 10.2\% \sim 10\%$

Combining uncertainty: Approach 1 [6]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1
COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

U_i: relative

Example: CO_2 emissions due to fuel consumption

$E = C \times \delta \times NCV \times EF$

C: Gasoil consumption = 500 m^3

δ: Density = 0.87 t/m^3

$E = 500 \times 0.87 \times 43.0 \times 0.0741 = 1\ 386\ tCO_2$

Uncertainty in activity data: 5%

Uncertainty in density: 5%

Uncertainty in emission factor: 2%

Uncertainty in NCV: 4%

(kg of greenhouse gas per TJ on a Net Calorific Basis)

Fuel	CO_2		
	Default Emission Factor	Lower	Upper
Gas/Diesel Oil	74 100	72 600	74 800
		2%	**1%**

TABLE 1.2
DEFAULT NET CALORIFIC VALUES (NCVS) AND LOWER AND UPPER LIMITS OF THE 95% CONFIDENCE INTERVALS [1]

Fuel type English description	Net calorific value (TJ/Gg)	Lower	Upper
Gas/Diesel Oil	43.0	41.4	43.3
		[41.4 – 43.0] / 43.0	[41.4 – 43.0] / 43.0
		4%	**1%**

Uncertainty in emissions= $\sqrt{5^2 + 5^2 + 4^2 + 2^2} = 8\%$

Combining uncertainty: Approach 1 [7]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1
COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

***U_i*: relative**

$$\sigma_f^2 = \left|\frac{\partial f}{\partial a}\right|^2 \sigma_a^2 + \left|\frac{\partial f}{\partial b}\right|^2 \sigma_b^2$$

U = $\sqrt{4u_{DE}^2 + u_{EF}^2}$

Examples: enteric fermentation, transportation, waste treatment

$E = DE^2 \times EF$ ➡ $E = DE \times DE \times EF$ ➡ **U =** $\sqrt{u_{DE}^2 + u_{DE}^2 + u_{EF}^2}$! independent?

$E = \frac{D \times EF}{\eta}$ ➡ $E = D \times EF \times SC$ ➡ **U =** $\sqrt{u_D^2 + u_{EF}^2 + u_{SC}^2}$

$E = DDOC \times e^{-k}$ ➡ $\sigma_f^2 = \left|\frac{\partial f}{\partial a}\right|^2 \sigma_a^2 + \left|\frac{\partial f}{\partial b}\right|^2 \sigma_b^2$ ➡ **U =** $\sqrt{u_{DDOC}^2 + (u_k \times k)^2}$

WOW! It was useful! :)

Combining uncertainty: Approach 1 [8]

Linear Error Propagation (LEP)

E1 + E2 + … + En

EQUATION 3.2

COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

x_i: CO_2e

Example: CO_2 emissions due to vent and flare

$$E = E_{vent} + E_{flare}$$

E_{vent} = 7 310 tCO_2e

E_{flare} = 5 282 tCO_2e

$$E = 7\,310 + 5\,282 = 12\,592\ tCO_2e$$

Uncertainty in vent emissions: 40% (± 2 924)

Uncertainty in flare emissions: 10% (± 528)

Uncertainty in emissions = $\frac{\sqrt{(0.4 \times 7\,310)^2 + (0.1 \times 5\,282)^2}}{|12\,592|}$

Uncertainty in emissions = $\frac{\sqrt{(2\,924)^2 + (528)^2}}{|12\,592|}$ **= 24%**

Combining uncertainty: Approach 1 [9]

Linear Error Propagation (LEP)

E1 + E2 + … + En

EQUATION 3.2

COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

x_i: CO_2e

Example: adding multiple emission sources

$$E = E_1 + E_2 + E_3 + E_4 + E_5$$

$$U_E = \frac{\sqrt{(0.4\times200)^2+(0.3\times500)^2+(0.1\times300)^2+(0.8\times100)^2+(0.2\times800)^2}}{|1\ 900|} = 13\%$$

E_1 = 200 tCO_2e ± 40%
E_2 = 500 tCO_2e ± 30%
E_3 = 300 tCO_2e ± 10%
E_4 = 100 tCO_2e ± 80%
E_5 = 800 tCO_2e ± 20%

E = 1 900 tCO_2e

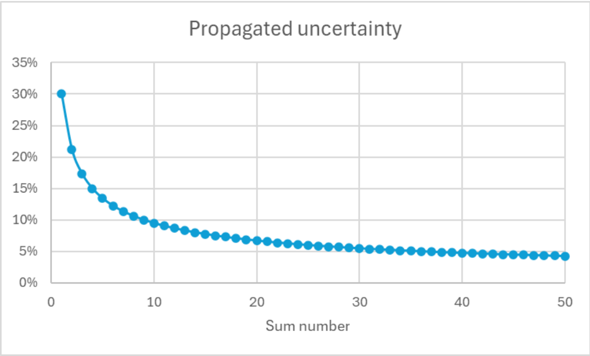
Contribution to uncertainty

$$\frac{(U_i \times E_i)^2}{\sum(U_i \times E_i)^2}$$

=> 10%
=> 36%
=> 1%
=> 10%
=> 41%

Contribution to variance

$$\frac{(U_i \times E_i)^2}{(\sum E_i)^2}$$



Addition reduces overall uncertainty

Combining uncertainty: Approach 1 [10]

Linear Error Propagation (LEP)

E1 + E2 + … + En

EQUATION 3.2

COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

x_i: CO_2e

Example: subtracting

$E = E_1 - E_2$

E_1 = 500 tCO_2e ± 30%
E_2 = 100 tCO_2e ± 20%

$$U_E = \frac{\sqrt{(0.3\times500)^2+(0.2\times100)^2}}{|400|} = 38\%$$

Subtraction increases overall uncertainty!

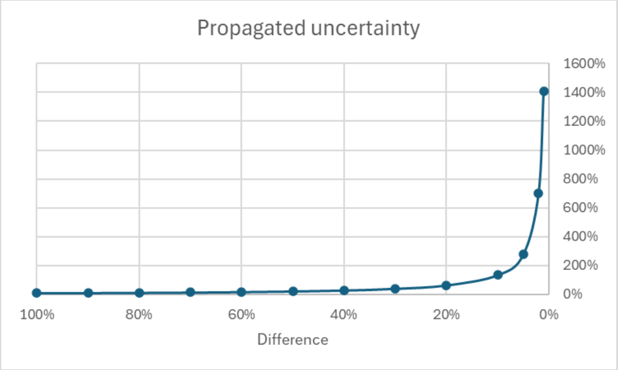
E = 400 tCO_2e

IF
E_1 = 500 tCO_2e ± 30%
E_2 = 400 tCO_2e ± 20%

$$U_E = \frac{\sqrt{(0.3\times500)^2+(0.2\times400)^2}}{|100|} = 170\%$$

E = 100 tCO_2e

Careful if similar values!



