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Supervisory Body
Article 6.4 Mechanism
UNFCC
Bonn, Germany

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Dear members of the Supervisory Body,

Addressing your call for input from June 5th 2023, I present here some scientific considerations relevant to the definitions being discussed under *"Removal activities under the article 6.4 mechanism"*. My comments are based on scientific research published in two articles annexed to this document. In the following, I summarize the main points from these scientific publications following the structured consultation from document A6.4-SB005-A02. In preparing these comments, I used as a reference the definitions presented in information note A6.4-SB005-AA-A09. My comments concern mostly ambiguities in the definition of removals and tonne-year accounting.

Cross-cutting questions:
1. Discuss the role of removals activities and this guidance in supporting the aim of balancing emissions with removals through mid-century.
Note A6.4-SB005-AA-A09 presents definitions of **Removals** (§2.1, ¶ 11, 12), differentiating between the process of separation of carbon from the atmosphere versus the amount of carbon removed. This is an important distinction that is consistent with scientific understanding of transferring carbon from the atmosphere to a natural reservoir and storing this carbon in that reservoir for a certain amount of time. However, there are two ambiguities related to the process of removals that have to do with the rate at which the removals occur and the amount of time over which those removals are stored. Paragraph 12 mentions that for *land-based activities the quantity of removals are expressed in units of tCO_2 or tCO_2eq, but sometimes in units of tC*. This definition introduces ambiguity by not explicitly accounting for the amount of time that it takes to remove this amount of carbon and by not accounting for how long this carbon remains out of the atmosphere. For example, carbon in permafrost soils has accumulated over millions of years, and the amount of carbon present in these soils, expressed in tC, does not reflect the time it took for this carbon to accumulate. A project may claim that this carbon stock is due to some additional activity that should be credited under A6.4 Mechanism, but it would be difficult to attribute what portion of the carbon was removed from the atmosphere during the period of the project and what amount was already there. Tonne-year accounting partially addresses this issue (however

see comment below), but a more specific definition of removal as a process and as a quantity would help to establish clear distinctions between carbon removed from the atmosphere attributable to a project, and carbon that is not. These definitions also address the issue of determining for how long new carbon entering a reservoir would be removed from the atmosphere. I propose the following refinement to the definitions of Removals including a clear specification of units for reporting.

- **Removal as a process of separation (Removal rate)**: Removal refers to the process of separating greenhouse gases (GHG) from the atmosphere. As an active process of transferring a GHG from the atmosphere to a different reservoir (land, ocean, or a technological reservoir), it is quantified as an amount of mass transferred over a period of time. Therefore, it is quantified in units of mass per time, e.g. tonnes of CO_2 per year, or Mg C $year^{-1}$. Notice that the units of this removal rate are equivalent to the units of emissions when quantified as a mass of GHG emitted per unit of time. This removal rate can be distinguished between a gross or a net flux:
  - **Gross removal rate**: The total amount of mass of GHG removed from the atmosphere per unit time. For biological systems on land and in the ocean, this definition is equivalent to gross primary production. This definition can also be applied to technological reservoirs as the total mass of GHG removed from the atmosphere per unit time.
  - **Reversal or release rate**: The total mass of GHG returning from a reservoir to the atmosphere per unit time. As most reservoirs are not completely sealed and cannot store carbon and other elements to infinity, the mass of GHGs released by the reservoir per unit time must be quantified. Geological reservoirs of carbon very likely have a release rate close to zero, but they may also incur in operating and maintenance activities that release GHGs to the atmosphere that must also be quantified and reported. For land-based reservoirs, events such as fires would be included in the release rate for the time at which the event occurs.
  - **Net removal rate**: The net amount of mass of GHG removed from the atmosphere per unit time after accounting for inefficiencies, losses and reversals, calculated as gross removal rate minus release rate.
- **Removal as a standing stock (Mass stored at a given time)**: For any reservoir it is possible to quantify the mass of carbon or other element present at a particular time. This standing stock is commonly quantified in mass units such as tonnes of C. It is a static quantity without any reference to the time that was necessary to accumulate the standing stock, but it should be reported based on the time at which it was quantified as the stock may change over time.
- **Removal as an accumulated quantity over a period of time (Carbon Sequestration, CS)**: As the process of removal operates over time, the mass of the element (e.g. carbon) accumulates in a reservoir over time. During the time the element is removed from the atmosphere, it does not contribute to the greenhouse effect and therefore there is a value in storing the element for as long as possible. To quantify both the amount and the time the element remains stored, the accumulated removal over time can be quan-

tified as the integral of the remaining mass that started entering the reservoir at some initial time t_0 until some later time t_0+T, where T is some time horizon. In many natural reservoirs, carbon is not permanently removed from the atmosphere, but the time it remains stored can be quantified under this definition, which was introduced by Sierra et al. (2021) and expanded in Crow & Sierra (2022), called Carbon Sequestration CS. This definition allows one to compare different activities in which there are differences in the amount or the time that the removal remains in a reservoir. The concept is related to the Absolute Global Warming Potential (AGWP), which quantifies the amount and the time that a GHG contributes to warming in the atmosphere. AGWP is based on the idea that the integral of the mass of an emission over a time horizon between t_0 and $t_0 + T$ accounts for both the mass of an emission and its atmospheric lifetime. The concept of CS quantifies both the mass and the amount of time a GHG remains removed from the atmosphere. Because it is an integral, it is reported in units of mass-time as in tonne-year accounting, but CS is not the same as traditional tonne-year accounting.

To avoid ambiguities, it is important to provide mathematical equations to each of these terms. Here I use the symbols presented in Sierra et al. (2021), although they can be replaced for other symbols if necessary.

- $s(t)$: gross removal rate at time t in units of mass per time.
- $r(t)$: release (reversal) rate from reservoir back to the atmosphere at time t in units of mass per time.
- $s(t) - r(t)$: Net removal rate at time t in units of mass per time.
- $M(t) = \int_{t_0}^{t}[s(t) - r(t)]\mathrm{d}t$: Mass stored of carbon or other element at time t that results from the balance between removal and release of a GHG since an initial time t_0 until the time of observation t. The units of $M(t)$ are units of mass. Notice that under this definition, baseline element stock is not considered, and only the balance between removal and release during a specific timeframe is considered.
- $\mathrm{CS}(T) = \int_{t_0}^{t_0+T} M(t)\mathrm{d}t$: Accumulated removal over a period of time. For carbon, this can be defined as Carbon Sequestration. It integrates over a selected time horizon T the mass stored in the system that results from the balance between removal and release. The units of CS are mass multiplied by time (e.g, MgC $\cdot$ year). Notice that CS implies two integrals,

$$\mathrm{CS}(T) = \int_{t_0}^{t_0+T}\int_{t_0}^{t}[s(t) - r(t)]\mathrm{d}t\,\mathrm{d}t, \tag{1}$$

  which is fundamentally different from previous proposals for carbon accounting using the so-called tonne-year methods.

Adopting CS as the definition of removals would simplify accounting methods as it is no longer necessary to make a distinction between short-term and long-term removals (as in §4.6 in A6.4-SB005-AA-A09) because the amount of time an amount of removal remains in a reservoir is explicitly considered in the computation of CS. For activities in which the removal only remains stored for a short time, the net removal is small as time progresses because the gross removal rate is almost balanced by the release rate. For a geological reservoir on the contrary, the nearly

zero release rate makes the mass stored and the accumulated removal increase over time.

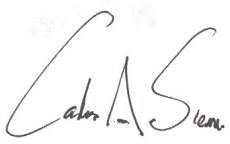
On tonne-year accounting

At a fundamental level, tonne-year is simply a unit of measurement that quantifies a concept involving mathematical integration of mass over time. Because it is a unit of measurement, it is very likely that very different methodologies would produce very different outcomes but in the same units of measurement. It would be preferable if methods that are currently called tonne-year accounting are referred more specifically. For instance, it'd be preferable to refer to the 'Moura-Costa' method or the 'Lashof' method as two separate methodologies, instead of using their unit of reporting without distinguishing the fundamental differences between the two methods.

The two scientific publications enclosed in this letter provide additional details on the approach to define and quantify removals. In these publications, an additional metric is presented and discussed, the Climate Benefit of Sequestration (CBS). This new metric could also be helpful in refining definitions of removals, however CBS is reported in units of W m^{-2}, which are more appropriate to compare potential warming or cooling in the atmosphere related to removal activities. If the focus of the new methodologies being discussed under A6.4 Mechanism are based on carbon accounting, CS as defined above is more appropriate, but for a more comprehensive accounting of potential warming or cooling, CBS is a more appropriate metric.

I hope you find these definitions useful.

Best regards,



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The climate benefit of carbon sequestration

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Abstract. Ecosystems play a fundamental role in climate change mitigation by photosynthetically fixing carbon from the atmosphere and storing it for a period of time in organic matter. Although climate impacts of carbon emissions by sources can be quantified by global warming potentials, the appropriate formal metrics to assess climate benefits of carbon removals by sinks are unclear. We introduce here the climate benefit of sequestration (CBS), a metric that quantifies the radiative effect of fixing carbon dioxide from the atmosphere and retaining it for a period of time in an ecosystem before releasing it back as the result of respiratory processes and disturbances. In order to quantify CBS, we present a formal definition of carbon sequestration (CS) as the integral of an amount of carbon removed from the atmosphere stored over the time horizon it remains within an ecosystem. Both metrics incorporate the separate effects of (i) inputs (amount of atmospheric carbon removal) and (ii) transit time (time of carbon retention) on carbon sinks, which can vary largely for different ecosystems or forms of management. These metrics can be useful for comparing the climate impacts of carbon removals by different sinks over specific time horizons, to assess the climate impacts of ecosystem management, and to obtain direct quantifications of climate impacts as the net effect of carbon emissions by sources versus removals by sinks.

1 Introduction

Terrestrial ecosystems exchange carbon with the atmosphere at globally significant quantities, thereby influencing Earth's climate and potentially mitigating warming caused by increasing concentrations of CO_2 in the atmosphere. Carbon fixed during the process of photosynthesis remains stored in the terrestrial biosphere over a range of timescales, from days to millennia – timescales of relevance for affecting the concentration of greenhouse gases in the atmosphere (Archer et al., 2009; IPCC, 2014; Joos et al., 2013). During the time carbon is stored in the terrestrial biosphere, it is removed from the radiative forcing effect that occurs in the atmosphere; thus, it is of scientific and policy relevance to understand the timescale of carbon storage in ecosystems, i.e., for how long newly fixed carbon is retained in an ecosystem before it is released back to the atmosphere.

Timescales of element cycling and storage are unambiguously characterized by the concepts of *system age* and *transit time* (Bolin and Rodhe, 1973; Rodhe, 2000; Rasmussen et al., 2016; Sierra et al., 2017; Lu et al., 2018). In a system of multiple interconnected compartments, system age characterizes the time that the mass of an element observed in the system has remained there since its entry. Transit time characterizes the time that it takes element masses to traverse the entire system, from the time of entry until they are released back to the external environment (Sierra et al., 2017). Both metrics are excellent system-level diagnostics of the dynamics and timescales of ecosystem processes. Because system age and transit time both can be reported as mass or probability distributions, they provide different information about an ecosystem over a wide range in the time domain.

System age and transit time are closely related to the complexity of the ecosystem and its process rates, which are affected by the environment (Luo et al., 2017; Rasmussen et al., 2016; Sierra et al., 2017; Lu et al., 2018). Mean system ages of carbon are consistently greater than mean transit time (Lu et al., 2018; Sierra et al., 2018b), suggesting that once a

mass of carbon enters an ecosystem a large proportion gets quickly released back to the atmosphere, but a small proportion remains for very long times. Furthermore, differences in transit times across ecosystems suggest that not all carbon sequestered in the terrestrial biosphere spends the same amount of time stored; e.g., one unit of photosynthesized carbon is returned back to the atmosphere faster in a tropical than in a boreal forest (Lu et al., 2018). Therefore, not all carbon drawn down from the atmosphere should be treated equally for the purpose of quantifying the climate mitigation potential of sequestering carbon in ecosystems as it is currently recommended in accounting methodologies (IPCC, 2006).

Global warming potentials (GWPs; see definition in Sect. 2) quantify the radiative effects of greenhouse gases emitted to the atmosphere (Fig. 1) but do not consider the avoided radiative effect of storing carbon in ecosystems (Neubauer and Megonigal, 2015). GWPs are computed using the age distribution of CO_2 and other greenhouse gases in the atmosphere (Rodhe, 1990; Joos et al., 2013) but do not consider age or transit times of carbon in ecosystems in the case of sequestration. Transit time distributions, in particular, can better inform us about the time newly sequestered carbon will be removed from radiative effects in the atmosphere.

For more comprehensive accounting of the contribution of carbon sequestration to climate change mitigation, it is necessary to quantify the avoided warming effects of sequestered carbon in ecosystems over the timescale the carbon is stored. The GWP metric is inappropriate to quantify avoided warming potential as a result of sequestration. A metric that can capture this avoided warming effect could have applications for (1) comparing different carbon sequestration activities considering the time carbon is stored in ecosystems and (2) providing better accounting methods for the effect of removals by sinks in climate policy. Currently, the Intergovernmental Panel on Climate Change (IPCC) recommends that countries and project developers report only emissions by sources and removals by sinks of greenhouse gases (GHGs), treating all removals equally in terms of their fate (IPCC, 2006).

Problems with applying GWPs to compute climate benefits of sequestering carbon in ecosystems are well documented (Moura Costa and Wilson, 2000; Fearnside et al., 2000; Brandão et al., 2013; Neubauer and Megonigal, 2015). Several approaches have been proposed to deal with the issue of timescales (Brandão et al., 2013), many of which deal with time as some form of delay in emissions. However, to our knowledge, no solution proposed thus far explicitly accounts for the time carbon is sequestered in ecosystems, from the time of photosynthetic carbon fixation until it is returned back to the atmosphere by autotrophic and heterotrophic respiration, and fires.

Therefore, the main objective of this paper is to introduce a metric to assess the climate benefits of carbon sequestration while accounting for the time carbon is stored in ecosystems. We first present the theoretical framework for the development of the metric, then provide simple examples for its computation, and discuss potential applications for ecosystem management and for climate change mitigation.

2 Theoretical framework

2.1 Absolute global warming potential (AGWP)

The direction of carbon flow, into or out of ecosystems, is of fundamental importance to understand and quantify their contribution to climate change mitigation. The absolute global warming potential (AGWP) of carbon dioxide quantifies the radiative effects of a unit of CO_2 emitted to the atmosphere during its life time – in the direction land → atmosphere. It is expressed as (Lashof and Ahuja, 1990; Rodhe, 1990)

$$\mathrm{AGWP}(T, t_0) = \int_{t_0}^{t_0+T} k_{\mathrm{CO_2}}(t)\, M_{\mathrm{a}}(t)\, \mathrm{d}t, \tag{1}$$

where $k_{\mathrm{CO_2}}(t)$ is the radiative efficiency or greenhouse effect of one unit of CO_2 (in mole or mass) in the atmosphere at time t, and $M_{\mathrm{a}}(t)$ is the amount of gas present in the atmosphere at time t (Rodhe, 1990; Joos et al., 2013). The AGWP quantifies the amount of warming produced by CO_2, while it stays in the atmosphere since the time the gas is emitted at time t_0 over a time horizon T. The function $M_{\mathrm{a}}(t)$ quantifies the fate of the emitted carbon in the atmosphere and can be written in general form as

$$M_{\mathrm{a}}(t) = h_{\mathrm{a}}(t - t_0) M_{\mathrm{a}}(t_0) + \int_{t_0}^{t} h_{\mathrm{a}}(t - \tau)\, Q(\tau)\, \mathrm{d}\tau, \tag{2}$$

where $h_{\mathrm{a}}(t - t_0)$ is the impulse response function of atmospheric CO_2 released into the atmosphere, $M_{\mathrm{a}}(t_0)$ is the content of atmospheric CO_2 at time t_0, and $Q(\tau)$ is the perturbation of new incoming carbon to the atmosphere between t_0 and t.

For a pulse, or instantaneous emission of CO_2, $M_{\mathrm{a}}(t_0) = E_0$, and

$$M_{\mathrm{a}}(t) = h_{\mathrm{a}}(t - t_0) E_0, \tag{3}$$

assuming no additional carbon enters the atmosphere after the pulse. If the pulse is equivalent to 1 kg or mole of CO_2, then $E_0 = 1$ and $M_{\mathrm{a}}(t) = h_{\mathrm{a}}(t - t_0)$. For a pulse emission of any arbitrary size, and assuming constant radiative efficiency (see details about this assumption in Sect. 2.2),

$$\mathrm{AGWP}(T, E_0, t_0) = k_{\mathrm{CO_2}}\, E_0 \int_{t_0}^{t_0+T} h_{\mathrm{a}}(t - t_0)\, \mathrm{d}t. \tag{4}$$

The AGWP can be computed for any other greenhouse gas using their respective radiative efficiencies and fate in the atmosphere (impulse response function). To compare different gases, the global warming potential (GWP) is defined as the AGWP of a particular gas divided by the AGWP of CO_2 (Shine et al., 1990; Lashof and Ahuja, 1990). Our interest in this paper is on carbon fixation and respiration in the form CO_2; therefore, we primarily concentrate here on AGWP.

