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Documentation for the MARKAL Family of Models

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Authors:

Richard Loulou
Gary Goldstein
Ken Noble

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INTRODUCTION

This documentation replaces the documentation of the early version of the MARKAL model (Fishbone et al, 1983) and the many notes and partial documents which have been written since then. It is divided in three parts, each related to a different type of modeling paradigm.

Part I deals with the Standard MARKAL model, an intertemporal partial equilibrium model based on competitive, far sighted energy markets.

Part II discusses MARKAL-MACRO, a General Equilibrium model obtained by merging MARKAL with a set of macroeconomic equations.

Part III describes the SAGE model, a time-stepped version of MARKAL where a static partial equilibrium is computed for each time period separately.

Each Part is further divided in two main chapters, the first describing the properties of the model, its mathematical properties, and its economic rationale, and the second constituting a reference guide containing a full description of the parameters, variables, and equations of the model.

Note that section 2.11 in Part I deals with the GAMS environment and control sequences for running the models. It appplies to all three parts of the documentation.

Although the three parts are presented as separate files with their own tables of contents, the numbering of chapters, sections, and pages is continuous across all thtree parts.

PART I: The Standard MARKAL Model

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Part I is entirely devoted to documenting the Standard MARKAL model. It is organized in three chapters. Chapter 1 contains descriptions of Standard MARKAL concepts and theory. Chapter 2, a comprehensive reference manual, is intended for the technically minded programmer or modeler looking for an in-depth understanding of the relationship between the input data and the model mathematics, or contemplating augmenting the capabilities of MARKAL. It includes a full description of the attributes, sets, variables, and equations of the system, accompanied by text explaining their purpose and function. Section 2.11 describes the control statements required to run a MARKAL model. A full listing of the MARKAL source code (written in the Generalized Algebraic Modeling System (GAMS) language) can be made available.

1. Standard MARKAL: Concepts and Theory

1.1 Model Summary

MARKAL (an acronym for MARKal ALlocation) is a mathematical model of the energy system of one or several regions that provides a technology-rich basis for estimating energy dynamics over a multi-period horizon. Reference case estimates of end-use energy service demands (e.g., car, commercial truck, and heavy truck road travel; residential lighting; steam heat requirements in the paper industry) are developed by the user on the basis of economic and demographic projections, for each region in a multi-region formulation of the model. In addition, the user provides estimates of the existing stock of energy related equipment, and the characteristics of available future technologies, as well as new sources of primary energy supply and their potentials.

MARKAL computes energy balances at all levels of an energy system: primary resources, secondary fuels, final energy, and energy services. The model aims to supply energy services at minimum global cost by simultaneously making equipment investment and operating decisions and primary energy supply decisions, by region. For example, in MARKAL, if there is an increase in residential lighting energy service (perhaps due to a decline in the cost of residential lighting), either existing generation equipment must be used more intensively or new equipment must be installed. The choice of generation equipment (type and fuel) incorporates analysis of both the characteristics of alternative generation technologies and the economics of primary energy supply. MARKAL is thus a vertically integrated model of the entire energy system.

MARKAL computes an intertemporal partial equilibrium on energy markets, which means that the quantities and prices of the various fuels and other commodities are in equilibrium, i.e. their prices and quantities in each time period are such that at those prices the suppliers produce exactly the quantities demanded by the consumers. Further, this equilibrium has the property that the total surplus is maximized over the whole horizon. Investments made at any given period are optimal over the horizon as a whole.

In Standard MARKAL several options are available to model specific characteristics of an energy system such as the internalization of certain external costs, endogenous technological learning, the fact that certain investments are by nature "lumpy", and the representation of uncertainty in some model parameters.

Organization of Chapter1

Section 1.2 provides a general overview of the representation in MARKAL of the energy system of a typical region or country, focusing on the basic elements of such a Reference Energy System (RES), namely technologies and commodities. Sections 1.3 and 1.4 present a streamlined representation of the optimization problem formulated by MARKAL to compute the dynamic equilibrium, section 1.5 expands on the economic rationale of the model, while sections 1.6 and 1.7 state in detail the elastic demand feature and other economic and mathematical properties of the MARKAL equilibrium. Sections 1.8, 1.9, 1.10, and 1.11 respectively describe the additional options available in Standard MARKAL: Damage Costs, Lumpy Investments, Endogenous Technological Learning (ETL), and Stochastic Programming.