Combining uncertainty: Approach 1 [11]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1
COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

***U_i*: relative**

E1 + E2 + … + En

EQUATION 3.2
COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

***x_i*: CO_2e**

Example: combining Eq 3.1 and 3.2

Source	Emission (tCO_2e)	Uncertainty in AD U_{AD}	Uncertainty in EF U_{EF}	Combined uncertainty Ui $\sqrt{(U_{AD}^2+U_{EF}^2)}$	Contribution to variance $\frac{(Ui \times Ei)^2}{(\sum Ei)^2}$
a	100	3%	5%	5.8%	0.14%
b	5	3%	75%	75.1%	0.06%
c	28	3%	45%	45.1%	0.65%
d	3.2	3%	100%	100.0%	0.04%
e	21	3%	10%	10.4%	0.02%
	157.2				**0.90%**
					9%

Combining uncertainty: Approach 1 [12]

Linear Error Propagation (LEP)

Enter Emissions Data

Data Calculated using simple equations

TABLE 3.2
APPROACH 1 UNCERTAINTY CALCULATION

A	B	C	D	E	F	G	H	I	J	K	L	M
IPCC category	Gas	Base year emissions or removals	Year *t* emissions or removals	Activity data uncertainty	Emission factor / estimation parameter uncertainty	Combined uncertainty	Contribution to Variance by Category in Year *t*	Type A sensitivity	Type B sensitivity	Uncertainty in trend in national emissions introduced by emission factor / estimation parameter uncertainty	Uncertainty in trend in national emissions introduced by activity data uncertainty	Uncertainty introduced into the trend in total national emissions
		Input data	Input data	Input data Note A	Input data Note A	$\sqrt{E^2+F^2}$	$\frac{(G \bullet D)^2}{(\sum D)^2}$	Note B	$\left\lvert\frac{D}{\sum C}\right\rvert$	$I \bullet F$ Note C	$J \bullet E \bullet \sqrt{2}$ Note D	K^2+L^2
		Gg CO_2 equivalent	Gg CO_2 equivalent	%	%	%		%	%	%	%	%
E.g., 1.A.1. Energy Industries Fuel 1	CO_2											
E.g., 1.A.1. Energy Industries Fuel 2	CO_2											
Etc...	…											
Total		$\sum C$	$\sum D$				$\sum H$					$\sum M$
					Percentage uncertainty in total inventory:		$\sqrt{\sum H}$				Trend uncertainty:	$\sqrt{\sum M}$

Enter Uncertainties

AD uncertainties based on source of data

EF uncertainties based on data used

List of source/sinks

Approach 1 uncertainty calculation												
A	B	C	D	E	F	G	H	I	J	K	L	M
IPCC category	Gas	Base year emissions or removals	Year *t* emissions or removals	Activity data uncertainty	Emission factor / estimation parameter uncertainty	Combined uncertainty	Contribution to Variance by Category in	Type A sensitivity	Type B sensitivity	Uncertainty in trend in national	Uncertainty in trend in national	Uncertainty introduced into the trend in total national emissions
		Input data	Input data	Input data	Input data	$\sqrt{E^2+F^2}$	$\frac{(G \bullet D)^2}{(\sum D)^2}$	Note B	$\left\lvert\frac{D}{\sum C}\right\rvert$	$I \bullet F$	$J \bullet E \bullet \sqrt{2}$	K^2+L^2
		Gg CO_2 equivalent	Gg CO_2 equivalent	%	%	%		%	%	%	%	%
1.A.1. Energy Industries	CH4	35.5346662	32.9951217	5	25	25.50	0.0	3.20506E-05	0.00010495	0.000801264	0.000742109	1.19275E-06
1.A.2. Manufacturing Industries and Construction	CH4	57.0302899	51.8776096	5	25	25.50	0.0	4.80131E-05	0.000165011	0.001200328	0.001166804	2.80222E-06
1.A.3. Transport	CH4	81.7067834	37.1466612	5	25	25.50	0.0	-4.94664E-05	0.000118155	-0.00123666	0.000835483	2.22736E-06
1.A.4. Other Sectors	CH4	1041.24025	428.554682	5	25	25.50	0.0	-0.000772946	0.001363136	-0.019323647	0.009638828	0.00046631
1.A.5. Other	CH4	330.338228	97.5658895	5	25	25.50	0.0	-0.000367351	0.000310335	-0.009183772	0.002194401	8.91571E-05
1.B.1. Solid Fuels	CH4	24867.6834	12364.38	10	25	26.93	2.7	-0.011678579	0.039328314	-0.291964463	0.556186352	0.394586505
1.B.2. Oil and Natural Gas	CH4	12570.348	4022.34735	10	25	26.93	0.3	-0.012988732	0.012794183	-0.324718297	0.180937071	0.138180196
2.B. Chemical Industry .	CH4	40.53	37.5018	10	25	26.93	0.0	3.61373E-05	0.000119285	0.000903433	0.001686942	3.66196E-06
4.A. Enteric Fermentation.	CH4	14054.9863	7346.85	15	30	33.54	1.5	-0.005462727	0.023368679	-0.163881819	0.495724537	0.272600067
4.B. Manure Management.	CH4	1903.28061	1199.63088	15	30	33.54	0.0	-8.88245E-05	0.003815756	-0.002664735	0.080944413	0.006559099
4.C. Rice Cultivation.	CH4	522.9	338.94	10	30	31.62	0.0	5.3609E-06	0.001078092	0.000160827	0.015246523	0.000232482
4.F. Field Burning of Agricultural Residues.	CH4	64.3314	[illegible]	[illegible]	[illegible]	[illegible]	0.0	-1.24107E-05	0.000119565	-0.000372321	0.003381819	1.15753E-05
6.A. Solid Waste Disposal on Land.	CH4	1959.72	[illegible]	[illegible]	[illegible]	[illegible]	0.4	0.00787088	0.011891742	0.236126385	0.252261939	0.119391756
6.B. Wastewater Handling.	CH4	787.08	[illegible]	[illegible]	[illegible]	[illegible]	0.0	0.000761896	0.002376612	0.022856865	0.050415547	0.003064164
1.A.1. Energy Industries	CO2 (1)	102607.31	[illegible]	[illegible]	[illegible]	[illegible]	11.2	0.094441853	0.305249301	0.472209267	2.158438506	4.881838378
1.A.2. Manufacturing Industries and Construction	CO2 (1)	33991.06	[illegible]	[illegible]	5	7.07	1.1	0.02618491	0.095945987	0.130924551	0.678440577	0.477422855
1.A.3. Transport	CO2 (1)	23987.07	[illegible]406.48	5	5	7.07	0.1	-0.022453294	0.026739124	-0.11226647	0.189074157	0.048352797
1.A.4. Other Sectors	CO2 (1)	4[illegible]532.52	11784.04	5	5	7.07	0.2	-0.053800014	0.037482383	-0.269000072	0.265040472	0.14260749
1.A.5. Other	CO2 (1)	8370.16	4124.19	5	5	7.07	0.0	-0.004052209	0.013118122	-0.020261045	0.092759127	0.009014766
1.B.2. Oil and Natural Gas	CO2 (1)	3408.21	5171.49583	10	15	18.03	0.2	0.009456387	0.016449366	0.141845811	0.232629165	0.074236563
2.A. Mineral Products.	CO2 (1)	5744.63	2507.20146	10	15	18.03	0.0	-0.003809586	0.007974844	-0.057143788	0.112781331	0.015985041
2.B. Chemical Industry .	CO2 (1)	1355.56	171.93456	10	15	18.03	0.0	-0.002233954	0.000546885	-0.033509311	0.007734125	0.001182691
2.C. Metal Production.	CO2 (1)	12932.6799	10507.4715	10	15	18.03	0.9	0.006887639	0.033421905	0.103314586	0.47265712	0.234078657
5.A. Changes in Forest and Other Woody Biomass Stocks.	CO2 (1)	97.19		50	80	94.34	0.0	-0.000199385	0	-0.015950798	0	0.000254428
5.A. Changes in Forest and Other Woody Biomass Stocks Removals	CO2 (1)	-7810.79	-7721.7341	50	80	94.34	12.9	-0.008539362	0.024561101	-0.683148991	1.736732102	3.482930938
5.B. Forest and Grassland Conversion.	CO2 (1)	6.26	280.43888	25	75	79.06	0.0	0.00087917	0.000892013	0.065937785	0.031537424	0.005342401
1.A.1. Energy Industries	N2O	388.516902	328.741673	5	50	50.25	0.0	0.000248607	0.001045653	0.012430334	0.007393886	0.000209183
1.A.2. Manufacturing Industries and Construction	N2O	112.709781	114.844426	5	50	50.25	0.0	0.000134069	0.000365294	0.006703468	0.002583021	5.16085E-05
1.A.3. Transport	N2O	57.3319301	21.6195922	5	50	50.25	0.0	-4.88495E-05	6.87671E-05	-0.002442474	0.000486257	6.20212E-06
1.A.4. Other Sectors	N2O	194.497577	46.1816455	5	50	50.25	0.0	-0.000252117	0.000146893	-0.01260587	0.001038693	0.000159987
1.A.5. Other	N2O	27.4386549	13.5195061	5	50	50.25	0.0	-1.3288E-05	4.30025E-05	-0.000664398	0.000304074	5.33886E-07
4.B. Manure Management.	N2O	375.1	198.4	15	30	33.54	0.0	-0.000138451	0.000631066	-0.004153541	0.013386927	0.000196462
4.D. Agricultural Soils(2).	N2O	25217.694	9798.17	20	30	36.06	3.0	-0.020551916	0.031165777	-0.616557485	0.881501284	1.157187646
4.F. Field Burning of Agricultural Residues.	N2O	24.304	21.297	20	30	36.06	0.0	1.78812E-05	6.7741E-05	0.000536437	0.001916004	3.95884E-06
6.B. Wastewater Handling.	N2O	452.6	384.4	15	30	33.54	0.0	0.000294175	0.00122269	0.008825264	0.025937172	0.000750622
Keep Blank!	…									0		
Total		314388.7626	202771.1719			$\sum H$	34.6				$\sum M$	11.4670044
					Percentage uncertainty in total inventory:		5.880740472				Trend uncertainty:	3.386296561