The impulse response function $h_a(t - t_0)$ plays a central role within the AGWP framework. The function encodes information about the fate of a gas once it enters the atmosphere and determines for how long the gas will remain. Therefore, it can be interpreted as a density distribution for the transit time of a gas, since the time of emission until it is removed by natural sinks (e.g., CO_2) or by chemical reactions (e.g., CH_4).

The function typically is assumed to be static; i.e., the time at which the gas enters the atmosphere is not relevant, only the time it remains there $(t - t_0)$. However, this function can be time-dependent, expressing different shapes depending on the time the gas enters the atmosphere, i.e., $h_a(t_0, t - t_0)$. For example, when natural sinks saturate, faster accumulation of CO_2 and longer transit times of carbon in the atmosphere are observed (Metzler et al., 2018). In this situation, the specific time of an emission would lead to different response functions in the atmosphere. Because current research on impulse response functions primarily considers the static time-independent case (see Millar et al., 2017, for an exception), we will consider only the static case for the remainder of this paper.

2.2 The radiative efficiency of CO_2 and its impulse response function

The radiative efficiency of CO_2 is a function of the concentration of this gas and the concentration of other gases in the atmosphere with overlapping absorption bands (Lashof and Ahuja, 1990; Shine et al., 1990). Therefore, k_{CO_2} changes as the concentration of GHGs change in the atmosphere. For most applications however, the radiative efficiency of CO_2 has been assumed constant in the limit of a small perturbation at a specific background concentration (Lashof and Ahuja, 1990; Shine et al., 1990; Joos et al., 2013; Myhre et al., 2013).

Here, we use a constant value of $k_{CO_2} = 6.48 \times 10^{-12}\,\mathrm{W\,m^{-2}}$ per megagram of carbon based on results reported by Joos et al. (2013) for an atmospheric background of 389 ppm ($\sim$ present day). This radiative efficiency represents the change in radiative forcing caused by a change of 1 Mg of carbon in the atmosphere in the form of CO_2 in units of rate of energy transfer (watt) per square meter of surface.

Joos et al. (2013) have also derived impulse response functions (IRFs) of CO_2 in the atmosphere using coupled carbon–climate models that include multiple feedbacks among Earth system processes. One function was obtained by emitting a pulse of 100 Gt of carbon to a pre-industrial atmosphere with a background concentration of 280 ppm (PI100 function from here on), and another function was obtained by emitting 100 Gt of carbon to a present-day atmosphere with a background of 389 ppm (PD100 from here on). The functions they report are averages from the numerical output of multiple models fitted to a sum of exponential functions that include an intercept term. This intercept implies that a proportion of the added CO_2 never leaves from the atmosphere–ocean–terrestrial system to long-term geological reservoirs. Following Millar et al. (2017), we added a timescale of 1 million years that corresponds to the intercept term in the IRFs. The addition of this timescale has no effect on the results presented here, which are focused on much shorter timescales, but they avoid the mathematical problem that the integrals of the original functions go to infinity with time (Lashof and Ahuja, 1990; Millar et al., 2017).

2.3 Carbon sequestration CS and the climate benefit of carbon sequestration (CBS)

GWPs are useful to quantify the climate impacts of increasing or reducing emissions of GHGs to the atmosphere. However, it is also necessary to quantify the climate benefits of carbon flows in the opposite direction, atmosphere $\rightarrow$ land. Furthermore, it is also important to quantify not only how much and how fast carbon enters ecosystems, but also for how long the carbon stays (Körner, 2017).

Carbon taken up from the atmosphere through the process of photosynthesis is stored in multiple ecosystem reservoirs for a particular amount of time. Carbon sequestration can be defined as the process of capture and long-term storage of CO_2 (Sedjo and Sohngen, 2012). We define here carbon sequestration CS over a time horizon T as

$$\mathrm{CS}(T, S_0, t_0) := \int_{t_0}^{t_0+T} M_s(t - t_0)\,\mathrm{d}t, \tag{5}$$

where $M_s(t - t_0)$ represents the fate of a certain amount of carbon S_0 taken up by the sequestering system at a time t_0. Notice that this definition of carbon sequestration is very similar to that of AGWP for an emission, with the exception that the radiative efficiency term is omitted.

To obtain the fate of sequestered carbon over time, we represent carbon cycling and storage in ecosystems using the theory of compartmental dynamical systems (Luo et al., 2017; Sierra et al., 2018a). In their most general form, we can write carbon cycle models as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \dot{\boldsymbol{x}}(t) = \boldsymbol{u}(\boldsymbol{x}, t) + \mathbf{B}(\boldsymbol{x}, t)\,\boldsymbol{x}, \tag{6}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is a vector of n ecosystem carbon pools, $\boldsymbol{u}(\boldsymbol{x}, t) \in \mathbb{R}^n$ is a time-dependent vector-valued function of carbon inputs to the system, and $\mathbf{B}(\boldsymbol{x}, t) \in \mathbb{R}^{n\times n}$ is a time-dependent compartmental matrix. The latter two terms can

depend on the vector of states, in which case the compartmental system is considered nonlinear. In case the input vector and the compartmental matrix have fixed coefficients (no time dependencies), the system is considered autonomous, and it is considered non-autonomous otherwise (Sierra et al., 2018a). This distinction of models with respect to linearity and time dependencies (autonomy) is fundamental to distinguish important properties of models. For instance, models expressed as autonomous linear systems have a steady-state solution given by $\boldsymbol{x}^* = -\mathbf{B}^{-1}\boldsymbol{u}$, where $\boldsymbol{x}^*$ is a vector of steady-state contents for all ecosystem pools. Non-autonomous models have no steady-state solution.

The fate of the fixed carbon for the general nonlinear non-autonomous case can be obtained as

$$M_s(t-t_0) = \| \boldsymbol{\Phi}(t,t_0)\boldsymbol{\beta}(t_0)S_0 \|, \tag{7}$$

where $\boldsymbol{\beta}(t_0)S_0 = \boldsymbol{u}(t_0)$, and $\boldsymbol{\beta}(t_0)$ is an n-dimension vector representing the partitioning of the total sequestered carbon among n ecosystem carbon pools (Ceballos-Núñez et al., 2020). The $n \times n$ matrix $\boldsymbol{\Phi}(t,t_0)$ is the state-transition operator, which represents the dynamics of how carbon moves in a system of multiple interconnected compartments (see details in Appendix B). Throughout this document, we use the symbol $\| \ \|$ to denote the 1-norm of a vector, i.e., the sum of the absolute values of all elements in a vector.

Because ecosystems and most reservoirs are open systems, the sequestered carbon S_0 returns back to the atmosphere, mostly as CO_2 due to ecosystem respiration and fires. Carbon release $r(t)$ from ecosystems can be obtained according to

$$r(t) = -\mathbf{1}^\intercal \mathbf{B}(t)\boldsymbol{\Phi}(t,t_0)\boldsymbol{\beta}(t_0)S_0, \tag{8}$$

where $\mathbf{1}^\intercal$ is the transpose of the n-dimensional vector containing only 1s. The state-transition matrix captures the entire fate and dynamics of the sequestered carbon, from the time it enters t_0 until release at any t.

The link between the time it takes sequestered carbon S_0 to appear in the release flux $r(t)$ is established by the concept of transit time (Metzler et al., 2018). In particular, we define the forward transit time (FTT) as the age that fixed carbon will have at the time it is released back to the atmosphere, or how long a mass fixed now will stay in the system. The backward transit time (BTT) is defined as the age of the carbon in the output flux since the time it was fixed, or how long the mass leaving the system now had stayed. This implies that

$$r(t) = p_{\mathrm{BTT}}(t-t_0,t) = p_{\mathrm{FTT}}(t-t_0,t_0), \tag{9}$$

where $p_{\mathrm{BTT}}(t-t_0,t)$ is the backward transit time distribution of carbon leaving the system at time t with an age $t-t_0$, while $p_{\mathrm{FTT}}(t-t_0,t_0)$ is the forward transit time distribution of carbon entering the system at time t_0 and leaving with an age $t-t_0$. For systems in equilibrium, both quantities are equal (Metzler et al., 2018). For systems not in equilibrium, semi-explicit formulas for their distributions are given in Appendix B.

For the atmosphere, carbon sequestration is a form of negative emission, and we can represent its fate in the atmosphere as

$$M_a'(t) = -h_a(t-t_0)S_0 + \int_{t_0}^{t} h_a(t-\tau)r(\tau)\,\mathrm{d}\tau, \tag{10}$$

where the prime symbol represents a perturbed atmosphere as an effect of sequestration. The first term in the right-hand side represents the response of the atmosphere to an instantaneous sequestration S_0 at t_0, and the second term represents the perturbation in the atmosphere of the carbon returning back from the terrestrial biosphere. Notice that the integral in this equation can be written as a convolution $(h_a \star r)(t)$ between the impulse response function of atmospheric CO_2 and the carbon returning from ecosystems to the atmosphere.

We define now the climate benefit of sequestration for a pulse of CO_2 into an ecosystem as

$$\begin{aligned}\mathrm{CBS}(T,S_0,t_0) &:= \int_{t_0}^{t_0+T} k_{CO_2} M_a'(t)\,\mathrm{d}t,\\ &= -k_{CO_2}\int_{t_0}^{t_0+T} (h_a(t-t_0)S_0 - (h_a \star r)(t))\,\mathrm{d}t.\end{aligned} \tag{11}$$

This metric integrates over a time horizon T the radiative effect avoided by sequestration of an amount of carbon S_0 taken up at time t_0 by an ecosystem. It captures the timescale at which the carbon is stored and gradually returns back to the atmosphere. It can also be interpreted as the atmospheric response to carbon sequestration in the form of a negative emission of CO_2 during a time horizon of interest. It relies on knowledge of the atmospheric response to perturbations in the form of an impulse response function and the transit time of carbon in an ecosystem.

2.4 Ecosystems in equilibrium: the linear, steady-state case

The computation of CS and CBS is simplified for systems in equilibrium. For linear systems at a steady state, the time at which the carbon enters the ecosystem is irrelevant (Kloeden and Rasmussen, 2011; Rasmussen et al., 2016); one only needs to know for how long the carbon has been in the system to predict how much of it remains. Mathematically, this implies

$$\boldsymbol{\Phi}(t,t_0) = e^{a\cdot\mathbf{B}} \ \text{ for all } \ t_0 \le t \ \text{ and } \ a = t-t_0. \tag{12}$$

Therefore, for linear systems at a steady state, we have the special cases

$$M_s(a) = \| e^{a\cdot\mathbf{B}}\boldsymbol{u} \|, \tag{13}$$

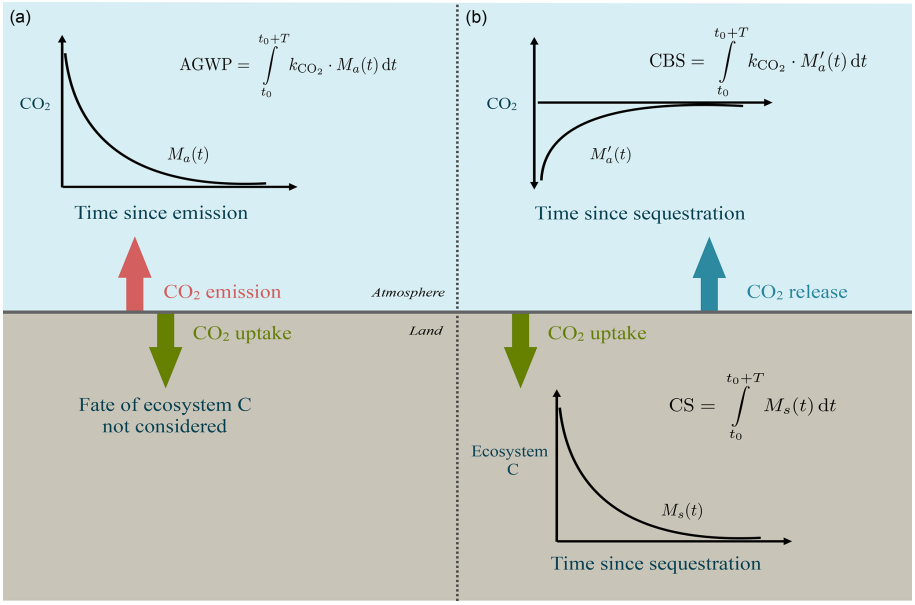

Figure 1. Contrast between current approach to quantification of climate effects of emissions and sequestration **(a)**, and the proposed approach for sequestration **(b)**. Plots and equations represent the concepts of absolute global warming potential (AGWP) of an emission of CO_2, carbon sequestration (CS), and climate benefits of sequestration (CBS). AGWP integrates over a time horizon T the fate of an instant emission at time t_0 of a gas ($M_a(t)$) and multiplies by the radiative efficiency k of the gas. A similar idea can be used to define CS as the integral of the fate $M_s(t)$ of an instant amount of carbon uptake S_0 over T. The CBS captures the atmospheric disturbance caused by CO_2 uptake and subsequent release by respiration as the integral over T of the fate of sequestered carbon $M'_a(t)$ multiplied by the radiative efficiency of CO_2.

and

$$M_{s1}(a) = \left\| e^{a\cdot\mathbf{B}} \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} \right\|, \tag{14}$$

where M_{s1} represents the fate of one unit of fixed carbon, which can also be interpreted as the proportion of carbon remaining after the time of fixation.

The amount of released carbon returning to the atmosphere is therefore

$$r(a) = -\mathbf{1}^\intercal \mathbf{B}\, e^{a\cdot\mathbf{B}}\, \boldsymbol{u}, \tag{15}$$

which for one unit of fixed carbon is equal to the transit time density distribution $f(\tau)$ of a linear system (Metzler and Sierra, 2018, see also Appendix B)

$$r_1(a) = -\mathbf{1}^\intercal \mathbf{B}\, e^{a\cdot\mathbf{B}} \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|}, \tag{16}$$

where $r_1(a) = f(\tau)$, with mean (expected value) transit time given by

$$\mathbb{E}(\tau) = -\mathbf{1}^\intercal \mathbf{B}^{-1} \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} = \frac{\|\boldsymbol{x}^*\|}{\|\boldsymbol{u}\|}. \tag{17}$$

We can now derive the steady-state expression of CS as

$$\mathrm{CS}(T) = \int_0^T \|e^{a\cdot\mathbf{B}}\,\boldsymbol{u}\|\,\mathrm{d}a. \tag{18}$$

Furthermore, it is possible to find a closed-form expression for this integral:

$$\mathrm{CS}(T) = \|\mathbf{B}^{-1}\left(e^{T\cdot\mathbf{B}} - \mathbf{I}\right)\boldsymbol{u}\|, \tag{19}$$

where $\mathbf{I} \in \mathbb{R}^{n\times n}$ is the identity matrix. Similarly, for one unit of carbon entering a steady-state system at any time, we define CS_1 as

$$\mathrm{CS}_1(T) = \int_0^T \left\| e^{a\cdot\mathbf{B}} \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} \right\| \mathrm{d}a, \tag{20}$$

which by integration gives

$$\mathrm{CS}_1(T) = \left\| \mathbf{B}^{-1}\left(e^{T\cdot\mathbf{B}} - \mathbf{I}\right) \frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} \right\|. \tag{21}$$

These steady-state expressions can be very useful to compare different systems or changes to a particular system if the steady-state assumption is justified. Furthermore, it can be shown that in the long term, as the time horizon T goes to infinity (∞), the term $(e^{T\cdot\mathbf{B}} - \mathbf{I})$ converges to $-\mathbf{I}$, and therefore Eq. (19) converges to the expression

$$\lim_{T\to\infty} \mathrm{CS}(T) = \|\boldsymbol{x}^*\|, \tag{22}$$

which means that the total amount of carbon at a steady state is equal to the long-term carbon sequestration of an instantaneous amount of fixed carbon at an arbitrary time.

Similarly, for one unit of carbon entering a system at a steady state, the long-term CS_1 from Eq. (21) can be obtained simply as

$$\lim_{T\to\infty} \mathrm{CS}_1(T) = \mathbb{E}(\tau) \tag{23}$$

by using the definition of mean transit time of Eq. (17). This means that long-term sequestration of one unit of CO_2 converges to the mean transit time of carbon in an ecosystem.