1.2 The basic structure of the MARKAL model generator

It is useful to distinguish between a model's *structure* and a particular instance of its implementation. A model's structure exemplifies its fundamental approach for representing and analyzing a problem—it does not change from one implementation to the next. MARKAL models for different regions have identical structure, however, because MARKAL is data[1] driven, each (regional) model will vary as data inputs vary. For example, in a multi-region model one region may, as a matter of user data input, have undiscovered domestic oil reserves. Accordingly, MARKAL generates technologies and processes that account for the cost of discovery and field development. If, alternatively, user supplied data indicate that a region does not have undiscovered oil reserves no such technologies and processes would be included in the representation of that region's reference energy system (RES, see subsections 1.2.3 and 1.2.4).

The structure of MARKAL is ultimately defined by variables and equations determined from the data input provided by the user. This information collectively defines each MARKAL regional model database, and therefore the resulting mathematical representation of the RES for each region. The database itself contains both qualitative and quantitative data. The *qualitative data* includes, for example, lists of energy carriers, the technologies that the modeler feels are applicable (to each region) over a specified time horizon, as well as the environmental emissions that are to be tracked. *Quantitative data*, in contrast, contains the technological and economic parameter assumptions specific to each technology, region, and time period. When constructing multi-region models it is often the case that a technology may be available for use in two distinct

[1] Data in this context refers to parameter assumptions, technology characteristics, projections of energy service demands, etc. It does not refer to historical data series.

regions; however, cost and performance assumptions may be quite different (i.e., consider a residential heat pump in Canada versus the same piece of equipment in the United States). This section discusses both qualitative and quantitative assumptions in the MARKAL modeling system.

1.2.1 Summary of the key elements of MARKAL

The MARKAL energy economy is made up of producers and consumers of energy carriers.[2] MARKAL, like most computable economic equilibrium models, assumes perfectly competitive markets for energy carriers—producers maximize profits and consumers maximize their collective utility. The result is a supply-demand equilibrium that maximizes the net total surplus (i.e. the sum of producers' and consumers' surpluses) as will be fully discussed in section 1.5. MARKAL may, however, depart from perfectly competitive market assumptions by the introduction of user-defined, explicit special assumptions, such as limits to technological penetration, speed of introduction of new technologies, agnet-specific discount rates, etc.

Operationally, a MARKAL run configures the *energy system* (of a *set of regions*) over a certain *time horizon* in such a way as to *minimize the net total cost (or* equivalently *maximize the net total surplus*) of the system, while satisfying a number of *constraints*. MARKAL is generally run in a dynamic manner, which is to say that all investment decisions are made in each period with full knowledge of future events.[3] In addition to time-periods (each usually 5 or 10 years long), MARKAL recognizes three seasons (Winter, Summer, Intermediate), and two diurnal divisions (Day, Night). These time divisions result in six *time-slices*. Time-slices are recognized only for technologies producing electricity (seasonal and diurnal) or low-temperature heat (seasonal), both of which may not be easily stored and thus require a finer time disaggregation than other energy carriers. As a result, these two energy carriers are disaggregated into 6 or 3 time-slices respectively during each time period. In addition, a peak requirement is imposed for these two energy carriers, which forces enough additional capacity to be installed to meet the peak demand.

1.2.2 Time horizon

The time horizon is divided into a user-chosen number of time-periods, with each model period having the same user-defined number of years. In MARKAL, each of the years belonging to a given period is considered identical. In other words, if each period has 5 years, any model input or output related to period ***t*** applies to each of the 5 years in that period. Similarly, the energy flows and emissions levels reported in the results represent

[2] An *energy carrier*, also denoted in this report as an *energy source* or *energy form*, is anything in the energy system containing usable energy and utilized as such either to produce another energy carrier (e.g. coal or gas used to produce electricity) or to produce usable heat or mechanical movement via certain technologies (e.g. gasoline, electricity, wood). An *energy service* is a commodity representing a demand for some useful service, such as transportation of persons, heating of dwellings, etc.
[3] This is often referred to as perfect foresight. Note that the SAGE variant of the model assumes that foresight is limited to the current period only (see Part III).

annual flows in each of the 5 years embodied in a period. The important exception is that the period's investments are assumed to occur at the beginning of each period with the resulting installed capacity available throughout that period.