Combining uncertainty: Approach 1 [14]

Linear Error Propagation (LEP)

AD x EF

EQUATION 3.1

COMBINING UNCERTAINTIES – APPROACH 1 – MULTIPLICATION

$$U_{total} = \sqrt{U_1^2 + U_2^2 + \ldots + U_n^2}$$

U_i: relative

E1 + E2 + … + En

EQUATION 3.2

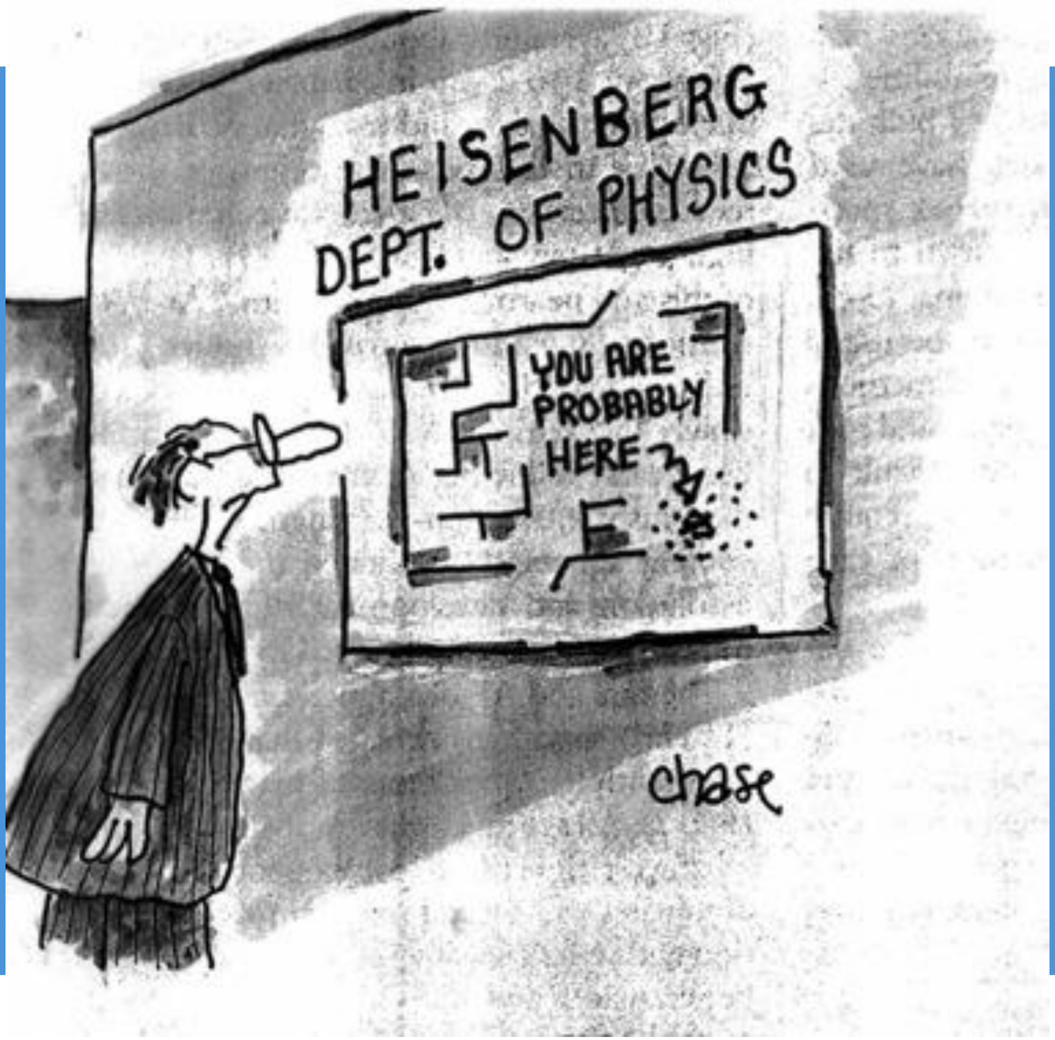
COMBINING UNCERTAINTIES – APPROACH 1 – ADDITION AND SUBTRACTION

$$U_{total} = \frac{\sqrt{(U_1 \bullet x_1)^2 + (U_2 \bullet x_2)^2 + \ldots + (U_n \bullet x_n)^2}}{|x_1 + x_2 + \ldots + x_n|}$$