2.5 Dynamic ecosystems out of equilibrium: the continuous sequestration and emissions case

In addition of considering isolated pulses of emissions E_0 or sequestrations S_0, we can also consider permanently ongoing emissions $e : t \longmapsto E(t)$ and sequestration $s : t \longmapsto S(t)$, respectively. Hence,

$$\mathrm{CS}(T, s, t_0) := \int_{t_0}^{t_0+T} M_\mathrm{s}(t)\,\mathrm{d}t, \tag{24}$$

where

$$M_\mathrm{s}(t) = \int_{t_0}^{t} \|\boldsymbol{\Phi}(t,\tau)\,\boldsymbol{\beta}(\tau)\,s(\tau)\|\,\mathrm{d}\tau. \tag{25}$$

Here $s(\tau)$ is a scalar flux of sequestration at time τ. This leads to

$$r(t) = -\mathbf{1}^\mathsf{T}\mathbf{B}(t) \int_{t_0}^{t} \boldsymbol{\Phi}(t,\tau)\,\boldsymbol{\beta}(\tau)\,s(\tau)\,\mathrm{d}\tau. \tag{26}$$

The fate of sequestered carbon, for the atmosphere in the form of a balance between simultaneous sequestration and return of carbon, can now be obtained as

$$\begin{aligned} M'_\mathrm{a}(t) &= -\int_{t_0}^{t} h_\mathrm{a}(t-\tau)\,s(\tau)\,\mathrm{d}\tau + \int_{t_0}^{t} h_\mathrm{a}(t-\tau)r(\tau)\,\mathrm{d}\tau \\ &= -\int_{t_0}^{t} h_\mathrm{a}(t-\tau)\,[s(\tau)-r(\tau)]\,\mathrm{d}\tau \\ &= -(h_\mathrm{a}\star(s-r))(t). \end{aligned} \tag{27}$$

We can now define the climate benefit of sequestration for a dynamic ecosystem with continuous sequestration and respiration as

$$\begin{aligned} \mathrm{CBS}(T, s, t_0) &:= \int_{t_0}^{t_0+T} k_{\mathrm{CO}_2} M'_\mathrm{a}(t)\,\mathrm{d}t, \\ &= -k_{\mathrm{CO}_2} \int_{t_0}^{t_0+T} (h_\mathrm{a}\star(s-r))(t)\,\mathrm{d}t. \end{aligned} \tag{28}$$

This expression of CBS accounts for the dynamic behavior of inputs and outputs of carbon in ecosystems, and it can be used to represent time dependencies resulting from environmental changes and disturbances or produced by emission scenarios or scheduled management activities. This time-dependent CBS is computed for a time horizon T starting at any initial time t_0. In other words, it can be used to analyze specific time windows of interest, accounting for the fate of all carbon sequestered during specific time intervals.

3 Example 1: CS and CBS for linear systems in equilibrium

3.1 The fate of a pulse of inputs through the system

A simple ecosystem carbon model, the terrestrial ecosystem model (TECO), will now demonstrate an application of the theory to compute CS and CBS assuming a linear system at a steady state (i.e., in equilibrium). We used a modified version of the TECO model, originally described by Weng and Luo (2011) with parameter values obtained through data assimilation using observations from the Duke Forest in North Carolina, USA. It contains eight main compartments: foliage x_1, woody biomass x_2, fine roots x_3, metabolic litter x_4, structural litter x_5, fast soil organic matter (SOM) x_6, slow SOM x_7, and passive SOM x_8 (Fig. 2). The model represents the dynamics of carbon at a temperate forest dominated by loblolly pine. We chose this model due to its simplicity and tractability, but the framework presented in Sect. 2 can be applied to more complex models and for other ecosystems (see reference in Sect. "Executable research compendium (ERC)" for an example with a nonlinear model). In addition to its simplicity and tractability, there are two advantages of using this model over others: (1) it provides reasonable predictions of net ecosystem carbon fluxes and biometric pool data (Weng and Luo, 2011); (2) it is commonly used to express complex ecosystem-level concepts such as the matrix generalization of carbon cycle models, their traceability, and transient behavior (e.g., Luo and Weng, 2011; Luo et al., 2012; Xia et al., 2013; Luo et al., 2017; Sierra, 2019).

The model is commonly expressed as

$$\frac{\mathrm{d}\boldsymbol{X}(t)}{\mathrm{d}t} = \boldsymbol{b}U(t) + \xi(t)\mathbf{A}\mathbf{C}\boldsymbol{X}(t), \tag{29}$$

where $\boldsymbol{X}$ is a vector of ecosystem carbon pools, $\mathbf{C}$ is a diagonal matrix with cycling rates for each pool, $\mathbf{A}$ is a matrix of

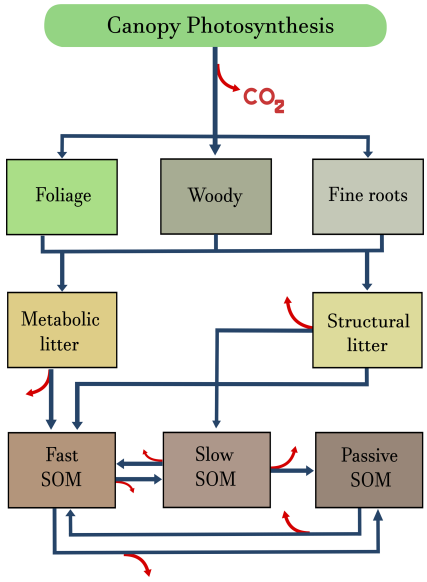

Figure 2. Graphical representation of the terrestrial ecosystem model (TECO) described in Weng and Luo (2011) and Luo et al. (2012). Carbon enters the ecosystem through canopy photosynthesis and is allocated to three biomass pools: foliage, woody biomass, and fine roots. From these pools, carbon is transferred to metabolic and structural litter pools, from where it can be respired as CO_2 or transferred to the soil organic matter (SOM) pools. Blue arrows represent transfers among compartments, and red arrows release to the atmosphere in the form of CO_2.

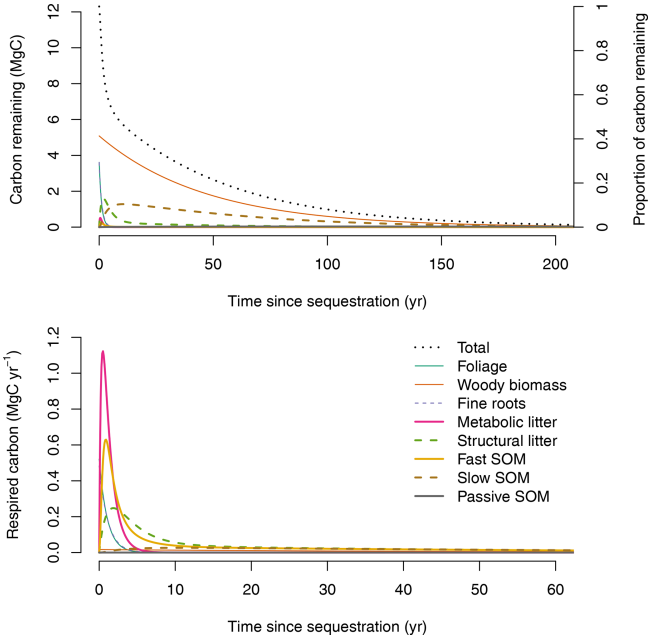

Figure 3. Fate of carbon ($M_s(t)$, left axis; and $M_{s1}(t)$, right axis) entering the ecosystem according to the TECO model parameterized for the Duke Forest and calculated using Eq. (13) for the upper panel, and respired carbon ($r(t)$) returning back to the atmosphere calculated using Eq. (15).

transfer coefficients among pools, and $\boldsymbol{b}$ is a vector of allocation coefficients to plant parts. We modified the entries of matrix $\mathbf{A}$ to allow autotrophic respiration to be computed from the vegetation pools and not from the GPP flux as in the original model (see details in Appendix C). The function $U(t)$ determines the carbon inputs to the system as gross primary production (GPP), and $\xi(t)$ is a time-dependent function that modifies ecosystem cycling rates according to changes in the environment.

For this steady-state example, we assume constant inputs ($U(t) = U$) and constant rates ($\xi(t) = 1$). Furthermore, defining $\mathbf{B} := \mathbf{AC}$, and $\boldsymbol{u} := \boldsymbol{b}U$, we can write this model as a linear, autonomous compartmental system of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{u} + \mathbf{B}\boldsymbol{x}, \tag{30}$$

with values for $\mathbf{B}$ and $\boldsymbol{u}$ as described in Appendix C.

The fate of a pulse of carbon input entering the ecosystem at an arbitrary time when the system is in equilibrium can be obtained by applying Eqs. (13) and (14) (Fig. 3). Carbon enters the ecosystem through foliage, wood, and fine-root pools. A large proportion of this carbon is quickly transferred from these pools to the fine and metabolic litter pools. Subsequently, the carbon moves to the SOM pools with important respiration losses during these transfers. Most carbon is returned back to the atmosphere with a mean transit time of 30.4 years for the whole system. Half of the sequestered carbon is returned back to the atmosphere in 7.6 years and 95 % in 124 years.

Ecosystem-level CS, i.e., the area under the curve of the amount of remaining carbon over time (area under dotted line in Fig. 3, upper panel), increases towards an asymptote as the time horizon of integration increases (Fig. 4a). Here, CS is reported in units of Mg C ha^{-1} yr, because this is the amount of carbon retained in organic matter over a fixed time horizon. For relevant time horizons of 50, 100, 500, and 1000 years, CS was 233.51, 317.68, 371.64, and 373.42 Mg C ha^{-1} yr, respectively. In the long term (i.e., as the time horizon goes to infinity), CS converges to the steady-state carbon stock predicted by the model of 373.67 Mg C ha^{-1}.

A similar computation can be made for one unit of fixed carbon (CS_1). In this case CS_1 was 18.98, 25.83, 30.21, and 30.36 years for time horizons of 50, 100, 500, and 1000 years, respectively. In the long term, CS_1 converges to the mean transit time of carbon: 30.4 years (Fig. 4b).

Due to sequestration at t_0, the CBS shows a rapid negative increase in radiative forcing, which decreases as the time horizon increases due to the return of carbon to the atmosphere as an effect of respiration (Fig. 4c). The shape of the curve, however, depends strongly on the IRF for atmospheric CO_2. CBS is larger over the long term (> 200 years) for the present-day (PD100) curve proposed by Joos et al. (2013) than for the pre-industrial curve (PI100). During the

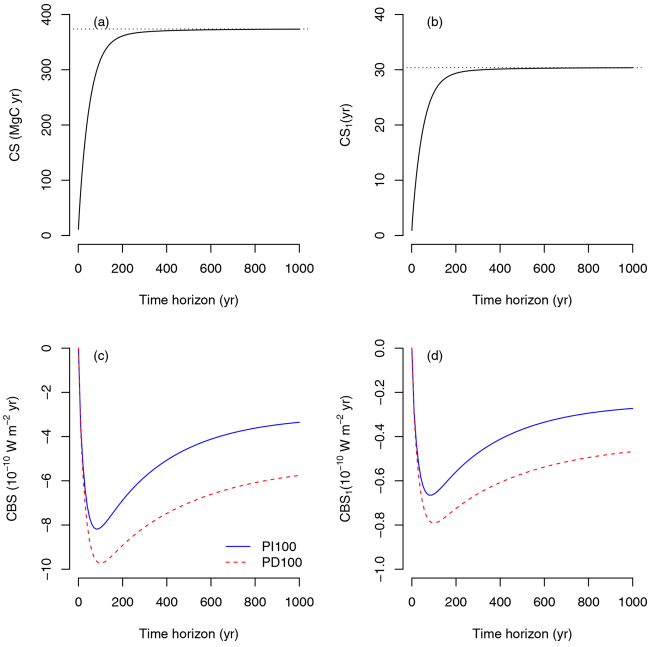

Figure 4. Carbon sequestration (CS) and climate benefit of sequestration (CBS) for instantaneous carbon uptake at any given time. **(a)** CS due to the uptake of 12.3 Mg C ha^{-1}, which corresponds to GPP of 1 year. **(b)** CS due to the uptake of one unit of carbon (CS_1). **(c)** CBS due to the uptake of 12.3 Mg C ha^{-1} for two different impulse response functions (pre-industrial atmosphere with a pulse of 100 Gt of carbon: PI100, and present-day atmosphere with a pulse of 100 Gt of carbon: PD100). **(d)** CBS due to the uptake of one unit of carbon (CBS_1) for two different impulse response function. Dotted lines in **(a)** and **(b)** represent steady-state carbon storage and mean transit time, respectively.

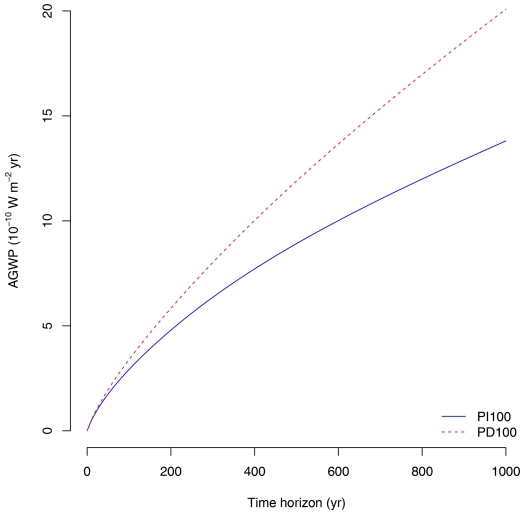

Figure 5. Absolute global warming potential (AGWP) due to the emission of 1 Mg of CO_2–C to the atmosphere for the two different IRFs (pre-industrial atmosphere with a pulse of 100 Gt of carbon: PI100, and present-day atmosphere with a pulse of 100 Gt of carbon: PD100) reported by Joos et al. (2013).

pre-industrial period, perturbations of CO_2 in the atmosphere are lower than in the present-day period due to higher uptake of carbon from the oceans and the land biosphere (Joos et al., 2013). Therefore, the benefits of carbon sequestration are larger under present-day conditions based on these IRF curves. Impulse response functions depend strongly on the magnitude and timing of the pulse (Joos et al., 2013; Millar et al., 2017). Therefore, estimates of climate impacts of emissions (AGWP, Fig. 5) and climate benefits of sequestration (CBS, Fig. 4c, d) depend strongly on the choice of the IRF. For the purpose of this paper, we will use the present-day curve (PD100) from here on.

Because AGWP and CBS are based on similar concepts and share similar units, it becomes possible to directly compare one another (Fig. 6) and obtain an estimate of the climate impact of emissions versus sequestration. This can be done either as the ratio of the absolute value of CBS to AGWP, i.e., |CBS|/AGWP (unitless), or as the net radiative balance CBS + AGWP (W m^{-2} yr). It is possible to compute these relations using the CBS for one unit of sequestered carbon, which provides a direct estimate of the impact of one unit of sequestration versus one unit of emission, or corresponding to the amount of GPP sequestered in 1 year (12.3 Mg C ha^{-1} yr^{-1} for Duke Forest).

In our example, the emission of 1 Mg of carbon to the atmosphere has a predominant warming effect that cannot be compensated for by the sequestration of 1 Mg of carbon at the Duke Forest (Fig. 6). However, the sequestration of the equivalent of GPP in 1 year can have a significant climate benefit compared to the emission of 1 Mg of carbon, depending on the time horizon of analysis. When one integrates in time horizons lower than 200 years, CBS outweighs AGWP in this example. However, because the lifetime of an emission of CO_2 is much longer in the atmosphere than the transit time of carbon through a forest ecosystem, AGWP outweighs CBS on longer timescales.

The time of integration in the computation of GWP has been a heavily debated topic in the past, and this is related to the topic of "permanence" of sequestration in carbon accounting and climate policy (Moura Costa and Wilson, 2000; Noble et al., 2000; Sedjo and Sohngen, 2012). One problem in these previous debates is that the timescale of carbon in ecosystems was not considered explicitly while the timescale of carbon in the atmosphere was. With the approach proposed here, both are explicitly taken into account and can better inform management and policy debates about sequestration of carbon in natural and man-made sinks.

3.2 Carbon management to maximize the climate benefit of carbon sequestration

In the context of climate change mitigation, management of ecosystems may be oriented to increase carbon sequestration and its climate benefit. In the recent past, scientists and pol-

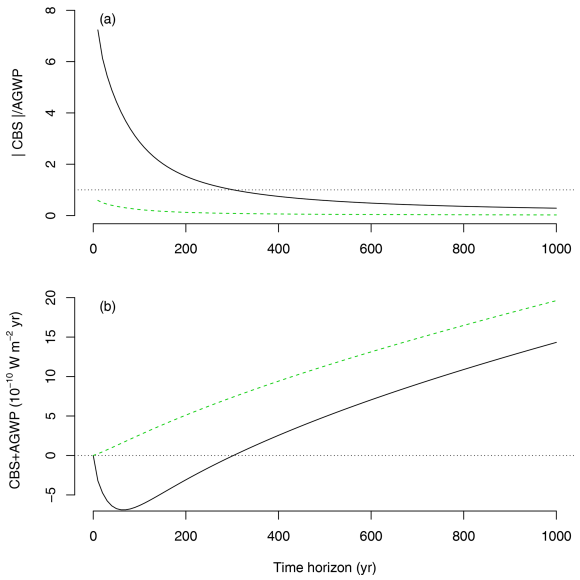

Figure 6. Relations between CBS and AGWP for the IRF PD100 as a function of time horizon T. **(a)** Ratio between the absolute value of CBS and AGWP, based on a total sequestration of 12.3 Mg of carbon (back line, GPP equivalent for 1 ha and 1 year at Duke Forest) versus a sequestration of 1 Mg of carbon (dashed green line). **(b)** Radiative balance (net difference) between CBS and AGWP for the sequestration of 12.3 Mg of carbon (black line) and 1 Mg of carbon (dashed green line).

icy makers have advocated increasing the amount of inputs to ecosystems as an effective form of carbon management (e.g., Silver et al., 2000; Grace, 2004; Lal, 2004; Chabbi et al., 2017; Minasny et al., 2017). Although increases in carbon inputs can increase the amount of stored carbon in an ecosystem with related climate benefits, it does not necessarily increase the amount of time the sequestered carbon will stay in the system. Therefore, strategies that focus on increasing carbon inputs alone do not take full advantage of the potential of ecosystems to mitigate climate change.