The initial period is a past period, over which the model has no freedom, and for which the quantities of interest are all fixed by the user at their historical values. This calibration to an initial period is one of the important tasks required for setting up a MARKAL model. The main variables to be calibrated are: the capacities and operating levels of all technologies, as well as the extracted, exported and imported quantities for all energy carriers. The associated emission levels are also validated. Note carefully that the initial period's calibration also influences the model's decisions over several future periods, since the profile of existing (residual) capacities is provided over the remaining life of the technologies at and after the initial period. Since such technologies are already in place, the model need not make any investment decision; only the operating and maintenance decisions are to be made. In this respect, the model is free to cease using the existing capacity of some technologies, if such a decision reduces total cost.

1.2.3 The RES concept

The MARKAL energy economy consists of:

- *Demands*, that represent the energy services (e.g., space heating, vehicle-kilometers traveled, tonnes of steel production) that must be satisfied by the system;
- Energy *sources* (mining or imports), that represent methods of acquiring various energy carriers;
- *Sinks,* that represent exports;
- *Technologies* (also called processes), that either transform an energy carrier to another form or into a useful energy service; and
- *Commodities* consisting of energy carriers, energy services, materials, and emissions that are either produced or consumed by the energy sources, sinks, technologies and demands.

The structural boundaries of the RES consist of the energy services and the energy sources, both of which are specified not as fixed assumptions, but as supply curves (for energy sources) and demand curves (for energy services). Timewise, the boundaries are the intial period (when the initially existing system is described), and the end of the horizon (when the remaining capacities are valued).

It is helpful to summarize the relationships among these various entities using a network diagram referred to as a Reference Energy System (RES). In the MARKAL RES a node represents a source, sink, technology, or demand, and a link (arc) represents a commodity (energy carrier, material, energy service). An emission is represented by an open ended link pointing away from the emitting node.

is a snapshot of a small portion of a hypothetical RES, at a given time period. It contains a single energy service demand, residential space heating. In MARKAL, this demand is specified via a demand curve linking the level of demand to its own price (see section 1.3). There are three end-use space heating technologies using the energy carriers gas, electricity, and heating oil, respectively. These energy carriers in turn are produced by other technologies, represented in the diagram by a gas plant, three electricity-generating plants (gas fired, coal fired, oil fired), and an oil refinery. To complete the production chain on the primary energy side, the diagram also represents an extraction source for natural gas, an extraction source for coal, and two sources of crude oil (one extracted domestically and one imported, each using a pipeline). in MARKAL. each source is described as a supply curve, i.e. a series of quantities available at various prices. Note that in the RES every time a commodity enters/leaves a process its name is changed (e.g., wet gas becomes dry gas, crude becomes pipeline crude). This simple rule enables the inter-connectively between the processes to be properly maintained throughout the network.