x_i: CO_2e

Bonus track!: one way of dealing with asymmetric uncertainties

Source	Emission (tCO_2e)	Uncertainty in AD U_{AD}	Uncertainty in EF (-) U_{EF}	Uncertainty in EF (+)	Combined uncertainty Ui (-) $\sqrt{(U_{AD}^2+U_{EF}^2)}$	Combined uncertainty Ui (+)	Contribution to variance (-) $\frac{(Ui \times Ei)^2}{(\Sigma Ei)^2}$	Contribution to variance (+)
a	100	3%	-5%	5%	6%	6%	0.1%	0.1%
b	5	3%	-50%	100%	50%	100%	0.0%	0.1%
c	28	3%	-30%	60%	30%	60%	0.3%	1.1%
d	3.2	3%	-100%	900%	100%	900%	0.0%	3.4%
e	21	3%	-10%	10%	10%	10%	0.0%	0.0%
	157.2						**0.5%**	**4.8%**
							-7%	**22%**



End of day 2
Thank you!

Diego M. Ezcurra

TUESDAY

2. Uncertainty associated with input data

- Overview of uncertainty analysis in national GHG inventories
- Key concepts and causes of uncertainties associated with input data

WEDNESDAY

3. Approach 1 - Propagation of errors for uncertainty analysis

- Key concepts of the method to combine uncertainties
- Application to different cases

THURSDAY

4. Approach 2 - Monte Carlo simulation for uncertainty analysis

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- Selection of the probabilistic functions
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Practical exercises and typical challenges

Energy, IPPU, Agriculture, LULUCF and Waste

4

Session 4. Methods to combine uncertainties: Approach 2 Monte Carlo simulation

By the end of this session, you will:

1. Understand the basics of the simulation
2. Learn how to select probability density functions
3. Identify typical problems and avoid misinterpretation
4. Understand how to deal with correlation

Learning objectives

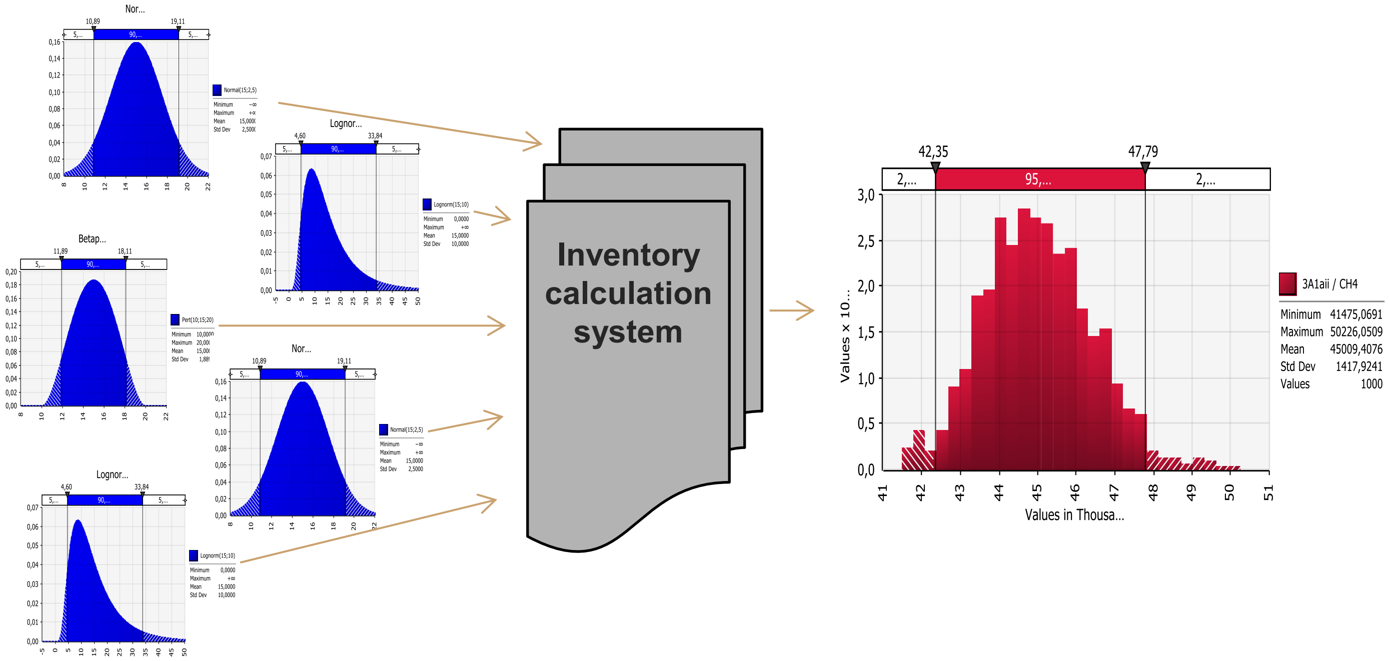
Combining uncertainty: Approach 2 [1]

Monte Carlo Simulation (MCS)

Numerical simulation method, nondeterministic, which simulates the behavior of a random static system where input parameters are defined by a known Probability Density Function.

Combining uncertainty: Approach 2 [2]

Monte Carlo Simulation (MCS)



Applicable even if:

- **Large std. dev.**
- **Skewed distributions**
- **Correlated variables**
- **Complex equations**

Combining uncertainty: Approach 2 [3]

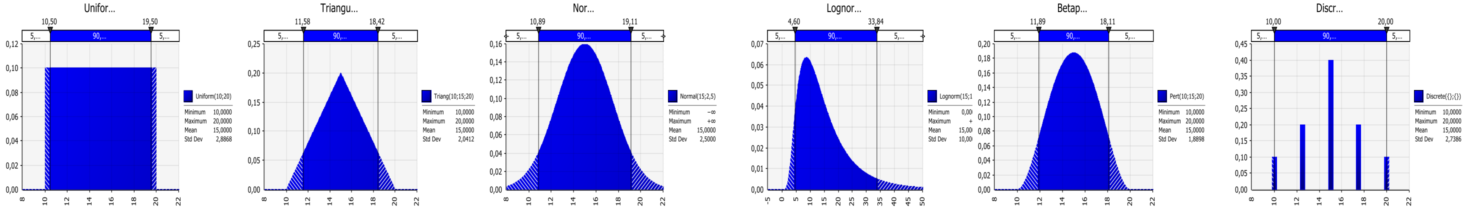
Monte Carlo Simulation (MCS)

Steps

1. Build the calculation model in which uncertainty needs to be evaluated

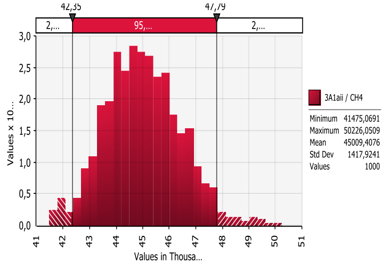
2. Identify relevant key variables → Sensitivity