We can conceptualize any management activity that increases or reduces carbon inputs to an ecosystem by a factor γ, so the new inputs are given by the product $\gamma\,\boldsymbol{u}$. For example, if we increase carbon inputs to an ecosystem by 10 %, $\gamma = 1.1$. Increasing carbon inputs by a proportion $\gamma > 1$ increases carbon storage at a steady state by an equal proportion since

$$\begin{aligned} -\mathbf{B}^{-1}(\gamma\,\boldsymbol{u}) &= \gamma\,(-\mathbf{B}^{-1}\,\boldsymbol{u}), \\ &= \gamma\,\boldsymbol{x}^*. \end{aligned} \tag{31}$$

Similarly, a decrease in carbon inputs by a proportion $\gamma < 1$ decreases steady-state carbon storage by an equal proportion. However, the time carbon requires to travel through the ecosystem is still the same since the transit time does not change, as we can see from the mean transit time expression

$$-\mathbf{1}^{\intercal}\,\mathbf{B}^{-1}\,\frac{\gamma\,\boldsymbol{u}}{\|\gamma\,\boldsymbol{u}\|} = \mathbb{E}(\tau). \tag{32}$$

Both the transit time distribution (Eqs. B4 and 16) and the mean transit time (Eq. 17) only take into account the proportional distribution of the carbon inputs to the different pools ($\boldsymbol{u}/\|\boldsymbol{u}\|$) but not the total amount of inputs. Therefore, a unit of carbon that enters an ecosystem stays there for the same amount of time independent of how much carbon is entering the system. Although these results only apply to linear systems at a steady state, they provide some intuition about what might be the case in systems out of equilibrium.

Carbon management can also be oriented to modify process rates in ecosystems as encoded in the matrix $\mathbf{B}$. A proportional decrease in process rates by a factor $\xi < 1$ not only increases carbon storage as

$$\begin{aligned} -(\xi\,\mathbf{B})^{-1}\,\boldsymbol{u} &= \frac{1}{\xi}\,(-\mathbf{B}^{-1}\,\boldsymbol{u}), \\ &= \frac{\boldsymbol{x}^*}{\xi}, \end{aligned} \tag{33}$$

but also increases the mean transit time as

$$-\mathbf{1}^{\intercal}\,(\xi\,\mathbf{B})^{-1}\,\frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} = \frac{\mathbb{E}(\tau)}{\xi}. \tag{34}$$

A proportional change in the opposite direction ($\xi > 1$) causes the opposite effect; a proportional increase in process rates decreases carbon storage and decreases mean transit time.

Based on these results, it is now clear that carbon management to increase carbon inputs alone can only increase CS but not CS_1; i.e., the new carbon inputs have a sequestration benefit only through increase of carbon storage but not through a longer transit time in ecosystems. Management to decrease process rates, on the contrary, can increase both CS and CS_1 because the new carbon entering the system stays there for longer.

We can see these effects of carbon management on CS by running simulations using the TECO model at a steady state (Fig. 7). Now, we modified carbon inputs and process rates by either increasing them by 10 % and 50 % ($\gamma, \xi = 1.1, 1.5$) or decreasing them by 10 % and 50 % ($\gamma, \xi = 0.9, 0.5$). The simulations showed that increasing or decreasing carbon inputs increase or decrease CS for any time horizon (Fig. 7a), but it does not modify the behavior of one unit of sequestered carbon (CS_1) (Fig. 7b). On the contrary, decreasing or increasing process rates increase or decrease both CS (Fig. 7c) and CS_1 (Fig. 7d).

The resultant effects of changes in management of inputs or process rates on CBS can differ substantially. Increases or decreases of carbon inputs have similar proportional effects on CBS, but differences in processes rates are not equally proportional. While an increase in inputs by 50 % would increase CBS by 50 %, a decrease in process rates by 50 % would have an increase in CBS by more than 100 % for time horizons longer than 300 years (Fig. 8). Similarly, while a decrease in inputs by 50 % would reduce CBS by 50 %, an

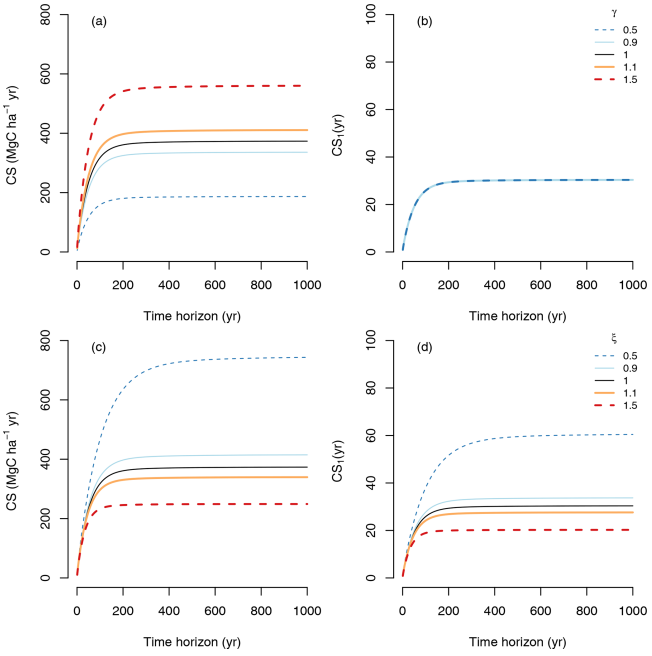

Figure 7. Different carbon management strategies and their effect on the CS and CS_1. Management to increase or decrease carbon inputs in the vector $\boldsymbol{u}$ by specific proportions γ is shown in panel **(a)** and **(b)**. Management to increase or decrease process rates in the matrix **B** by a proportion ξ is shown in panels **(c)** and **(d)**. Since CS_1 quantifies carbon sequestration of one unit of carbon, management of the amount of carbon inputs does not modify CS_1 in panel **(b)**, and all lines overlap.

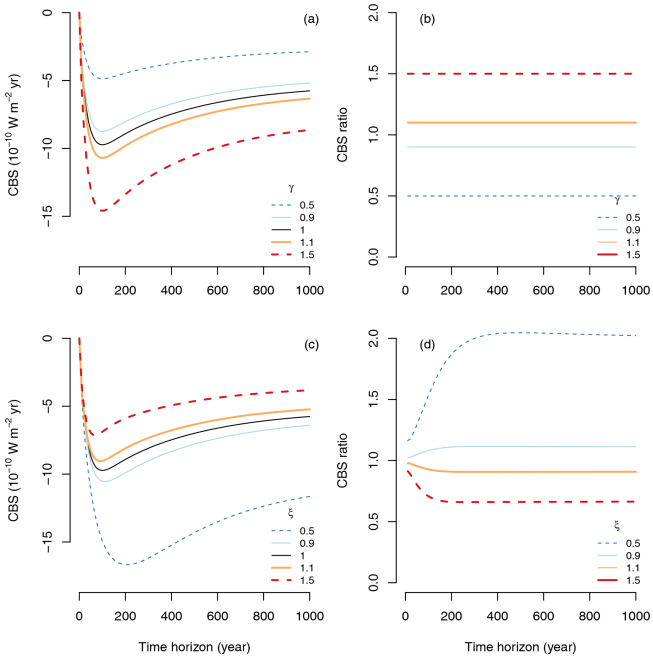

Figure 8. Effects of different management strategies on CBS. **(a)** Effect of increasing or decreasing carbon inputs by a proportion γ on CBS; **(b)** same effect of γ expressed as a ratio with respect to the reference case of $\gamma = 1$. **(c)** Effects of decreasing or increasing process rates in the matrix **B** by a proportion ξ on CBS; **(d)** same effect of ξ expressed as a ratio with respect to the reference case $\xi = 1$.

increase in process rates by 50 % would decrease CBS by only ∼ 40 %.

These results show that management of transit time, e.g., by decreasing process rates, may lead to stronger climate benefits than managing carbon inputs alone. Furthermore, one could think about optimization scenarios in which both inputs and transit times are managed to achieve larger climate benefits given certain constraints. The concept of CBS is thus a useful mathematical framework to formally pose such an optimization problem.

We can also use these results to infer differences in CS and CBS for different ecosystem types. Without management, we would expect large variability of CS and CBS in the terrestrial biosphere. Inputs and process rates vary considerably for terrestrial ecosystems as previously reported in other studies. For instance, gross primary productivity can range from about 1 to $> 30\,\mathrm{Mg\,C\,ha^{-1}\,yr^{-1}}$ from high- to low-latitude ecosystems (Jung et al., 2020). Based on simulations from the CABLE model, Lu et al. (2018) found a range of mean transit times between 13 and 341 years from low- to high-latitude ecosystems. These large ranges of variability for GPP and mean transit time suggest that CS and CBS may vary among ecosystems by large proportions (> 20 times larger or smaller depending on the ecosystems being compared).

4 Example 2: CS and CBS for dynamic systems out of equilibrium

4.1 Pulses entering at different times and experiencing different environments

The steady-state examples above are useful to gain some intuition about potential long-term patterns in CS and CBS, but for real-world applications it is necessary to consider systems out of equilibrium and driven by specific time-dependent signals. We will consider now the case of the temperate ecosystem of our previous example driven by increases in atmospheric CO_2 concentrations that lead to higher photosynthetic uptake and increasing temperatures that lead to faster cycling rates. We will thus consider a non-autonomous version of the TECO model that follows the general form

$$\dot{\boldsymbol{x}}(t) = \gamma(t) \cdot \boldsymbol{u} + \xi(t) \cdot \mathbf{B} \cdot \boldsymbol{x}(t), \tag{35}$$

where the time-dependent function $\gamma(t)$ incorporates the effects of temperature and atmospheric CO_2 on primary production, and the function $\xi(t)$ incorporates the effects of tem-

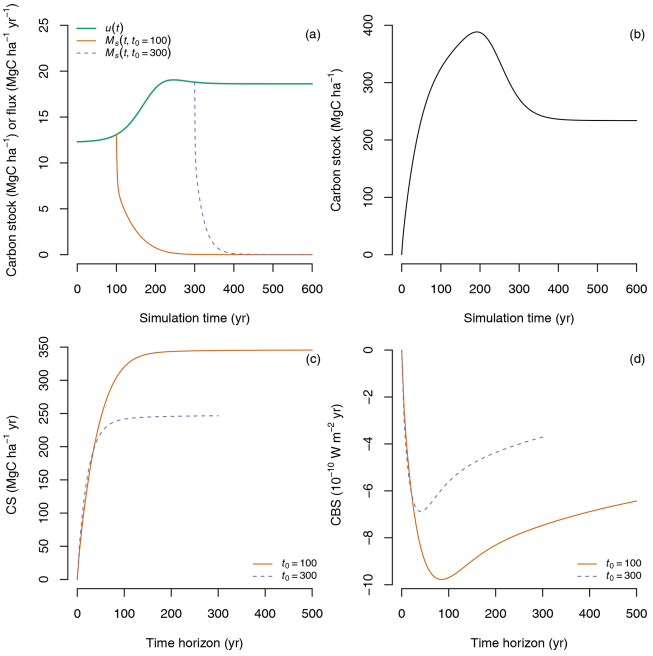

Figure 9. Prediction of CS and CBS for a non-steady-state case with time-dependent inputs $\boldsymbol{u}(t)$ controlled by CO_2 fertilization and temperature and process rates controlled by temperature modified by a time-dependent factor $\xi(t)$. **(a)** Predicted time-dependent inputs $\boldsymbol{u}(t)$, and the fate of carbon entering the ecosystem at simulation year 100 ($M_s(t, t_0 = 100)$) and simulation year 300 ($M_s(t, t_0 = 300)$). **(b)** Predicted carbon accumulation in the ecosystem ($\| \boldsymbol{x}(t) \|$) for the entire simulation period. **(c)** Carbon sequestration for the amount of inputs entering at simulation years 100 and 300 calculated for different time horizons T. **(d)** Climate benefit of sequestration for carbon entering the ecosystem at simulation years 100 and 300 integrated for different time horizons T.

perature on respiration rates. Specific shapes for these functions were taken from Rasmussen et al. (2016) and are described in detail in Appendix C. When applied to the CASA model in Rasmussen et al. (2016), these functions predicted an increase in primary production and an increase in process rates, which resulted in a decrease in transit times over a simulation of 600 years.

We used the same simulation setup here starting from an empty system ($\boldsymbol{x}(0) = 0$) and obtained similar results in terms of primary production and transit times as in Rasmussen et al. (2016). We used these simulation results to compute CS and CBS for carbon entering the ecosystem at different times during the simulation window. In particular, we considered the case of the amount of carbon sequestered at years 100 and 300 after the start of the simulation; i.e., we considered the cases $t_0 = 100$ and $t_0 = 300$ (Fig. 9a) and computed the fate of this carbon ($M_s(t, t_0, \boldsymbol{u}_0)$), its carbon sequestration ($\mathrm{CS}(T, \boldsymbol{u}_0, t_0)$) and the climate benefit of sequestration ($\mathrm{CBS}(T, \boldsymbol{u}_0, t_0)$) for different time horizons T.

Although more carbon enters the ecosystem at simulation year 300 than at year 100 due to the CO_2 fertilization effect, it is lost much faster because of higher temperatures that result in faster transit times for simulation times above 300 years (Fig. 9a). The slower transit times experienced by the carbon that enters at year 100 due to lower temperature result then in much higher values of CS for time horizons $T > 100$ years (Fig. 9c). Similarly for CBS, where differences are evident much earlier, lower temperatures lead to higher values of CBS for time horizons $T > 50$ years (Fig. 9d).

This simple example highlights the importance of time-dependent transit times in determining CS and CBS. If changes in climate lead to faster carbon processing rates, we would thus expect carbon to transit faster through the ecosystem, returning faster to the atmosphere, and therefore with lower values for carbon sequestration and its climate benefit.

4.2 Continuous inputs into a changing environment

In the previous example, we considered the case of two single pulses entering the ecosystem at different times under changing environmental conditions during a simulation. A consolidated view can be obtained by taking all single pulses and integrating them continuously in time to compute CS and CBS using Eqs. (24) and (28), respectively. In this case, CS increases monotonically, and CBS decreases monotonically with time horizon (Fig. 10, continuous black lines), which is somewhat obvious because as the ecosystem accumulates carbon, more of it is retained in the ecosystem and is isolated from atmospheric radiative effects. However, this simulation only considers carbon that enters the ecosystem from the beginning of the simulation until the end of the time horizon, from t_0 to $t_0 + T$. An important aspect to consider is the role of carbon already present in the ecosystem at t_0.

We will consider now the case of continuous sequestration and release of carbon with differences in the initial conditions in the simulation, which can vary according to land use changes. For example, when changing land use from agriculture to forest, or from natural forest to plantation, there are carbon legacies that have an influence on future carbon trajectories (Harmon et al., 1990; Janisch and Harmon, 2002; Sierra et al., 2012). These carbon legacies are usually dead biomass and detritus, which cause ecosystems to lose carbon via decomposition before photosynthesis from new biomass compensates for the losses. In these initial stages of recovery, ecosystems are usually net carbon sources, but they still may store more carbon than an ecosystem developing from bare ground.

The CS and CBS concepts can be very useful to compare contrasting trajectories of ecosystem development and assess their role in terms of carbon sequestration alone and their climate impact. For this purpose, we performed an additional simulation in which at the starting time there is no living biomass, but the detritus pools and the SOM pools are 1.5 and 1.0 times as large as in the equilibrium case, re-

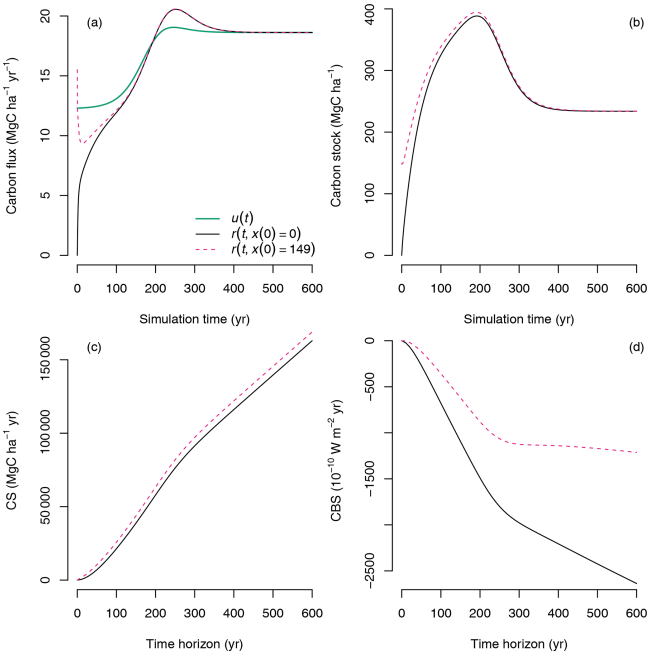

Figure 10. Computation of CS and CBS for continuous inputs and release of carbon in simulations with different initial conditions $\boldsymbol{x}_0$: in one simulation the ecosystem develops from empty pools ($\boldsymbol{x}(0) = \boldsymbol{0}$, i.e., bare ground, black lines), and in the second simulation the ecosystem develops from existing litter and SOM pools but empty biomass pools ($\|\boldsymbol{x}(0)\| = 149.04\,\mathrm{Mg\,C\,ha^{-1}}$, dashed magenta color lines). **(a)** Inputs $\boldsymbol{u}(t)$ and release fluxes $r(t)$ along the simulation time. **(b)** Carbon stocks predicted by the model along the simulation time. **(c)** Carbon sequestration CS for a sequence of time horizons. **(d)** Climate benefit of sequestration CBS for a sequence of time horizons.

spectively ($\|\boldsymbol{x}(0)\| = 149.04\,\mathrm{Mg\,C\,ha^{-1}}$). In this simulation, the ecosystem losses a significant amount of carbon in the early stages of development, and respiration is much larger than primary production ($r(t) > \|\boldsymbol{u}(t)\|$) (Fig. 10a, dashed magenta line). Because soils are already close to an equilibrium value, the ecosystem already has a large amount of carbon stored; therefore in the computation of the fate of carbon $M_s(t, t_0)$ there is already a larger amount of carbon to consider, which causes CS to be larger for the land-use-change case than for the bare ground case (Fig. 10c). On the contrary, because there are more emissions from the ecosystem in early development stages, CBS is lower for the land-use-change case than for the bare ground case (Fig. 10d).