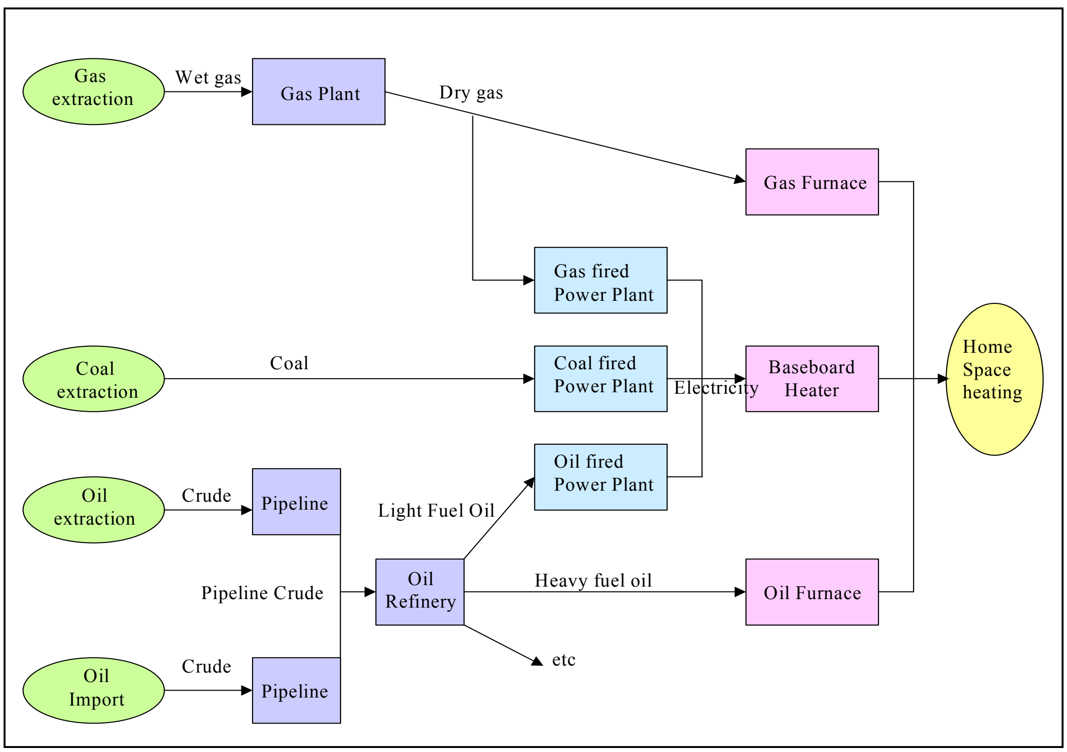

Figure 1-1. Partial view of a simple Reference Energy System

To organize the RES, and inform the modeling system of the nature of its components, the various technologies (nodes) and commodities (links) are classified into *sets*. Each MARKAL set regroups components of a similar nature. The entities belonging to a set are referred to as members, items or elements of that set. In this section, we briefly review the main component sets, and we informally describe the properties and features associated with each set. The complete description of all sets, and of the attributes of each set member, is covered in chapter 2.

In some incarnations of MARKAL, technology and process naming conventions play a critical role in allowing the easy identification of an element's position in the RES and its set membership(s). This is a very useful feature that allows the user to quickly explore the structure of MARKAL. For instance, in the SAGE model[4] each energy carrier for use in the transportation sector has a six-letter name starting with TRA and ending with the energy form (thus gasoline for vehicles is called TRAGSL). Similarly technology names have up to ten characters, the first three indicating the subsector, the next three the fuel/technology combination (standard, medium and high efficiency heat pumps), and the last three reserved to distinguish between vintages of technologies (the string '000' indicates that the technology is an existing technology; '005' refers to a technology first available in 2005, etc.). For instance, the standard residential electric water heater available for purchase in 2005 might be named RWHELC005.

1.2.4 The MARKAL RES: Overview of MARKAL set definitions

1.2.4.1 SRCENCP: the set of sources and sinks

SRCENCP contains all source nodes: Mining, Imports, Exports, and physical Renewables such as biomass (not wind, etc.). Each source node deals with a single commodity such as mined natural gas. However, it is possible to distinguish between several steps in the mining of the same resource (gas), each step having its own cost and other parameters. For example, in the case of natural gas there may be several steps, for production from located reserves, from enhanced recovery and from undiscovered reserves. Each step has its own production cost and volume available at that cost. This allows the modeling of supply curves. Each source has attributes that specify its cumulative resource availability, cost of procurement (or revenue from export), and bounds on its annual production activity in each period. Sources may also require the consumption of other commodities in order to operate (just like a technology).

Note again that in MARKAL, imports into, and exports from, a region may be exogenously specified by the user or endogenously determined. In the former case, a commodity is purchased from (or sold to) an unspecified region, at an exogenously specified price, whereas, in the latter case, trade is between specific regions and the unit value (shadow price) of the traded energy carrier is calculated endogenously.