3. Establish the **Probability Distribution Functions** for each of the inputs identified and obtain the parameters to define them



4. Run the simulation

5. Obtain the PDF of the result and determine uncertainty as the 95% CI.



Mean = 100

Standard deviation = 15

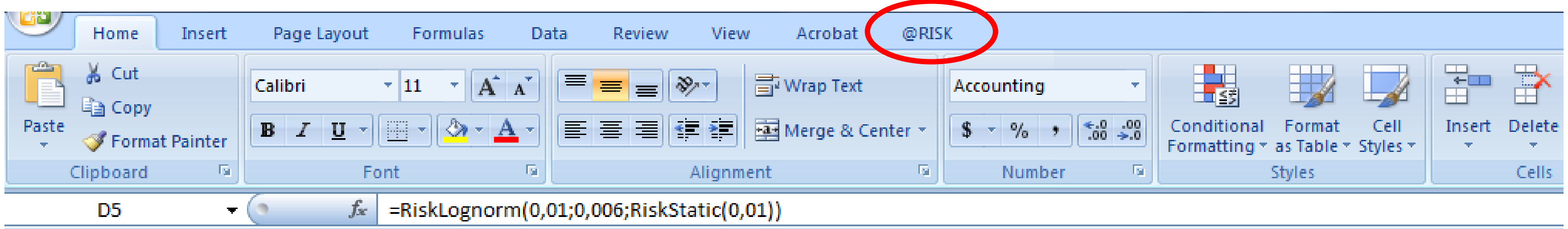
CI = [70 ; 130]

Uncertainty = 30%

Combining uncertainty: Approach 2 [4]

Monte Carlo Simulation (MCS)

Example



Direct N2O emissions from managed soils

Página 11.12 - Guía IPCC 2006 Vol. 4

Factor	Unit	Dafault	Range
EF_1	kg N2O–N / kg N	0,01	0,003 - 0,03
$EF3_{PRP}$	Kg N2O-N / Kg N	0,02	0,007 - 0,06

Indirect N2O emissions from soils (volatilization and leachate

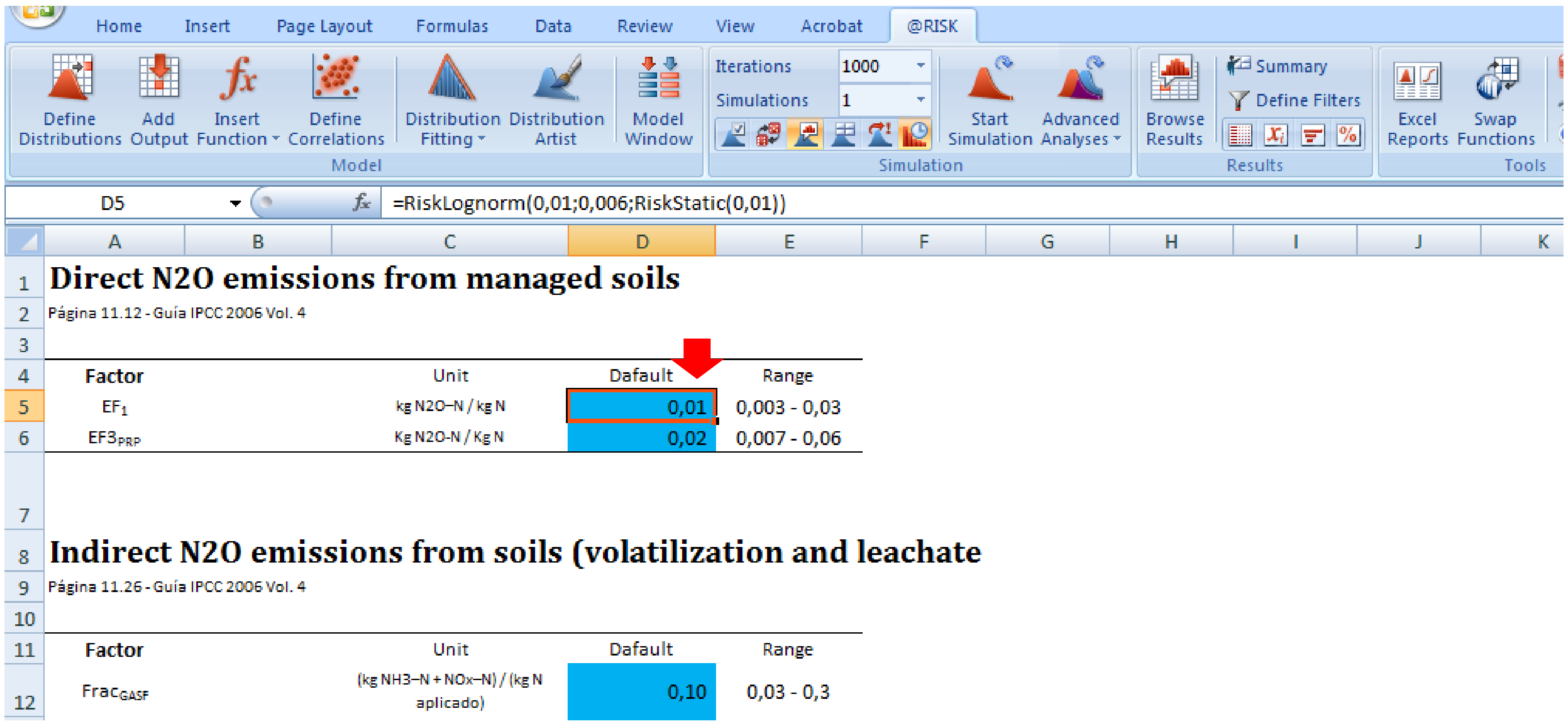
Página 11.26 - Guía IPCC 2006 Vol. 4

Factor	Unit	Dafault	Range
$Frac_{GASF}$	(kg NH3–N + NOx–N) / (kg N aplicado)	0,10	0,03 - 0,3

Combining uncertainty: Approach 2 [5]

Monte Carlo Simulation (MCS)