These contrasting results between CS and CBS for the continuous case with contrasting initial conditions can be very useful to address debates and controversies about the role of land use change and baselines in carbon accounting. The results show that carbon sequestration can still be high in ecosystems where emission fluxes are large, but climate impacts can differ significantly. By using two different metrics, these two different aspects of carbon sequestration can be discussed separately.

5 Discussion

The metrics introduced here, carbon sequestration (CS) and the climate benefit of sequestration (CBS), integrate both the amount of carbon entering an ecosystem and the time it is stored there, thus avoiding radiative effects in the atmosphere. Disproportionate attention is given to quantifying sources and sinks of carbon in ongoing debates about the role of ecosystems in climate change mitigation, with much less attention paid to the fate of carbon once it enters an ecosystem. The time carbon remains in an ecosystem, encapsulated in the concept of transit time, is critical for climate change mitigation because during this time carbon is removed from radiative effects in the atmosphere.

The CS and CBS concepts unify atmospheric and ecosystem approaches to quantifying the greenhouse effect. The CBS concept builds on that of the absolute global warming potential (AGWP) of a greenhouse gas. The main difference is that CBS quantifies avoided warming during the time carbon is stored in an ecosystem, while AGWP quantifies potential warming when the carbon enters the atmosphere. Both metrics rely on the quantification of the fate of carbon (or other GHGs for AGWP) once it enters the particular system. For atmospheric systems, a significant amount of work has been done in determining the fate of GHGs once they enter the atmosphere after emissions (e.g., Rodhe, 1990; O'Neill et al., 1994; Prather, 1996; Archer et al., 2009; Joos et al., 2013). For terrestrial ecosystems; however, robust methods to quantify the fate of carbon as it flows through terrestrial system components have been developed only recently (Rasmussen et al., 2016; Metzler and Sierra, 2018; Metzler et al., 2018).

Global warming potential (GWP), or the climate impact of an emission of a certain gas in relation to the impact of an emission of CO_2, is often used to assess climate impacts of actions, e.g., avoided deforestation, land use change, and even enhanced carbon sequestration. However, this metric has two limitations when applied to carbon sequestration and in comparison to the combined use of CBS and AGWP we advocate here: (1) it only quantifies the climate effects of emissions but not of sequestration and treats all fixed carbon equally independent of its transit time in the ecosystem and (2) it is a relative measure with respect to the emission of CO_2. GWPs are commonly reported in units of CO_2 equivalents, which only address indirectly the effect of a gas in producing warming. In contrast, CBS quantifies the effects of avoided warming in units of $\mathrm{W\,m^{-2}}$ over the period of time carbon is retained.

Other concepts have been proposed in the past to account for the temporary nature of carbon sequestration (see review by Brandão et al., 2013, and references therein), with special

interest in accounting for credits in carbon markets. In fact, "ton-year" accounting methods (Noble et al., 2000) resemble our definition of carbon sequestration; however, none of these previous concepts explicitly considers the time carbon is retained in the ecosystem. Instead, these approaches relate carbon sequestration to delay in fossil fuel emissions (Fearnside et al., 2000), or as the equivalence of the amount of carbon storage to AGWP (Moura Costa and Wilson, 2000). The concepts of sustained global warming potential (SGWP) and sustained global cooling potential (SGCP) proposed by Neubauer and Megonigal (2015) are notable exceptions. The CBS concept captures some of the ideas of the SGCP concept but differs in some fundamental assumptions related to the interpretation of the impulse response functions, the treatment of time-dependent fluxes and rates, and reporting. While SGCP reports values in reference to CO_2 as is commonly done for GWP, we report CBS for individual gases as it is done for AGWP. Appendix A elaborates on other aspects of the SGWP and SGCP concepts.

The concept of CBS improves our ability to address some of the existing debates about the role of ecosystems in mitigating climate change and enhances our potential to provide decision support. In combination with quantifications of AGWP, CBS provides the net climate effect of an ecosystem or some management. For example, CBS can be used to better understand the climate impacts of storing carbon in long-term reservoirs such as soils and wood products, as well as the climate benefits of increasing the transit time in these systems. CBS can be used to better quantify the climate benefits of using biofuels as fossil fuel substitution by computing the CBS of the whole bioenergy production system and adding the negative AGWP attributed to the avoided emission. Similarly, it can be incorporated in assessments of sequestration in industrial systems with associated carbon capture and storage.

Carbon management of ecosystems can maximize CS and/or CBS by not only increasing carbon inputs, but also by increasing the transit time of carbon. There are many ways in which the transit time of carbon can be increased – for instance, by increasing transfers of carbon to slow cycling pools such as the case of increasing wood harvest allocation to long-duration products (Schulze et al., 2019), or addition of biochar to soils, or by reducing cycling rates of organic matter such as the case of soil flipping (Schiedung et al., 2019). Independently of the management activity, CS and CBS can be powerful metrics to quantify their climate benefits, make comparisons among them, and compare against baselines or no-management scenarios.

The examples we provided in this paper illustrate the use and interpretation of CS and CBS metrics under the assumptions of linearity, steady state, or time dependencies in carbon cycle dynamics with subsequent consequences for carbon sequestration and its climate benefits. The computation of the CBS relies on a model, which can be as simple as a one-pool model or a state-of-the-science land surface model. The TECO model is an excellent tool to illustrate ecosystem-level concepts because of its simplicity and tractability, but other models with more accurate parameterizations and including more processes should be considered for practical applications. The formulas and formal theory developed in Sect. 2 are general enough to deal with the non-steady-state case as well as with models with nonlinear interactions among state variables. In Sierra (2020), we provide an example in the form of a Jupyter Notebook to compute CS and CBS for a nonlinear model (see Sect. "Executable research compendium (ERC)" for details).

The concepts of CS and CBS present improvements to the current guidelines for carbon inventories that treat all carbon removals by sinks equally (IPCC, 2006) by explicitly considering the transit time of carbon in ecosystems. Therefore, these new concepts have potential for being incorporated in revised policies for carbon accounting in the context of international climate agreements and carbon markets. CS and CBS can aid in the economic valuation of carbon by adding economic incentives to sequestration activities that retain carbon in ecosystems for longer times. In addition, the concepts can help in dealing with the issue of permanence of carbon by explicitly quantifying climate benefits of sequestration that can be compared directly with the climate impacts of emissions on a similar time horizon.

Two potential limitations to apply the concepts of CS and CBS are that they rely (1) on a model that tracks the fate of the fixed carbon and (2) on an impulse response function of CO_2 in the atmosphere. Reliable models may not be available for certain types of ecosystems or may include large uncertainties that propagate to CS and CBS estimates. Also, estimates of impulse response functions for atmospheric CO_2 seem to also have uncertainties, particularly related to the size of the emission pulse, the atmospheric background at which the pulse is applied, and the long-term behavior of the curve for timescales longer than 1000 years (Archer et al., 2009; Lashof and Ahuja, 1990; Joos et al., 2013; Millar et al., 2017). However, one advantage of the functions proposed by Joos et al. (2013) is that they are derived from coupled climate–carbon models that include multiple feedbacks. Therefore, when computing CS and CBS for small perturbations of the carbon cycle, it is not necessary to explicitly compute carbon–climate feedbacks. Also, when comparing two different systems with a CBS ratio as in Fig. (8) or a ratio CBS to AGWP (Fig. 6), uncertainties in the IRFs would tend to cancel each other out. Nevertheless, advances in our understanding of the fate of emitted CO_2 to the atmosphere will consequently derive better estimates of the climate benefits of carbon sequestration.

6 Conclusions

Analyses of carbon sequestration for climate change mitigation purposes must consider both the amount of carbon in-

puts and the transit time of carbon. Both concepts are encapsulated in the unifying concepts of carbon sequestration (CS) and climate benefit of sequestration (CBS) that we propose. Carbon management can be oriented to maximize CS and CBS, which can be achieved by managing both rates of carbon input and process rates in ecosystems. We believe the use of these metrics can help to better deal with current discussions about the role of ecosystems in mitigating climate change, provide better estimates of avoided or human-induced warming, and have the potential to be included in accounting methods for climate policy.

Appendix A: Comment on Neubauer and Megonigal (2015)

Neubauer and Megonigal (2015) proposed two metrics, the sustained global warming potential (SGWP) and the sustained global cooling potential (SGCP), to overcome issues with GWP. However, there is an important misconception in their study that we would like to address here. In particular, these authors state "... GWPs requires the implicit assumption that greenhouse gas emissions occur as a single pulse; this assumption is rarely justified in ecosystem studies". The use of pulse emissions in computing AGWP, as shown in Eq. (3), is done with the purpose of obtaining a representation of the fate of a unit of emissions under the assumption that the system is in equilibrium. This is a mathematical property of linear time-invariant dynamical systems by which an impulse response function can provide a full characterization of the dynamics of the system (Hespanha, 2009). In other words, the emission pulse is a mathematical method to obtain a description of the fate of incoming mass into the system, but it is not an assumption imposed on the system.

To use impulse response functions, it is necessary to assume that a system is in equilibrium and that all rates remain constant for all times. It is this assumption that is problematic and difficult to impose on ecosystems and not the pulse emission because it is simply a method. Therefore, we are of the opinion that the sustained-flux global warming potential metric proposed by these authors is unjustified on the argument that it removes the assumption of pulse emissions.

One interesting characteristic of the study of Neubauer and Megonigal (2015) is that it uses a model that couples an ecosystem compartment with the atmosphere, and their computation of SGWP and SGCP captures the interactions between these two reservoirs similarly as in the framework described here in Sect. 2. The SGCP is very similar in spirit to the CBS. However, their approach differs from the approach we present here in that our mathematical framework is general enough to deal with ecosystem models of any level of complexity and not restricted to a one-pool model and constant parameters and sequestration rates. Furthermore, we abstain from proposing a metric that is relative to CO_2. We are rather interested in an absolute metric that quantifies the effect of CO_2 sequestration on radiative forcing and not in equivalents to sequestration or emissions of other gases.

Appendix B: Fate and timescales of carbon in compartmental systems

Carbon cycling in the terrestrial biosphere is well characterized by a particular type of dynamical systems called *compartmental systems* (Anderson, 1983; Jacquez and Simon, 1993). These systems of differential equations generalize mass-balanced models and therefore generalize element and carbon cycling models in ecosystems (Rasmussen et al., 2016; Luo et al., 2017; Sierra et al., 2018a). In their most general form, we can write carbon cycle models as

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \dot{\boldsymbol{x}}(t) = \boldsymbol{u}(\boldsymbol{x},t) + \mathbf{B}(\boldsymbol{x},t)\,\boldsymbol{x}, \tag{B1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is a vector of ecosystem carbon pools, $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^n$ is a time-dependent vector-valued function of carbon inputs to the system, and $\mathbf{B}(\boldsymbol{x},t) \in \mathbb{R}^{n\times n}$ is a time-dependent compartmental matrix. The latter two terms can depend on the vector of states, in which case the compartmental system is considered nonlinear. In case the input vector and the compartmental matrix have fixed coefficients (no time dependencies), the system is considered autonomous, and it is considered non-autonomous otherwise (Sierra et al., 2018a). At a steady state, the autonomous linear system has the general solution $\boldsymbol{x}^* = -\mathbf{B}^{-1}\,\boldsymbol{u}$.

The probability density function (pdf) for system age of linear autonomous models at a steady state can be computed by the following expression (Metzler and Sierra, 2018)

$$f(a) = -\mathbf{1}^\intercal\,\mathbf{B}\,e^{a\cdot\mathbf{B}}\,\frac{\boldsymbol{x}^*}{\|\boldsymbol{x}^*\|}, \quad a \geq 0, \tag{B2}$$

where a is the random variable age, $\mathbf{1}^\intercal$ is the transpose of the n-dimensional vector containing ones, $e^{a\cdot\mathbf{B}}$ is the matrix exponential computed for each value of a, and $\|\boldsymbol{x}^*\|$ is the sum of the stocks of all pools at a steady state.

The mean, i.e., the expected value, of the age pdf can be computed by the expression

$$\mathbb{E}(a) = -\mathbf{1}^\intercal\,\mathbf{B}^{-1}\,\frac{\boldsymbol{x}^*}{\|\boldsymbol{x}^*\|} = \frac{\|\mathbf{B}^{-1}\,\boldsymbol{x}^*\|}{\|\boldsymbol{x}^*\|}. \tag{B3}$$

The pdf of the transit time variable τ for linear autonomous systems in equilibrium is given by (Metzler and Sierra, 2018)

$$f(\tau) = -\mathbf{1}^\intercal\,\mathbf{B}\,e^{\tau\cdot\mathbf{B}}\,\frac{\boldsymbol{u}}{\|\boldsymbol{u}\|}, \quad \tau \geq 0 \tag{B4}$$

and the mean transit time by

$$\mathbb{E}(\tau) = -\mathbf{1}^\intercal\,\mathbf{B}^{-1}\,\frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} = \frac{\|\boldsymbol{x}^*\|}{\|\boldsymbol{u}\|}. \tag{B5}$$

For the most general case of nonlinear non-autonomous systems, we follow the approach described in Metzler et al. (2018). For these systems, the age distribution of mass is given by

$$\begin{array}{l}\text{Mass in the}\\ \text{system at}\\ \text{time } t \text{ with}\\ \text{age } a\end{array} = \begin{cases}\boldsymbol{\Phi}(t,t-a)\,\boldsymbol{u}(t-a), & a < t-t_0,\\ \boldsymbol{\Phi}(t,t_0)\,\boldsymbol{f}^0(a-(t-t_0)), & a \geq t-t_0,\end{cases}$$

where $\boldsymbol{\Phi}$ is a state-transition matrix, and $\boldsymbol{f}^0$ is an initial age density distribution at initial time t_0. We obtain $\boldsymbol{\Phi}$ by taking advantage of an existing numerical solution $\boldsymbol{x}(t)$, which

we plug into the original system, obtaining a new compartmental matrix $\tilde{\mathbf{B}}(t) := \mathbf{B}(\boldsymbol{x}(t), t)$ and a new input vector $\tilde{\boldsymbol{u}} := \boldsymbol{u}(\boldsymbol{x}(t), t)$. Then, the new linear non-autonomous compartmental system,

$$\dot{\boldsymbol{y}}(t) = \tilde{\mathbf{B}}(t)\,\boldsymbol{y}(t) + \tilde{\boldsymbol{u}}(t), \quad t > t_0, \tag{B6}$$

has the unique solution $\boldsymbol{y}(t) = \boldsymbol{x}(t)$, which emerges from the fact that both systems are identical. The solution of the system is then given by

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}(t, t_0)\,\boldsymbol{x}^0 + \int_{t_0}^{t} \boldsymbol{\Phi}(t, s)\,\boldsymbol{u}(s)\,\mathrm{d}s, \tag{B7}$$

where $\boldsymbol{x}^0 = \int_0^\infty \boldsymbol{f}^0(a)\,\mathrm{d}a$ is the initial vector of carbon stocks. We obtain the state-transition matrix as the solution of the following matrix differential equation:

$$\frac{\boldsymbol{\Phi}(t, t_0)}{\mathrm{d}t} = \mathbf{B}(t)\,\boldsymbol{\Phi}(t, t_0), \quad t > t_0, \tag{B8}$$

with initial condition

$$\boldsymbol{\Phi}(t_0, t_0) = \mathbf{I}, \tag{B9}$$

where $\mathbf{I} \in \mathbb{R}^{n \times n}$ is the identity matrix. For the special case in which the time-dependent matrix can be expressed as a product between a time-dependent scalar factor $\xi(t)$ and a constant value matrix $\mathbf{B}$, i.e., $\mathbf{B}(t) = \xi(t)\mathbf{B}$, we obtain the state-transition matrix as

$$\boldsymbol{\Phi}(t, t_0) = \exp\left(\int_{t_0}^{t} \xi(\tau)\mathrm{d}\tau \cdot \mathbf{B}\right). \tag{B10}$$

These formulas can be applied to any carbon cycle model represented as a compartmental system to obtain the fate of carbon once it enters the ecosystem as well as timescale metrics such as age and transit time distributions.