Renewables for which there are physical (e.g., biomass, municipal solid waste), as opposed to "free" (the sun, wind) commodities are also modeled as source nodes in the same manner as mining.

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[4] The System for Analysis of Global Energy is a variant of MARKAL developed at the United States Department of Energy's Energy Information Administration. SAGE is described in Part III of this documentation, and in a set of public documents available at Hhttp://www.eia.doe.gov/bookshelf/docs.htmlH

1.2.4.2 PRC: the set of Process technologies

The set **PRC** contains all technologies that transform energy forms into other energy forms, *except those technologies producing electricity or low temperature heat.* Note that demand devices, discussed below, mainly deliver energy services rather than produce an energy form (although they can do both) and are thus not members of **PRC**. Each PRC technology may also have material inputs and outputs as well as emissions. The set PRC is usually quite large, having typically hundreds of elements. Apart from the inputs and outputs, a PRC member has up to four types of cost attributes attached to it: investment cost (per unit of new capacity), fixed annual maintenance (per unit of existing capacity), variable operating cost (per unit of activity), and delivery costs (per unit of each commodity required to operate the technology). Each PRC technology is further characterized by its life duration, the period when it first becomes available, and its discount rate (used to compute the annualized investment cost). Its operation may also be altered by bounds expressed in absolute terms or as maximum growth rate per period. Finally, each PRC technology may have emission coefficients attached to each of its three variables: investment, capacity, and activity.

1.2.4.3 CON: the set of conversion technologies

The set **CON** contains technologies that produce either electricity or low temperature heat. **CON** consists of three primary subsets: **ELE**, **HPL,** and **CPD**. **ELE** contains the electricity conversion technologies, **HPL** the set of technologies producing low temperature heat, and **CPD** the coupled production technologies (producing heat and power in combination). The two energy carriers *electricity* and *heat* are special, since they are not easily stored. They are associated with the dedicated sets *ELC* and *LTH*. The members of these sets are tracked for each of the relevant time-slices (six for electricity, three for heat), whereas all other commodities are tracked only via annual flows.

The **ELE** set is further composed of subsets, whose members may have somewhat different roles in MARKAL. For instance, power plants designated as base load plants (**BAS)** are constrained to operate at the same rate day and night in the same season. This would be typical for a nuclear, hydro or large coal-fired power plant. Another subset include all centralized plants (**CEN)** which incur transmission losses on the grid, whereas those in the subset of decentralized plants (**DCN)** do not.

Each **CON** technology has the same attributes mentioned earlier for **PRC** technologies, plus a few more describing the seasonal and diurnal availability of the capacity. For example, solar technologies (without battery storage) are not available during the night. And wind power may vary by season. In addition, each **CON** technology has a peaking attribute defining its ability to produce during the peaking demand time-slice.

1.2.4.4 DMD: the set of end-use technologies

The set **DMD** contains all *end-use technologies*, i.e. those which produce an energy service to satisfy a demand (Table 1-1 lists the energy services modeled in the SAGE model). By convention, and to facilitate understandability, the first three letters of a technology's name correspond to the main energy service demand being produced. For instance, technology RK1KER000 is an existing kerosene cooker. The first three letters RK1 indicate that this device services cooking demand, the next three letters KER that it uses kerosene, and the last three letters 000 that it is available at the start of the modeling horizon.

Note that in the actual model, **DMD** technologies, also commonly referred to as demand devices, do not have a distinct activity variable. It is implicitly assumed that a demand device operates at a level proportional to its capacity[5], and therefore output (activity) is explicitly derived directly from the capacity variable.

1.2.4.5 DM: the set of demands for energy services

The set **DM** contains the energy services that must be satisfied. Each energy service is expressed in MARKAL in units of *useful energy*. Table 1-1 contains the list of the 42 energy service demand categories in the current version of the SAGE model, grouped by economic sector. Note that the first letter of each energy service designates the sector (C for commercial, R for residential, A for agriculture, I for industry, and T for transport).