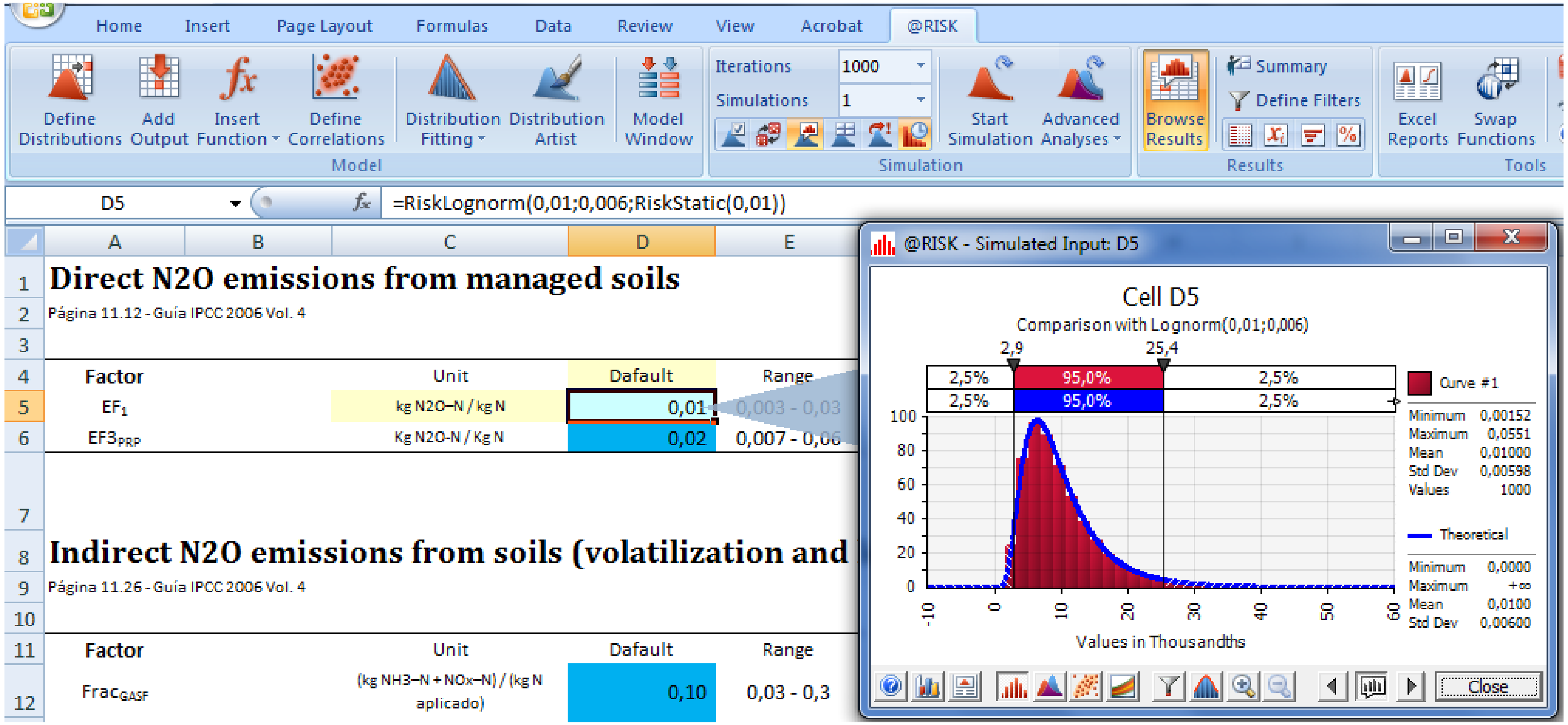
Example



Combining uncertainty: Approach 2 [6]

Monte Carlo Simulation (MCS)

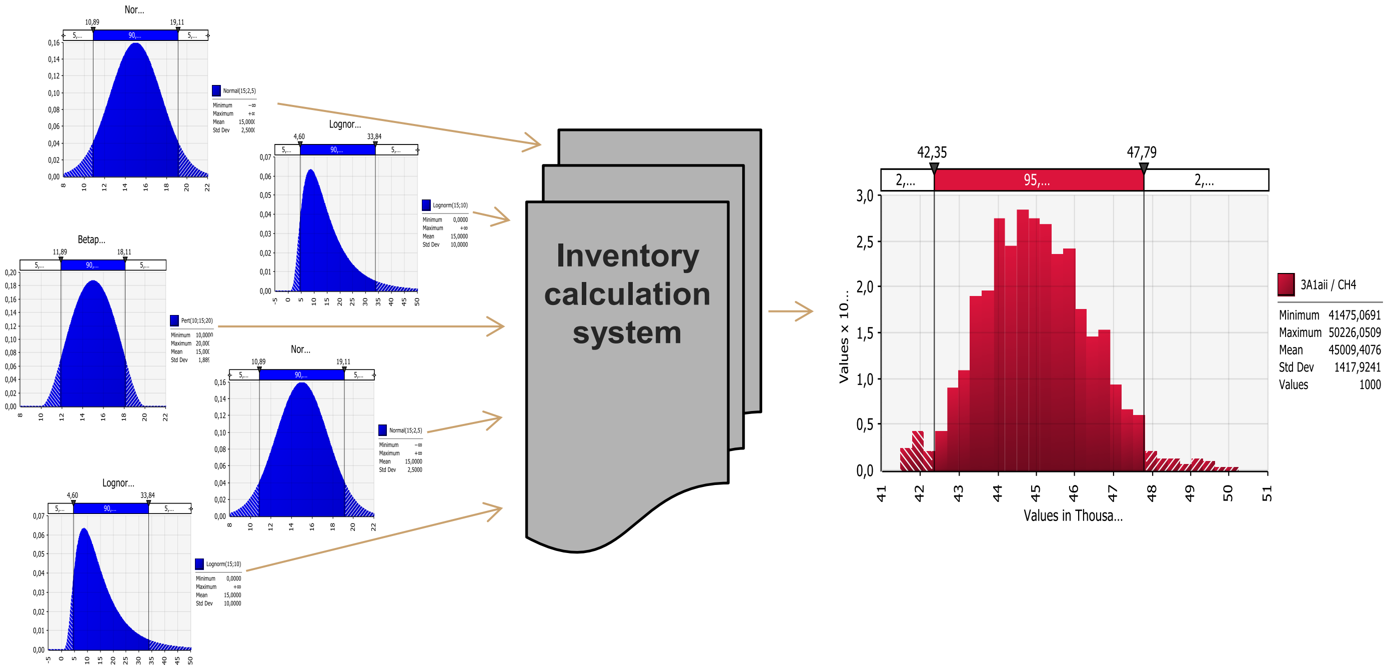
Example



Combining uncertainty: Approach 2 [7]

Monte Carlo Simulation (MCS)

Example



Combining uncertainty: Approach 2 [8]

Monte Carlo Simulation (MCS)

GOOD PRACTICE GUIDANCE FOR SELECTING PROBABILITY DENSITY FUNCTIONS

V1_Ch3. 3.2.2.4

2006 IPCC Guidelines

Recommendations for different cases and commonly applied criteria to follow

Combining uncertainty: Approach 2 [9]

Monte Carlo Simulation (MCS)

Domain (+, -, ∞)

Range (narrow or broad)

Shape (symmetry)

Underlying process (+, x)

Others

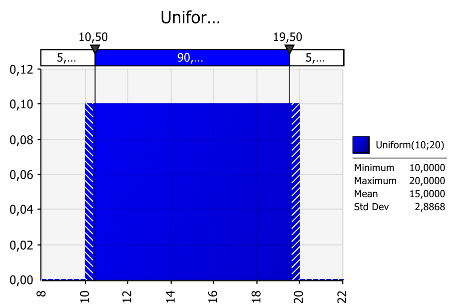
Combining uncertainty: Approach 2 [10]

Selecting PDF

Most used PDF

Combining uncertainty: Approach 2 [11]

Selecting PDF



Uniform

All values with same probability

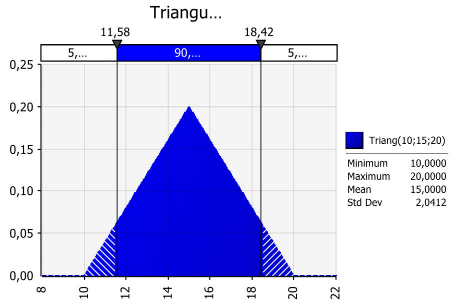
Parameters: Uniform **(min, max)**

Application to inventories: large uncertainty and lack of information

Combining uncertainty: Approach 2 [12]

Selecting PDF

Triangular



Intuitive and flexible.