Computation of the mass remaining in the system

From Eq. (B7), we can see from the first term that the initial amount of carbon in the system $\boldsymbol{x}^0$ changes over time according to the term $\boldsymbol{\Phi}(t, t_0)\,\boldsymbol{x}^0$. Rasmussen et al. (2016) showed that, under certain circumstances, Eq. (B7) is exponentially stable as long as $\mathbf{B}$ is invertible, and the state-transition operator acts as a term that exponentially "decomposes" the initial amount of carbon. Furthermore, the state-transition operator tracks the dynamics of the incoming carbon and how it is transferred among the different pools before it is respired. Therefore, this operator can be used to compute the fate of an amount of carbon sequestered at time t_s as

$$M_s(t - t_s) = M_s(a) = \|\boldsymbol{\Phi}(t, t_s)\,\boldsymbol{u}(t_s)\|, \quad a = t - t_s. \tag{B11}$$

Similarly, the fate of one unit of sequestered carbon at time t_s can be computed as

$$M_{s1}(a) = \left\| \boldsymbol{\Phi}(t, t_s) \cdot \frac{\boldsymbol{u}(t_s)}{\|\boldsymbol{u}(t_s)\|} \right\|, \tag{B12}$$

where the subscript 1 denotes that the function predicts the fate of one unit of carbon.

Appendix C: Detailed representation of the TECO model and the transient simulations used in examples

The terrestrial ecosystem model (TECO) described in Weng and Luo (2011) and Luo et al. (2012) has eight pools to simulate ecosystem-level carbon dynamics, with a parameterization for the Duke Forest, a temperate forest in North Carolina, USA. The annual amount of photosynthetically fixed carbon predicted by the model in this forest (GPP) is $U = 12.3\,\mathrm{Mg\,C\,ha^{-1}\,yr^{-1}}$. The vector of carbon allocation is given by

$$\boldsymbol{b} = \begin{pmatrix} 0.14 \\ 0.26 \\ 0.14 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

which shows that from all photosynthetically fixed carbon, 14 % is allocated to foliage, 26 % to woody biomass, and 14 % to roots.

Each pool in the model cycles at annual rates given by the diagonal elements of the matrix

$$\mathbf{C} = \begin{pmatrix} 0.942 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.021 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.872 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 3.978 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.347 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 3.833 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.036 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.004 \end{pmatrix},$$

with a matrix of transfer coefficients as

$$\mathbf{A} = \begin{pmatrix} -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.82 & 0.00 & 0.12 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.85 & 0.72 & 0.00 & -1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.45 & 0.28 & -1.00 & 0.42 & 0.45 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.28 & 0.30 & -1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & -1.00 \end{pmatrix}.$$

Matrix $\mathbf{A}$ was modified from the original publication (Luo et al., 2012) by decreasing the proportion that is transferred from vegetation to litter pools in a proportion consistent with the proportions of carbon that are respired by autotrophic respiration. In other words, autotrophic respiration is not computed here as in the original publication where the inputs enter the ecosystem in the form of net primary production, i.e.,

$U = \mathrm{GPP} - R_\mathrm{a}$. We compute autotrophic respiration as the proportion that leaves the autotrophic pools and that is not transferred to the litter pools. In this way, $U = \mathrm{GPP}$, and all carbon that is fixed enters the vegetation pools from where it is subsequently respired or added to the litter pools.

Defining $\mathbf{B} := \mathbf{AC}$ and $\boldsymbol{u} = \boldsymbol{b}U$, we obtained the steady-state solution as

$$\boldsymbol{x}^* = -\mathbf{B}^{-1} \cdot \boldsymbol{u} = \begin{pmatrix} 3.83 \\ 237.70 \\ 4.14 \\ 0.86 \\ 20.18 \\ 1.28 \\ 92.96 \\ 12.72 \end{pmatrix}. \quad \text{(C1)}$$

For the simulation with initial conditions as in a land-use-change case, the initial conditions $\boldsymbol{x}_0$ of the simulation were set as

$$\boldsymbol{x}_0 = \begin{pmatrix} 0 & 0 & 0 & 1.5 & 1.5 & 1.0 & 1.0 & 1.0 \end{pmatrix}^\mathsf{T} \circ \boldsymbol{x}^*, \quad \text{(C2)}$$

where the symbol $\circ$ represents entry-wise multiplication.

For the transient simulations, we derived time-dependent modifiers for inputs $\gamma(t)$ and for process rates $\xi(t)$ following the approach described in Rasmussen et al. (2016). Atmospheric CO_2 concentrations increase following a sigmoid curve given by

$$x_\mathrm{a}(t) = 284 + 1715 \exp\left(\frac{0.0305t}{(1715 + \exp(0.0305t) - 1)}\right), \quad \text{(C3)}$$

and surface air temperature increases with CO_2 concentrations according to

$$T_\mathrm{s}(t) = T_{\mathrm{s}0} + \frac{\sigma}{\ln(2)} \ln(x_\mathrm{a}(t)/285). \quad \text{(C4)}$$

The combined effect of CO_2 concentrations and air surface temperature on primary production is then computed as

$$\gamma(t) = (1 + \beta(x_\mathrm{a}(t), T_\mathrm{s}(t)) \ln(x_\mathrm{a}(t)/285)), \quad \text{(C5)}$$

with

$$\beta(x_\mathrm{a}(t), T_\mathrm{s}(t)) = \frac{3\rho x_\mathrm{a}(t)\Gamma(T_\mathrm{s}(t))}{(\rho x_\mathrm{a}(t) - \Gamma(T_\mathrm{s}(t)))(\rho x_\mathrm{a}(t) + 2\Gamma(T_\mathrm{s}(t)))}, \quad \text{(C6)}$$

where $\beta(x_\mathrm{a}(t), T_\mathrm{s}(t))$ is the sensitivity of primary production with respect to atmospheric CO_2 and air surface temperature, and $\rho = 0.65$ is the ratio of intracellular CO_2 to $x_\mathrm{a}(t)$. The response function with respect to temperature $\Gamma(T_\mathrm{s}(t))$ is given by

$$\Gamma(T_\mathrm{s}(t)) = 42.7 + 1.68(T_\mathrm{s}(t) - 25) + 0.012(T_\mathrm{s}(t) - 25)^2. \quad \text{(C7)}$$

The separate effect of air surface temperatures on process rates is computed with a power function of the form

$$\xi(T_\mathrm{s}(t)) = \xi_b^{0.1T_\mathrm{s}(t) - 1.5}, \quad \text{(C8)}$$

with $\xi_b = 2$.

Executable research compendium (ERC). Code to reproduce all results has been permanently stored and can be found in Sierra (2020, https://doi.org/10.5281/zenodo.4399181). The file TECO.R contains all code to reproduce all examples in this paper. The file nonlinear_CS_CBS.ipynb is a Jupyter Notebook that contains code with an example for computing CS and CBS for a nonlinear model out of equilibrium.

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The climate benefit of sequestration in soils for warming mitigation

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Abstract Soils are an enticing reservoir for nature-based climate solutions, but long timescales are required to store amounts of C of relevance to mitigate warming acknowledging its impermanence. Scientific clarity on the controlling factors in soil C persistence should help to disambiguate debates related to permanence in the climate policy domain. However, another contributing factor that is lacking in this debate is a way to compute the climate benefits of C in terrestrial ecosystems over time in the same units as greenhouse gas emissions. We use a case study approach here to demonstrate the use of the metrics of carbon sequestration (CS) and climate benefit of sequestration (CBS) with the aim of assessing the contribution of simultaneous emissions and uptake on radiative forcing. We show how this new computational framework quantifies the climate benefit achieved in two different agricultural systems, one a managed tropical perennial grass system in Hawai'i, USA and the other a boreal (cold-temperate, semi-humid) agricultural soil from long term amendment trials in Sweden. Using a set of computations, we show how C inputs and persistence interact to produce different levels of radiative forcing at relevant time frames, which could greatly help to clarify issues of carbon permanence discussed in climate policy. Temporary soil C storage could help to decrease peak warming provided that ambitious emission reductions are part of the portfolio of solutions; the CS and CBS framework gives us a way to quantify it based on biogeochemical understanding of soil C persistence.

Keywords Carbon dioxide removal · Sustainable development goals · Climate policy · Peak warming · Carbon accounting

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Introduction

Meeting climate objectives set in the Paris Agreement requires achieving net-zero CO_2 emissions by mid-century. Carbon dioxide removal (CDR) options, including nature-based climate solutions that aim to preserve or enhance storage in terrestrial and marine systems, may be critical to achieving net-zero objectives by offsetting unavoidable non-renewable

emissions elsewhere in the global system (Rogelj et al. 2021). Soils are an enticing reservoir for nature-based CDR and mediation of greenhouse gas emissions (Lal 2013; Chabbi et al. 2017; Bossio et al. 2020). However, long timescales are required to store amounts of carbon in soils of relevance to mitigate climate change. In addition, uncertainty in the magnitude and relevant timeframe for soil carbon management remains high across ecosystems (Lu et al. 2018; Cai et al. 2022). Carbon is stored impermanently in soils, thus it is challenging to know how long new C inputs from the implementation of climate-smart practices or mitigation strategies will stay belowground and thereby provide quantifiable climate benefits (Xiao et al. 2022). An approach that can quantifies both how much and for how long C inputs will remain stored can advance the valuation of protecting or improving soil C in a climate change mitigation portfolio.

Permanence—a policy term for when credits are traded as part of a climate change mitigation project and the buyer seeks assurance that the C will remain in the system for a contracted period—is an issue that remains highly debated in policy making. In contrast, the scientific concept of persistence—an ecosystem property resulting from physicochemical and biological influences in the soil environment that cause organic matter to remain longer in soil than outside it—has been well studied in soil science and biogeochemistry (Torn et al. 1997; Schmidt et al. 2011; Sierra et al. 2018; Cotrufo and Lavallee 2022; Heckman et al. 2022). These concepts both seek to introduce the aspect of time into their frameworks to assess how much and for long C resides in soils.

The debate over permanence and related uncertainties in how long soil C remains belowground is creating a barrier to incentive programs surrounding nature-based solutions that include soils and terrestrial ecosystems (Bradford et al. 2019; Dynarski et al. 2020). These debates distract from the diversity of potential climate, environmental, and societal co-benefits to the actions that increase C drawdown into terrestrial landscapes (Smith et al. 2015; Keesstra et al. 2016; Lal et al. 2021). The balance between C inputs and outputs determines the size of the soil C reservoir (Olson 1963), with the outputs depending strongly on how fast microbes can access and consume organic matter (Schimel and Schaeffer 2012; Wieder et al. 2013). The slower their rate of consumption and release, the longer C persists in soils (Sierra et al. 2018). Scientific clarity on soil C persistence should help to disambiguate debates related to permanence in the climate policy domain. However, another contributing factor in this debate, is that there has not been a way to compute the climate benefits of C in terrestrial ecosystems over time, even when there is a mathematical model for that system, in the same units as GHG emissions are expressed in global warming potentials (GWP).

Currently, the Intergovernmental Panel for Climate Change (IPCC) use annual GHG inventory reporting as the metric of the GHG contribution of ecosystems (Intergovernmental Panel on Climate Change 2006; IPCC 2019). This approach requires reporting of GHG emissions by sources and removal by sinks, but treats all removals equally regardless of their fates over time (Sierra et al. 2021). Other policy frameworks continue to rely on measures of organic C storage and/or annual GHG flux, but do not apply an appropriate accounting mechanism for time and ignores potential effects of disturbance (Anderson-Teixeira and DeLucia 2011; Körner 2017).

Approaches to consider multiple year time frames in valuing the full GHG implication of ecosystems have been put forward, but each remain problematic. For example, the ton-year accounting methods took a first step to address the issue of temporary C storage in valuations for offset markets (Fearnside et al. 2000 and references therein), but they mostly focus on contrasting the warming effects of emissions (fluxes in units of mass per year) to static stocks in ecosystems (units of mass). This inconsistency in units remains problematic and does not reflect the potential impacts of emissions versus sequestrations on the radiative forcing effect of GHG in the atmosphere. Another example is the concept and metric of greenhouse gas value (GHGV), which accounts for storage, flux, and probable disturbance over multiyear timeframes and is sensitive to the timing of emissions (Anderson-Teixeira and DeLucia 2011). These methods effectively track the radiative forcing effects expected due to losses as emissions upon a major disturbance or land use change such as deforestation, and account for all sources from soil organic matter and burning, etc. versus maintenance of the ecosystem through protective measures. However, the metric does not allow for simulation of scenarios that include a valuation of the uptake, or sequestration, of C in soil.

Other recent work that focused on C markets and trading rather than soil processes prioritize valuing time in the contractual agreement at the expense of accurately portraying the biophysical controls on C cycling over time. For example, Leifeld and Keel (2022) oversimplify the biophysical processes that control cycling of C in the ecosystem to permanent versus impermanent soil C. Therefore, the simultaneous emission and uptake calculation presented assumes that all C gained during the contracting period (i.e., "hold time") is lost immediately after it is over, which is not representative of how ecosystems function. Nonetheless, the conclusion that non-permanent soil C sinks can make a significant contribution to cooling is appealing.

We can quantify the degree of permanence of soil C using existing frameworks of soil C stabilization and its persistence. But we need to connect the concepts of permanence and persistence to the amount of warming that is avoided while C is stored in ecosystems, including soils (Fig. 1) to achieve meaningful climate benefits.

Here, we will conceptually and computationally join the policy-oriented concept of "permanence" and biogeosciences-oriented concept of "persistence" to the amount of potential warming that is avoided while C is stored in ecosystems, including soils. The connection is made through the metrics of carbon sequestration (CS) and climate benefit of sequestration (CBS), developed with the aim of assessing the contribution of simultaneous emissions and uptake, from and to C reservoirs, on radiative forcing (Sierra et al. 2021; Sierra and Crow 2021). These metrics are consistent with the concept of global warming potential previously developed to assess the contribution of different GHGs to warming (Lashof and Ahuja 1990; Rodhe 1990). Because different gases stay for different times in the atmosphere after their emission, their contribution to warming depends on how much gas is emitted and how long it remains in the atmosphere. Similarly for the CS and CBS concepts, different ecosystems drawdown different amounts of C and retain it for different amounts of time, thus avoided warming through C sequestration in ecosystems must quantify how much C is stored and for how long. Our new insight is the explicit accounting for how much time new inputs spend in an ecosystem, grounded on process-based understanding of soil C persistence, and the resulting atmospheric response.

The CBS computational structure moves beyond current approaches to allow ecosystems to be treated with different values for C sequestration and help address the issue of permanence more explicitly. Here, we aim to clearly communicate the computation of these benefits using a case study of two agricultural systems in very different bioclimatic zones. We demonstrate how CBS could be used to plan management strategies. Then, we discuss how the computational framework can be deployed to determine whether a nature-based solution will provide meaningful climate benefits on appropriate time frames.

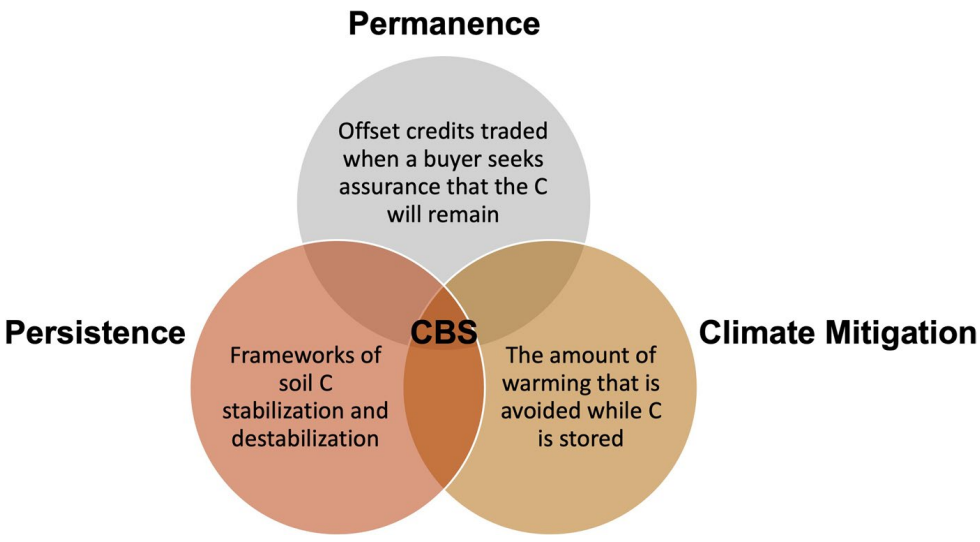

Fig. 1 The climate benefit of sequestration (CBS) metric, which quantifies the radiative effect of removing CO_2 from the atmosphere and retaining it temporarily, connects concepts of permanence, persistence, and the amount of warming that is avoided while C is stored in ecosystems, including soils