Each energy service has several attributes that describe (a) the amounts of service to be satisfied at each time period, (b) the seasonal/time-of-day nature of these requirements (or electricity and heat), and (c) the price-elasticity of the demand and the allowed interval of demand variation.

Table 1-1. Example of energy services in MARKAL

Transportation segments (15)	**Codes**
Autos	TRT
Buses	TRB
Light trucks	TRL
Commercial trucks	TRC
Medium trucks	TRM
Heavy trucks	TRH
Two wheelers	TRW
Three wheelers	TRE
International aviation	TAI
Domestic aviation	TAD
Freight rail transportation	TTF
Passengers rail transportation	TTP
Internal navigation	TWD
International navigation (bunkers)	TWI
Non-energy uses in transport	NEU
Residential segments (11)	**Codes**
Space heating	RH1, RH2, RH3, RH4

[5] When developing end-use technology cost estimates for MARKAL the modeler must explicitly make assumptions regarding capacity. For example, the number of cars needed to provide a billion vehicle kilometers per year depends on assumptions regarding kilometers traveled per year by an average car.

Space cooling Hot water heating Lighting Cooking Refrigerators and freezers Cloth washers Cloth dryers Dish washers Miscellaneous electric energy Other energy uses	RC1, RC2, RC3, RC4 RWH RL1, RL2, RL3, RL4 RK1, RK2, RK3, RK4 RRF RCW RCD RDW REA ROT
Commercial segments (8)	**Codes**
Space heating Space cooling Hot water heating Lighting Cooking Refrigerators and freezers Electric equipments Other energy uses	CH1, CH2, CH3, CH4 CC1, CC2, CC3. CC4 CHW CLA CCK CRF COE COT
Agriculture segments (1)	**Codes**
Agriculture	AGR
Industrial segments (6)	**Codes**
Iron and steel Non ferrous metals Chemicals Pulp and paper Non metal minerals Other industries	IIS INF ICH ILP INM IOI
Other segment (1)	**Codes**
Other non specified energy consumption	ONO

Industrial energy services such as IIS (iron and steel) are made up of a 'recipe' of more detailed services — steam, process heat, machine drive, electrolytic service, other, and feedstock. For example ISIS, IPIS, and IMIS refer, respectively, to the steam, process heat, and machine drive needed in the iron and steel industry.

Investment in iron and steel capacity only includes the energy cost [process heat, steam, feedstock, etc] of a million metric tons of steel capacity. The unit of output and capacity for iron and steel, non-ferrous metals and pulp and paper is million metric tons. The output of other industries is measured in units of Petajoules of energy service at base year technology efficiencies.

1.2.4.6 ENC: the set of energy carriers

Energy carriers are classified as fossil, nuclear, renewable, or synthetic, plus electricity and heat. ENC is the primary subset of all energy carriers excluding electricity and heat.

In the SAGE data base, a deliberate choice is made to strongly identify the fuels to the sector or subsector that produces or consumes it, thus allowing distribution costs, price markups, taxes, and carbon constraints to be sector and/or subsector specific. Thus a tax may be levied on industrial distillate use but not on agricultural distillate use. As before, this is accomplished by strict adherence to energy carrier naming conventions. In Table 1-2, we show as an example the subset of fuels used in the SAGE transportation sector. Note that all fuel names for that sector start with the three letters TRA, thus clearly identifying the sector. The same rule is applied to energy carriers in all sectors.

Note also that the renaming of fuels in each sector requires the creation of technologies whose only role is to rename a fuel. For instance, technology TRAELC000 transforms one unit of electricity (ELC) into one unit of electricity dedicated to the transportation sector (TRAELC). On the other hand, most technologies producing a transportation sector fuel are bona fide technologies, as for instance technology TRAMET100 that processes natural gas (GASMET) into methanol (TRAMET).