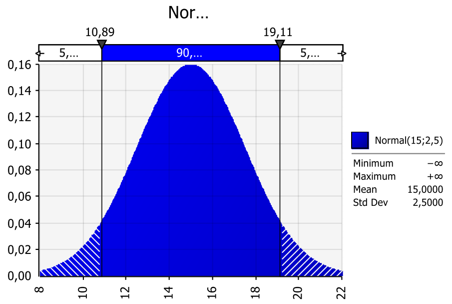
Parameters: Triang (***min, mean, max***)

Application to inventories: expert judgment, knowledge from experience. Limited information.

Combining uncertainty: Approach 2 [13]

Selecting PDF

Normal



Distribution around a most likely central value.

Parameters: Normal (***mean [μ], std. Dev. [σ]***)

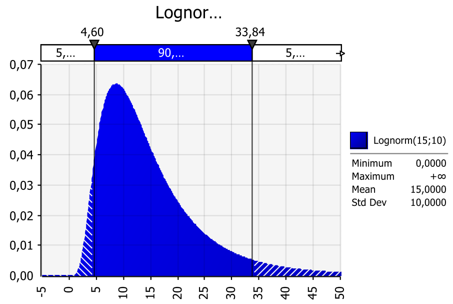
Application to inventories: most of situations (central limit). Additive processes.

The interval +/- 2*σ (1.96) accounts for approx. 95% of the values.
σ may be estimated as: (max - mean) / 2

Combining uncertainty: Approach 2 [14]

Selecting PDF

Log-normal



The natural logarithm of the variable adjusts to a Normal distribution

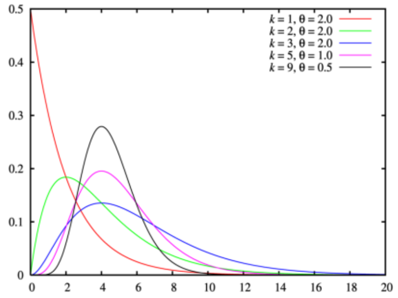
Parameters: Lognormal (***mean [μ], std. Dev. [σ]***)

Application to inventories: Generally good representation for skewed non-negative values (emission factors for N_2O). Multiplicative processes.

Combining uncertainty: Approach 2 [15]

Selecting PDF

Gamma



Similar to lognormal but with not so heavy tails

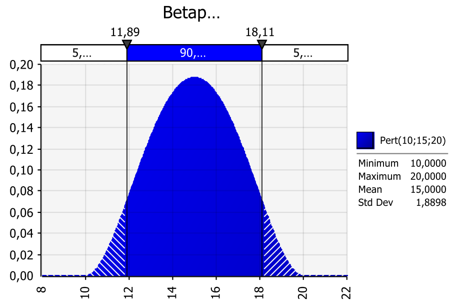
Parameters: Gamma (***shape [k], scale [θ]***)

Application to inventories: Good representation for skewed values. Very flexible depending on its parameters. Mean value: k.θ

Combining uncertainty: Approach 2 [16]

Selecting PDF

Beta pert



Version of the Beta using PERT (Program Evaluation and Review Technique).

Parameters: BetaPert (***min, mean, max***)

Application to inventories: similar to triangular but with lower standard deviation.

Combining uncertainty: Approach 2 [17]

Selecting PDF - Good practice guidance

- The minimum number of probability functions are used
- These probability functions are well known and well based (theoretical or empirical)
- Where empirical data are available, the first choice should be to assume a normal distribution
- If the variable must be non-negative and a normal distribution is assumed, the standard deviation should not exceed 30%
- Truncation of the negative tail of the normal distribution should be avoided (use instead lognormal, Weibull, or Gamma).

Combining uncertainty: Approach 2 [18]

Selecting PDF - Good practice guidance

- Where expert judgment is used, the distribution function adopted might be normal or lognormal, supplemented by uniform or triangular distributions
- If only the interval is known (upper and a lower value), assume that the probability density function is uniform and that the range corresponds to the 95% confidence interval
- If the distribution observed based on data does not seem correct, the data may be the problem (not representative, not random, small sample size, different timing, etc.)

Combining uncertainty: Approach 2 [19]

Selecting PDF - Good practice guidance

- When selecting the PDF from Goodness-of-Fit test, several functions will fit the data satisfactorily within a given probability limit
- Different functions can have radically different distributions at the extremes (few or no data to constrain them), and the choice of one function over another can systematically change the outcome of an uncertainty analysis.

"it must be knowledge of the underlying physical processes that governs the choice of a probability function"

Combining uncertainty: Approach 2 [20]

DEPENDENCE AND CORRELATION AMONG INPUTS

Dependence / Correlation

Relationship between two variables that make them not independent

One variable is determined (partially or totally) by another one

Combining uncertainty: Approach 2 [21]