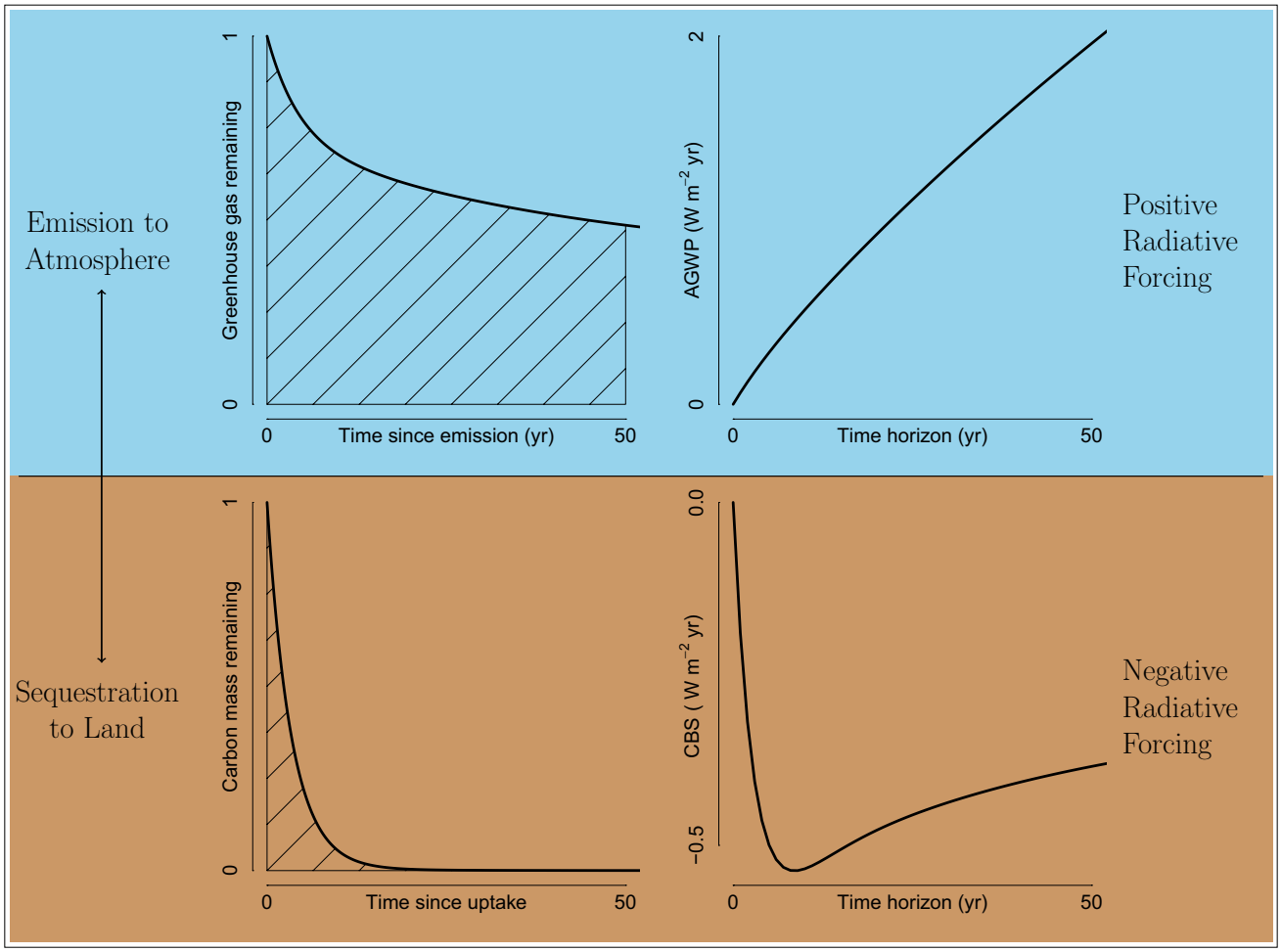

Fig. 2 Conceptual representation of absolute global warming potential (AGWP), carbon sequestration (CS) and climate benefit of sequestration (CBS). The concept of AGWP considers the fate of an emission (e.g., one year of emission is represented in this figure) of a greenhouse gas and computes the area under the curve of the amount of gas remaining after the emission occurs until a certain time horizon of interest. Then, AGWP is computed by multiplying this area under the curve by the radiative effect of the gas during the time it stays in the atmosphere. CS and CBS follow a similar approach; CS is the area under the curve of an amount of sequestered carbon (e.g., 1 year of uptake is represented in this figure) and its fate over time until a certain time horizon. CBS is computed as the radiative effect in the atmosphere of the sequestration pulse. From the point of view of the atmosphere, a sequestration pulse is a negative emission, and therefore CBS is expressed in negative numbers. Note that the units of CS are mass of C per hectare times year. This is because, as an area under a curve, it results from the multiplication of the mass per hectare and time

Computational approach summarized

To better understand the concepts of CS and CBS, it is important to review the concept of absolute global warming potential (AGWP) of an emission. For an amount of emitted C (Fig. 2, upper left), AGWP quantifies potential warming as the area under the curve of the amount of C remaining in the atmosphere for a given time horizon (Fig. 2, upper right). Many people are more familiar with the GWP of multiple greenhouse gases, presented relative to one another in CO_2 equivalents. The absolute value of these are AGWP, and the AGWP of 1 Mg CO_2-C to the atmosphere is 3.4×10^{-10} W m^{-2} year on a 100 year time horizon (Joos et al. 2013). Most CO_2 stays in the atmosphere for 300–1000 years, but some molecules stay shorter and some remain longer (Archer et al. 2009).

A similar approach can be taken to quantify the effects of CO_2 uptake on land. We quantified the area under the curve of an amount of C uptake over a given time frame since the initial uptake (Fig. 2, lower left). We defined this area under the curve as

C sequestration (or CS) because it is a metric that considers both the amount of C uptake and the time it remains stored in an ecosystem. In addition, we computed the amount of avoided warming of C uptake during the time of storage. We called this metric climate benefit of sequestration (or CBS), and it is similar to AGWP, but it considers C uptake as a negative emission that eventually returns to the atmosphere (Fig. 2, upper right). Details on the computational approach and equations are presented in detail in the Online Resource.

CS and CBS can be computed for any ecosystem over any time frame of interest. It only requires a model that describes how C is transferred and decomposed within an ecosystem, expressed in compartmental (matrix equation) form (Luo and Weng 2011; Luo et al. 2017; Sierra et al. 2018, Luo et al. 2022). The approach works regardless of whether there is simple linear model or a complex nonlinear model (Sierra and Crow 2021). AGWP and CBS can be added together to obtain the net climate effect of simultaneous emissions (which are +) and sequestration (which are −) in a particular system. The main insight of the CS metric is that it combines mass of C and the time it remains in soils, therefore directly addressing challenges of quantifying permanence. Every year that some portion of the initial input remains in the soil means that warming may be avoided as a result (and quantified by the CBS computation). The critical aspect is that the user may choose any time frame of interest and sum the areas under the curve for all uptake during that time. We now use a case study approach to demonstrate the computations.

Case study: Hawai‘i and Sweden

We explored the fate of one year's worth of new C inputs in two different agricultural systems, one a managed tropical perennial grass system in Hawai‘i, USA (Fig. 3 left) (Sumiyoshi et al. 2016; Crow et al. 2018; Crow and Sierra 2018) and the other a boreal (cold-temperate, semi-humid) agricultural soil from long term amendment trials in Sweden (Fig. 3 right) (Andrén and Kätterer 1997; Crow et al. 2018). For both sites, simple two or three-pool mathematical models for soil C were previously developed (Andrén and Kätterer 1997; Crow et al. 2018) but a more complex ecosystem model such as CLM or Daycent may also be adapted into matrix forms of the equations if available (e.g., Huang et al. 2018). In a series of experimental sets, we track an annual pulse of new inputs into the case study systems at steady state to effectively demonstrate how long fresh C remains in the different soils, how much warming it avoids while stored, and how this compares to warming produced by emissions of fossil fuels over the same time frame.

CS as a metric computed from any compartmental model, regardless of complexity, is the storage of a certain amount of C input over a time period as it flows through an ecosystem (see Online Resource for the model parameters and equations). The areas

Fig. 3 Experimental tropical managed perennial grass system on a Mollisol soil in Hawai‘i, USA (Crow et al. 2018) (left, photo credit Susan Crow) boreal agricultural Cambisol soils from long term amendment trials in Sweden (Andrén and Kätterer 1997) (right, photo credit Jenny Svennås-Gillner/SLU).

under the curves until a specific time horizon are used to compare how much C has remained until a certain time (e.g., a contracting period). First, we considered the fate of the same amount of input (1 Mg C ha^{-1}) in a tropical perennial grass system on a Mollisol in Hawai'i versus an arable Cambisol in Sweden. Considering one unit of input allows us to focus on differences in C cycling between the soils independent of productivity of the sites. On a 5-year time horizon, more C remained from 1 Mg of C input in Hawai'i than in Sweden, therefore CS was higher for the tropical Mollisol (Fig. 4, top). However, at a 20-year time horizon CS was higher for the Swedish Cambisol. Although one unit of C decomposed relatively fast first in the Swedish soil, because of differences in the processes that control

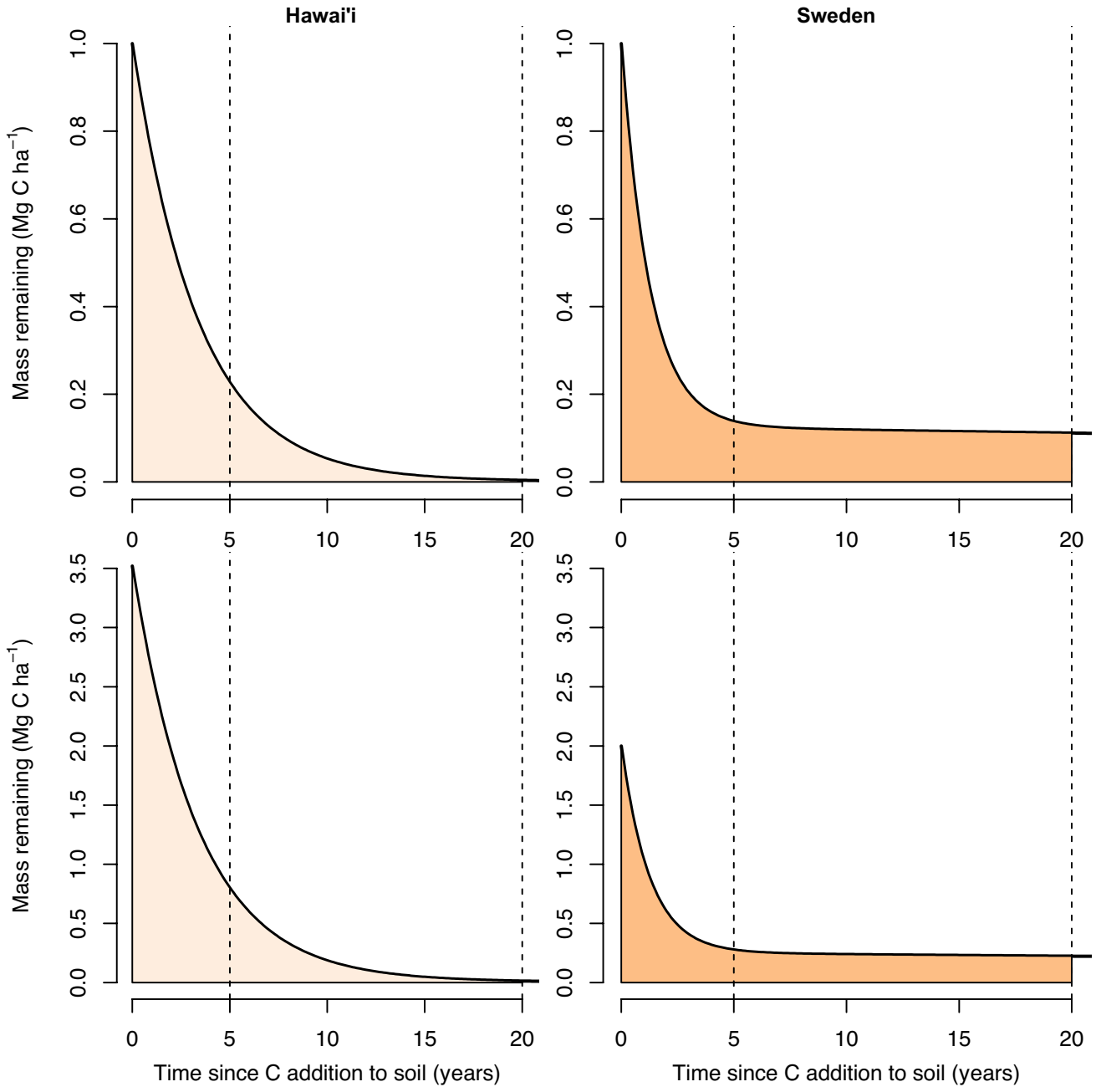

Fig. 4 Carbon sequestration (CS) of one unit of C input (top) or one year of productivity (bottom) over time as it flows through managed ecosystems in Hawai'i (left) and Sweden (right)

organic matter dynamics of the systems, more C remained after 20 years in comparison with the Hawai‘i soil.

Then, the differences in plant productivity and C input between the two sites were compared together with the inherent difference in C cycling between the soils. The more productive tropical perennial grasses (annual inputs of 3.5 Mg C ha^{-1} $year^{-1}$) (Crow et al. 2018) had a larger CS for all time horizons compared to the less productive Swedish cropland (annual inputs of 2.0 Mg C ha^{-1} $year^{-1}$) (Andrén and Kätterer 1997). Because soil from Hawai‘i has higher inputs (from higher crop productivity), the areas under the curve were higher for Hawai‘i at all time horizons shown here. At the 20-year time horizon, CS was 11.9 Mg C ha^{-1} year for the tropical Mollisol, which was more than in the Swedish Cambisol (6.9 Mg C ha^{-1} year). These values are the sum of all the mass remaining in the pools from 1 year pulse integrated over the 20-year time frame. Each year that the C remains in the soil is a year where the potential radiative effects are mitigated, therefore these values increase monotonically with increases in time horizon. In a real soil situation, each annual pulse would be integrated to calculate the stored C over time, thereby providing cumulative potential climate benefit (see example below).

The soil in Hawai‘i had almost twice as much CS from one year of inputs on a 20-year timeframe in comparison with the Swedish soil. Notice that the units of CS are mass of C per hectare times year. This is because, as an area under a curve, it results from the multiplication of the mass per hectare and time. Therefore, CS tells us about the amount of C stored over a time period, but it tells nothing about the greenhouse effect the C avoids while stored in soil.

The next step of the computation is the CBS, i.e., the radiative forcing effect avoided by C inputs to the soil stored over a period of time. Because most C that enters the soil returns to the atmosphere as heterotrophic respiration, CBS accounts for the temporary effect of storing C that enters at a particular time and returns to the atmosphere over a time horizon. We demonstrate the utility of this metric by using the productivity-based computation to compare the potential amount of avoided warming between the two soils for different time horizons. Values are negative because the system is pulling CO_2 out of the atmosphere and the more negative the higher the avoided warming and greater the climate benefit.

Our computations revealed that for our case study systems, the tropical soil had a larger climate benefit (or, more negative CBS) on short time horizons under about a decade (Fig. 5, top). On a 20-year time frame, the climate benefit starts to decline. Because inputs are larger in the tropical Mollisol, more warming is avoided for time horizons below ~40 years. But, beyond this time point, the temperate soil has a larger climate benefit. Because a larger proportion (albeit a small amount) of the original input stays in the Swedish Cambisol for a longer time than the tropical Mollisol, CBS is greater in the Cambisol for time horizons longer than 40 years. It is important to note that—as this example is just tracking one pulse of inputs—these curves all go back up to zero eventually. In reality, each year gets a pulse, and it gets summed up over time.

As a next step, we can now make the direct comparison between the radiative forcing effects of

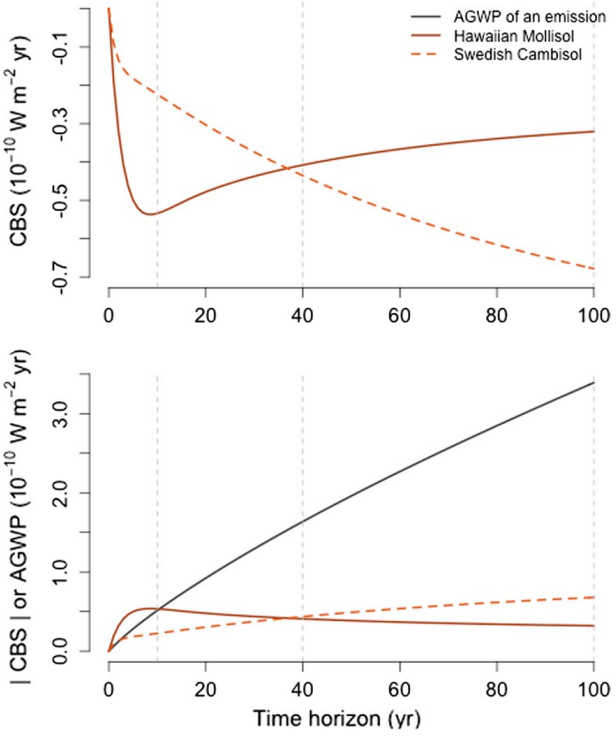

Fig. 5 Climate benefit of sequestration (CBS) of one year of productivity (top) over time as it flows through managed ecosystems in Hawai‘i (dark, solid line) and Sweden (light, dotted line). The absolute value of CBS over time compared directly to the absolute global warming potential (AGWP) of one Mg C of CO_2 emission (lower)

emissions and uptake in our case study soils. Generally speaking, the AGWP of CO_2 is much larger for 1 unit of emissions than for one unit of uptake for any timescale because fossil fuel emissions stay for much longer in the atmosphere–biosphere–ocean system (Sierra et al. 2021). The emission of one Mg C to the atmosphere leads to 3.4×10^{-10} W m^{-2} year on a 100 year time horizon; one order of magnitude higher than the potential warming avoided in either soil (Fig. 5, bottom).