DEPENDENCE AND CORRELATION AMONG INPUTS

Dependence / Correlation

Negative

Reduces uncertainty range when included

If not included: overestimation of uncertainty

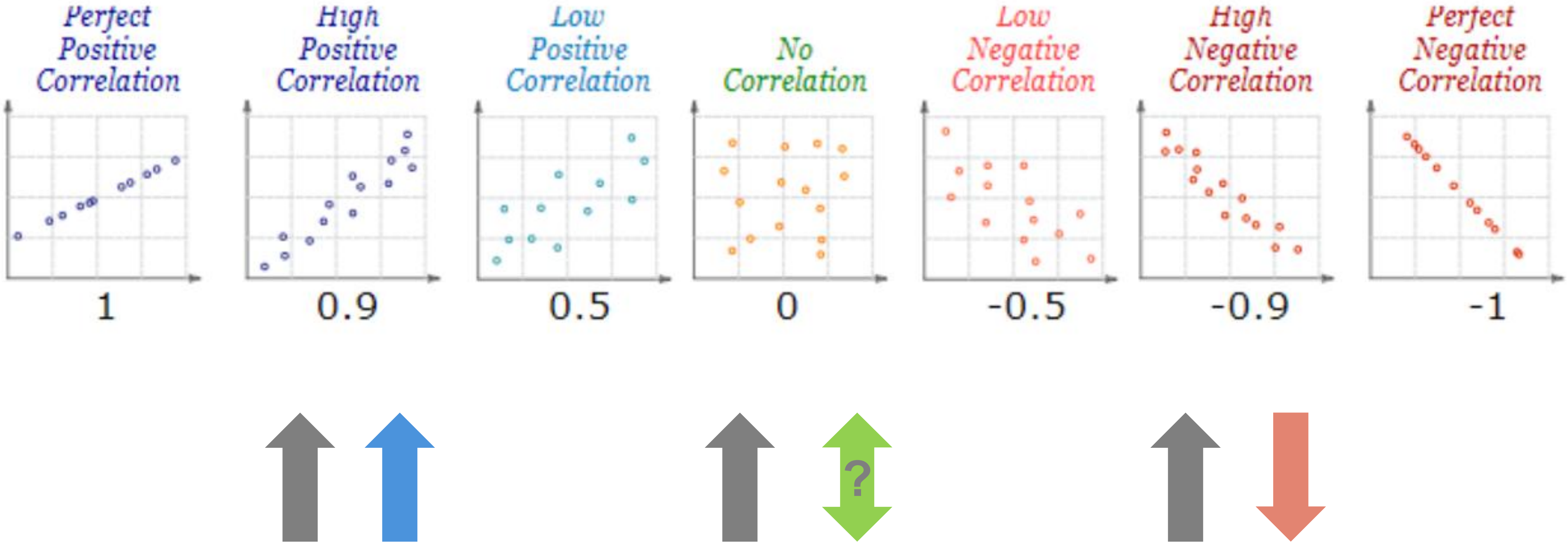
Positive

Increases uncertainty range when included

If not included: underestimation of uncertainty

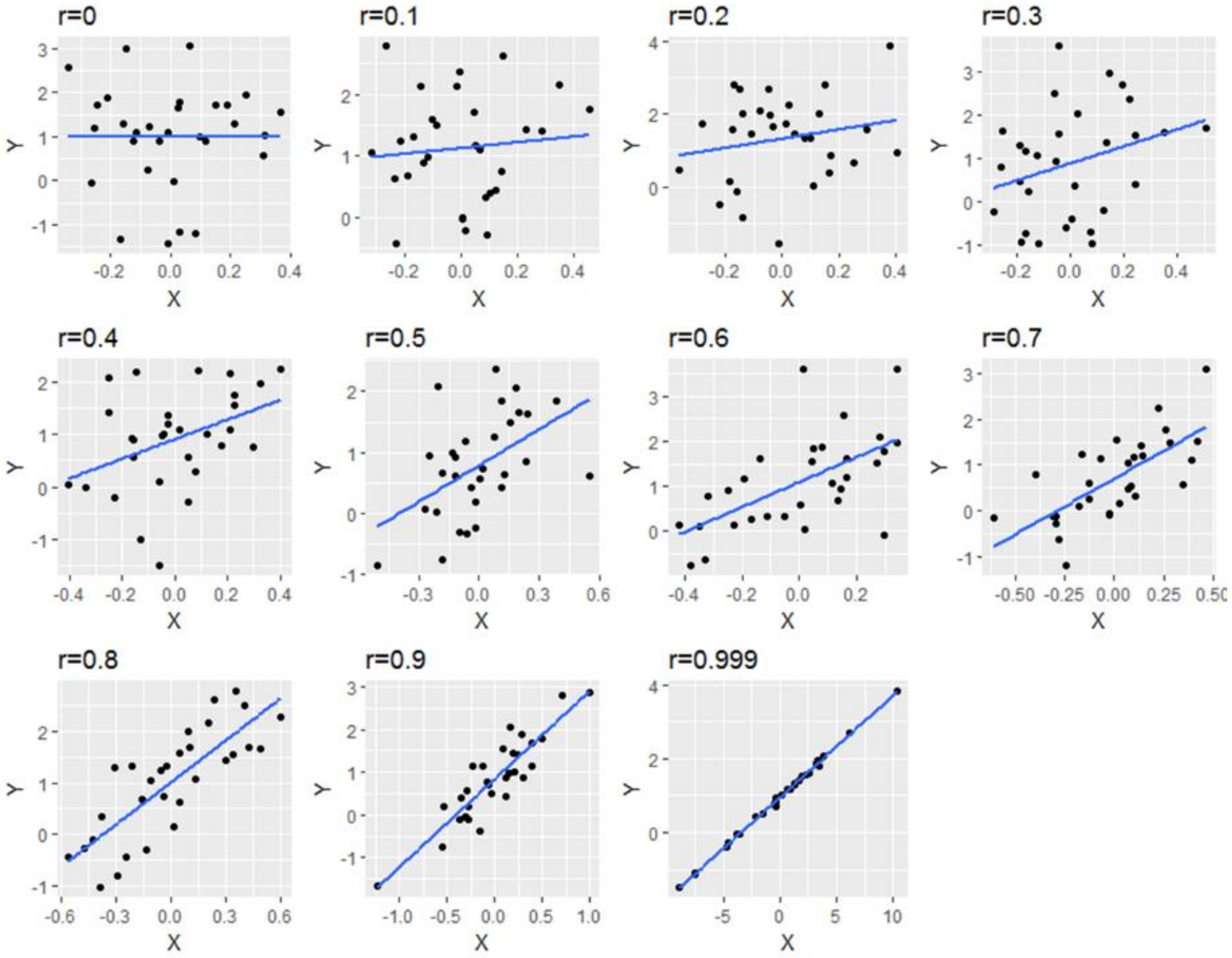
Combining uncertainty: Approach 2 [22]

DEPENDENCE AND CORRELATION AMONG INPUTS



Combining uncertainty: Approach 2 [23]

DEPENDENCE AND CORRELATION AMONG INPUTS



Correlation degree

< 0.2	Very low
0.2 – 0.4	Low
0.4 – 0.6	Moderate
0.6 – 0.8	High
> 0.8	Very high

* Indicative ranges

Combining uncertainty: Approach 2 [24]

DEPENDENCE AND CORRELATION AMONG INPUTS

$$E = \overbrace{FC \times NCV}^{FC\ (GJ)} \times EF$$

Are FC (GJ) and EF independent?

FC: Fuel consumption (tonnes)
NCV: Net calorific value (GJ/t)
EF: Emission factor (tCO_2/GJ)

$$EF = \frac{\%C}{NCV} x \frac{44}{12}$$

Are NCV and EF independent?

Combining uncertainty: Approach 2 [25]

DEPENDENCE AND CORRELATION AMONG INPUTS

Dependencies / Correlations

Are always important to uncertainty assessment?

Degree: strong or weak correlation (i.e. 0.8 or 0.2)

Sensitivity: impact to the overall uncertainty

Combining uncertainty: Approach 2 [26]

DEPENDENCE AND CORRELATION AMONG INPUTS

Exists between 2 variables to which uncertainty is NOT sensitive to and dependency is strong	Exists between 2 variables to which uncertainty is sensitive to and dependency is strong
Exists between 2 variables to which uncertainty is NOT sensitive to and dependency is weak	Exists between 2 variables to which uncertainty is sensitive to and dependency is weak

Combining uncertainty: Approach 2 [27]

DEPENDENCE AND CORRELATION AMONG INPUTS

Dependence / Correlation

Strategies

- Define the model so that the inputs are as statistically independent as possible
- Stratify or aggregate the category to minimise the dependency effect
- Model dependency explicitly
- Use sensitivity cases (independent, fully positive and fully negative correlated)

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The End!

Thank you!

Diego M. Ezcurra