The case study demonstrated that new C inputs to the soil do not remain for long timescales, and only small proportions are stabilized to provide warming mitigation using agricultural soils from Hawai'i, USA and Sweden as examples. Until now, there hasn't been a way to compute the climate benefits of C in terrestrial ecosystems, even when there is a model for that system, in the same units as GHG's emissions are expressed in global warming potentials (GWP). This series of experimental sets demonstrate how AGWP and CBS can be added together to obtain the net climate effect of simultaneous emissions and sequestration in a particular ecosystem and assists with fundamental policy-oriented questions surrounding permanence and soil C solutions (Table 1).

Computational exercise to demonstrate informed management options

The amount of input is affected by land use and management changes such as deforestation, afforestation, conversion of pasture to conservation, removal of crop residues, etc. The amount of time C inputs remain in the system is also affected by management choices that influence persistence such as site selection for climate/environmental factors or soil mineralogy, application of soil amendment such as biochar. Management decisions that factor in both inputs and persistence can maximize climate change mitigation potential, to the point that the warming benefits of a land-based action can be equal to or greater than emissions avoidance elsewhere. This way, one can select the most promising management techniques to enhance soil carbon at the same level of tackling the paramount issue of reducing fossil fuel combustion (Schlesinger and Amundson 2019).

Table 1 Policy-oriented questions concerning permanence using the CS and CBS computations in our case study soils in Hawai'i and Sweden

Question	Metric (unit)	Hawai'i	Sweden
How long do new C inputs stay in the soil on average?	Transit time (mean, year)	3.41	21.9
How long does half of the C in new inputs stay in the soil?	Transit time (median, year)	2.33	1.06
How much of one unit of C (1 Mg C ha^{-1}) remains in soil after 5 years?	Mass remaining (proportion)	0.23	0.14
How much of one unit of C remains in soil after 20 years?	Mass remaining (proportion)	0.004	0.11
What is the amount of one unit of C stored over 5 years?	CS-1 unit (Mg C ha^{-1} year)	2.60	1.69
What is the amount of one unit of C stored over 20 years?	CS-1 unit (Mg C ha^{-1} year)	3.38	3.47
What is the amount of ecosystem C inputs stored over 5 years?	CS-productivity (Mg C ha^{-1} year)	9.17	3.39
What is the amount of ecosystem C inputs stored over 20 years?	CS-productivity (Mg C ha^{-1} year)	11.9	6.93
What is the amount of warming mitigated by soil C storage in the ecosystem after 20 years?	CBS-productivity (absolute value, W m^{-2} year)	4.78×10^{-11}	3.03×10^{-11}
What is the amount of warming mitigated by soil C storage in the ecosystem after 40 years?	CBS-productivity (absolute value, W m^{-2} year)	4.08×10^{-11}	4.36×10^{-11}
What is the amount of warming mitigated by soil C storage in the ecosystem after 100 years?	CBS-productivity (absolute value, W m^{-2} year)	3.21×10^{-11}	6.78×10^{-11}

To consider whether we could potentially manage these soils to achieve values of CBS at least as large as AWGP of a unit of emissions, we performed a simple simulation experiment by modifying the decomposition rates of the two pools in the underlying compartmental models. For both soils, CBS was almost insensitive to changes in the decomposition rate of the slow pools because most of the C is lost early after it is added to the soil. Therefore, slowing down decomposition (i.e., it takes longer for the recent inputs to be processed by microorganisms) of the very small proportions of C inputs that can remain for longer time horizons makes no difference in terms of avoided warming. However, we observed important effects of modifying the decomposition rate of the fast pools, which is equivalent to slowing down decomposition of the fresh material before it is lost. In this case, we observed large avoided-warming potentials by slowing the decomposition rate of the fast pools (Fig. 6).

This analysis showed that slowing down the decomposition rate in the fast pool by about a decade in the Hawaiian case study soil may avoid a warming effect larger than the warming effect that could be generated by an emission over the course of a century. In both the Hawaiian and Swedish soils, when decomposition of the fast pool is slowed by a 100th of their original values (C remains on a century timescales), CBS is much larger than AGWP and the avoided warming of the NPP inputs to the soil is much larger than the warming produced by the emission of a ton of CO_2 at all time scales. This shows that managing soil C can be very effective to mitigate the effect of emissions, but efforts should concentrate on avoiding the quick losses from the decomposition of the fast pools. This implies that more C stays for much longer. However, care must be taken in implementing some types of management that may have other unintended impacts. For example, if decomposition of OM from the fast pools is slowed down, there would be less microbial activity and nutrient mineralization, which can negatively impact plant growth in nutrient limited ecosystems.

In this series of experiment sets and computations, the existing C stock was excluded because we focused on the fate of new C inputs for simplicity. Our aim was to provide a rigorous definition of C sequestration: the act of taking CO_2 from the atmosphere and keeping it in an ecosystem or a soil for a defined period of time. It is important to have this definition

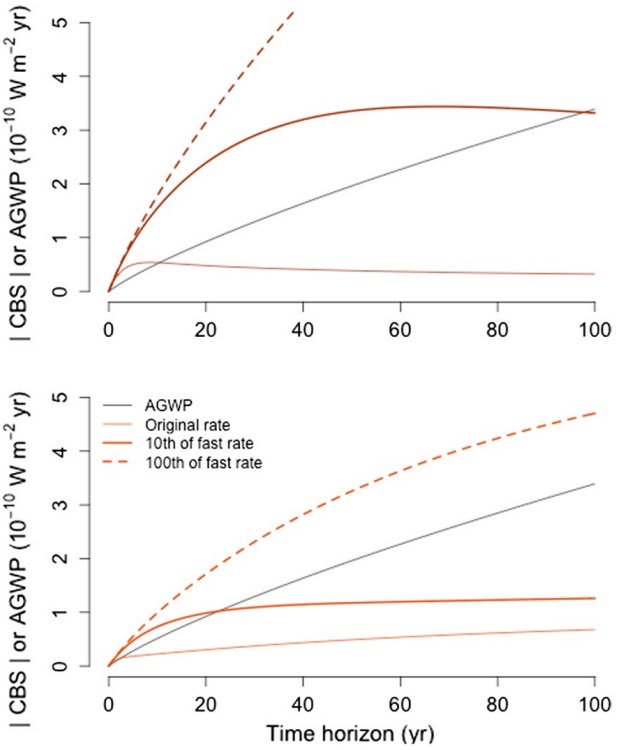

Fig. 6 For the Hawaiian (top) and Swedish (bottom) soils, AGWP (black line) and the absolute value of CBS (light line) obtained from the original productivity-based models. The thick lines represent the simulation in which the fast pool was slowed down by a 10th of its original value, i.e., the decomposition rate of the fast pool multiplied by 0.1, which is equivalent to retarding decomposition by about a decade. The dashed lines are the CBS obtained by multiplying the decomposition rate of the fast pool by 0.01, or, slowing down decomposition of this pool by a century

because previous approaches did not take into consideration the time new C is stored in an ecosystem and instead focused mostly on quantifying the effects of emissions from ecosystems. In this sense, these previous approaches provide an appropriate framework to quantify the effect of emissions of existing carbon stocks. For example, to quantify the value of conserving standing C stocks in ecosystems, an avoided emissions framework provides the best approach to quantify the effect of decomposition of existing carbon. The framework provided by Anderson-Teixeira and DeLucia (2011) is an example of an approach that is perfectly suitable for this application. However, this framework does not account for the fate of new inputs and for how long they stay in an ecosystem being restored or put into an improved practice. The CBS concept (not CS) as defined here can address the

simultaneous effects of emissions and sequestration. One can add the effect of emissions from standing C stocks and balance it with the effect of new inputs, and then quantify the atmospheric response in terms of radiative forcing. The existing soil C maybe added into the computational structure if desired. Mostly this change would increase the amount of C in respiration going back to the atmosphere; however, the amount respired from existing soil pools is small relative to losses from the new inputs, particularly from autotrophic respiration. An example of this case could be found in Sierra et al. (2021).

Exploring changes in steady state conditions allows us to understand the dynamics of soil C including transit time of new inputs and how that affects total C storage. Critically this is the starting point for assessing how a system might change with disturbance or land use/management change. This starting point is dependent on the system of interest and any potential management strategy under consideration. It is also important to establish the analog steady state in an undisturbed or restored system to understand the potential gains/benefits if an implementation is undertaken.

Transient, non-steady states in between the current and desired outcome are also important. The CS and CBS computational framework allows running dynamic simulations to better understand how long it may take and along what trajectory the system will follow to reach a desired, improved state. Implementation contracts will require this transient state computational prediction to know how much climate benefit will be achieved because of the contractual action. For example, assume you have a degraded agricultural system where you want to change from conventional tillage to zero-tillage ratoon harvest management (e.g., Crow et al. 2020). If a 20-year contract is desired, you would need to understand how much of the new C inputs will stay as a result of the alternative management system.

For non-steady-state cases with transient accumulation of C, the approach to computing CS and CBS is to consider a series of individual pulses (Fig. 7). The areas under the curve of each pulse accumulate the amount of C and the time it is retained in an ecosystem, providing a comprehensive quantification of CS that can reveal important differences between ecosystems or management strategies. In our case study, we can see how the individual pulses for the soil in Hawai‘i tend to reach a maximum faster than the soil in Sweden, mostly due to the differences in inputs and soil processes controlling decomposition and C cycling between the two soils. For time horizons of 20, 40, and 100 years, CS for the soil in Hawai‘i would be 203.3, 442.0, and 1160.8 Mg C ha^{-1} year, respectively. For the soil in Sweden, CS under continuous inputs would be 92.7, 276.3, and 1265.9 Mg C ha^{-1} year for time horizons of 20, 40, and 100 years, respectively. Notice that in the short term, CS is higher during the first 40 years in Hawai‘i while at longer time horizons CS is higher in Sweden.

A similar non-steady-state computation can be performed for CBS. For time horizons of 20, 40, and 100 years, CBS in Hawai‘i was estimated as -0.9, -1.8, and -3.9×10^{-9} W m^{-2} year, respectively. For the soil in Sweden, CBS was estimated as -0.4, -1.2, -4.6×10^{-9} W m^{-2} year respectively. Again, the climate mitigation potential for the soil in Hawai‘i is more important in the short term than for the soil in Sweden, but the roles reverse for longer time horizons (> 80 year).

Our case study shows how CDR and storage in soils is amenable to timeframes suitable for contracting periods of implementation (permanence) and represents the biophysical soil processes controlling decomposition and stabilization (persistance) of C inputs to the ecosystem. In Hawai‘i, a short contract period (e.g., 20 years) achieves maximum climate benefits, while longer periods do not have a marginal increase in benefits. In Sweden, longer contracting terms are required (~ 80 year) to achieve equal climate benefits in soil than in Hawai‘i.

These examples show how CS and CBS integrate biogeochemical understanding of soil carbon persistence with the policy-related concept of permanence. By selecting specific time horizons where known amounts of inputs stay for a known amount of time, less ambiguous contractual agreements can be developed in carbon trading markets. In particular, CBS can more directly estimate the temporary nature of C storage in natural reservoirs and can be contrasted with warming potential of emissions. It builds on the well-established framework of global warming potentials and allows comparisons of different management strategies in different ecosystems with different levels of productivity and soil carbon persistence.

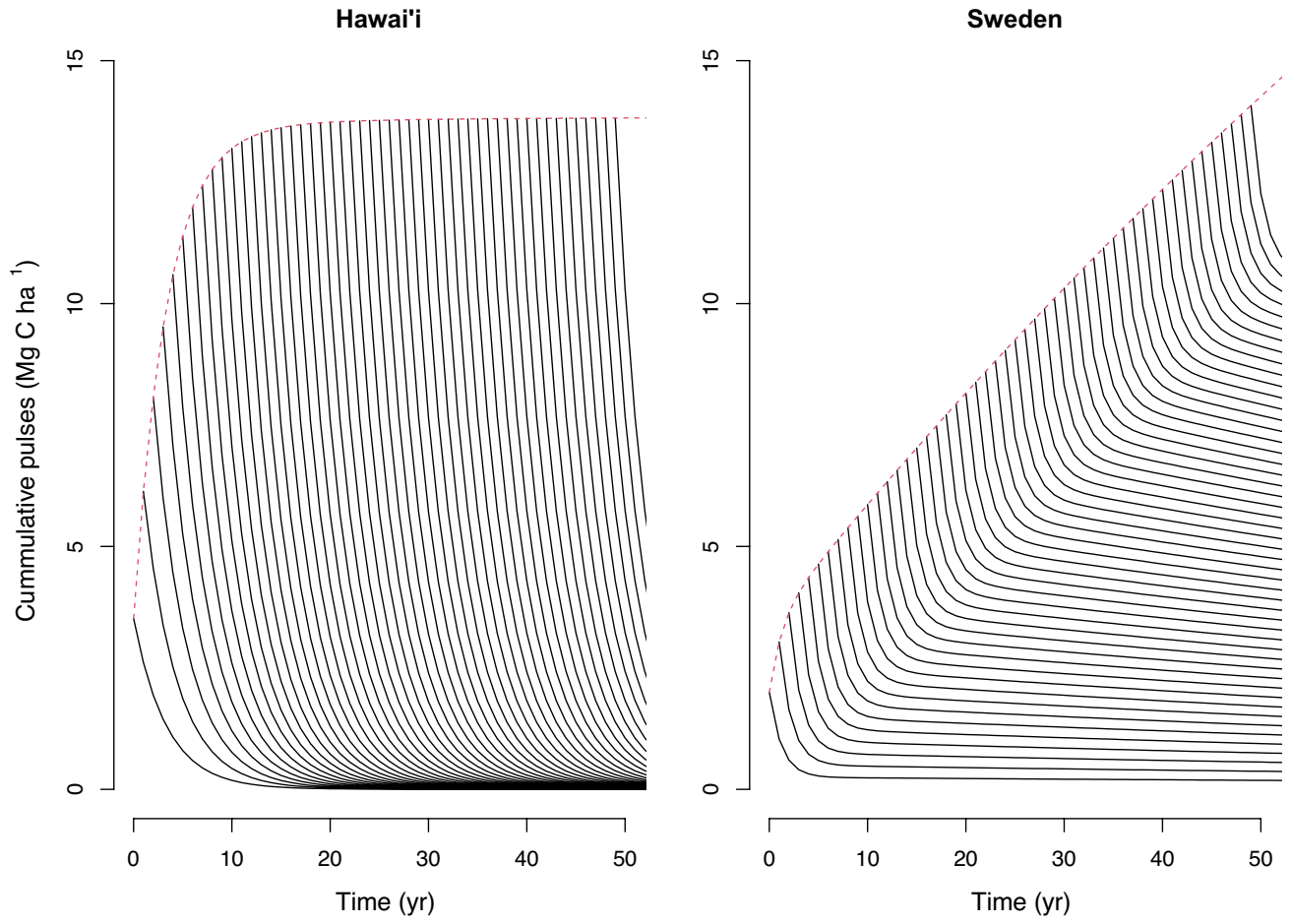

Fig. 7 Example of a trajectory of accumulated annual pulses for the two study case soils. In each case, the areas under the curve of individual pulses can be summed over the time period of interest to obtain CS. Similarly, the individual values of CBS obtained for each pulse can be summed over the period of interest. For this example, the soil in Hawai'i reaches a steady-state faster because decomposition is fast, while the Swedish soil continues accumulating carbon pulses after a 50 year time period because of the slow decomposition of the carbon pulses

Conclusion

Even though temporary storage in ecosystems has the potential to decrease peak warming if aggressive emission reductions are also pursued simultaneously (Matthews et al. 2022), soils are not yet included in nature-based climate solution policies and economic incentives (Smith et al. 2015; Amelung et al. 2020). Now, we have a computational framework to represent soils in a more accurate way and reduce uncertainty about how much and for how long C may remain in soil. This framework quantifies the climate benefit, specific to each system and adaptable to different models (simple or complex) available for a location regardless of scale. Further, there is often a need to directly compare the benefit of CDR to that of direct emission avoidance elsewhere in a system. CS allows you to compute how much carbon remains and for how long it stays in a soil. CBS allows you to assess how radiative forcing in the atmosphere responds to C drawdown and release in natural and managed lands. Then, CBS can facilitate direct, detailed comparisons of potential climate change mitigation and tradeoffs (e.g., soil C sequestration in improved management strategies, avoided import of food and fertilizer in a sustainable food system, and all aspects of bioenergy/fuel production and fossil offsets). Thus, geopolitical units and institutions may add rigor and clarity to their net-zero targets (Rogelj et al. 2021).

The CBS computational framework provides a critical missing piece that quantifies climate benefits

of sequestration alongside avoided emissions within complex systems. Food systems account for 1/3 of global emissions, with energy and transportation accounting for most of the rest (Crippa et al. 2021). This computational advance is critical to achieving multiple sustainability goals that include the food and energy sectors (Lal et al. 2021). Many soils will not achieve marketable levels of warming benefits from sequestration, but some will, especially in ecosystems with high productivity with potential to slow down decomposition through management. More importantly, with implementation of climate-smart practices and land-management decisions comes a multitude of co-benefits to the environment and society (including soil health, reduced dependance on imports, clean water, and local jobs) (Smith et al. 2015; Adhikari and Hartemink 2016; Amin et al. 2020). Investments back into the community build viable social-ecological-economic systems (Löbmann et al. 2022) that policy and incentives programming can support (Amelung et al. 2020).

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Data availability Original study data may be found in the references cited for each site, code for the analysis presented here may be downloaded at https://doi.org/10.5281/zenodo.6861402.

Declarations

Competing interests The authors have no relevant financial or non-financial interests to disclose.

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