Table 1-2. List of fuel names in the transportation sector (example from SAGE)

Energy Carrier's code	**Description**
TRANGA	Natural Gas
TRADST	Diesel fuel
TRAGSL	Gasoline
TRAHFO	Heavy fuel oil
TRACOA	Coal
TRALPG	Light Petroleum Gas
TRAAVG	Aviation Gasoline
TRAJTK	Let Fuel
TRAMET	Methanol
TRAETH	Ethanol
TRAELC	Electricity

Despite the fact that MARKAL does not attribute a specific variable to each energy carrier, there are nevertheless certain attributes that are attached to the electricity and low temperature heat energy carriers, describing the transmission/distribution costs and efficiency of the "grid" to which these energy carriers are attached.

1.2.4.7 MAT: the set of materials

In MARKAL, materials are separated from energy carriers for two reasons: first, for reporting purposes, so as not to mix commodities expressed in different units. Secondly, we shall see in section 1.4 that the balance equation for a material is expressed as an equality, whereas the balance equation for an energy carrier is expressed as an inequality. This former choice was made for convenience, since materials are often used for specific purposes which require an equality relationship between supply and consumption.

In addition to this difference, the set MAT of all materials is partitioned into two subsets, for added flexibility:

- **MWT** is the set of materials expressed in weight units
- **MVO** is the set of materials expressed in volume units

1.2.4.8 ENV: the set of environmental emissions

ENV is the set of environmental emissions. Contrary to energy carriers, a MARKAL emission variable is created for each emission commodity. This allows the user to easily model certain environmental policy instruments such as a cap or a tax on the emissions of a particular substance. Caps may be applied at each period or cumulatively, for the entire energy system or particular sector(s). There is also the possibility to transform an emission into another emission (for instance methane may be converted into CO2-equivalent) via a special environmental table.

Negative emission coefficients are perfectly acceptable in MARKAL, thus permitting technologies to absorb an emitted substance (for instance, a geological CO2 sequestering technology would have a negative CO2 emission coefficient).

Emission rates are usually associated with individual sources or technologies by tying them to the activity, capacity or investment of the corresponding RES component. There is also the possibility of associating emissions with energy carriers or materials.

1.2.4.9 System-Wide attributes

Several attributes are not attached to any particular technology or commodity, but are defined for the entire RES. Examples are: general discount rate, period length, and the fractions of the year covered by the six time-slices used for electricity (and heat) production. Also in this category are a few switches that allow the user to activate or deactivate the elastic demands, endogenous technology learning, stochastics, etc. Finally, several parameters apply to entire electricity or heat grids, and will be discussed in subsequent sections.

1.2.5 Some details on the MARKAL Electricity Sector

1.2.5.1 Load pattern -- Demand

For electricity, the year is divided into six time-divisions (time-slices), using two indices: Z = Winter/Summer/Intermediate, and Y=Day/Night. The demand for electricity in each season (Z) and time-of-day (Y) is calculated for all demand categories (DM). If a particular demand is uniformly distributed over the year, the demand in each time division (Z)(Y) is governed by the general breakdown of time divisions in SAGE as given by the constants QHR(Z)(Y), representing the fractions of each year covered by each division. Thus, we have:

$$ELC_{DM}(Z)(Y) = DEMAND_{DM} \text{ x } QHR(Z)(Y) \;\; ; \text{ if } DM_{DM} = UNIFDIST$$

Alternatively, a demand may have a non-uniform distribution throughout the year, as specified by six user parameters FR(Z)(Y). In this case, we have:

$$ELC_{DM}(Z)(Y) = DEMAND_{DM} \text{ x } FR_{DM}(Z)(Y) \text{ ; if } DM_{DM} \neq UNIFDIST$$

The relative load in each time division for such non-uniform demands equals:

$$LOAD_{DM}(Z)(Y) = DEMAND_{DM} \text{ x } \frac{FR_{DM}(Z)(Y)}{QHR(Z)(Y)}$$

The total electricity demand in each time-slice is then:

$$ELC(Z)(Y) = \sum_{DM} ELC_{DM}(Z)(Y)$$

1.2.5.2 Load Management – Supply

A conversion plant can produce electricity in each time division up to a level governed by the annual availability factor (AF):

$$ELC_{CON}(Z)(Y) \leq CAP_{CON} \text{ x CAPUNIT x AF x } QHR(Z)(Y)$$

Where the CAPUNIT factor allows for capacity units to be converted into electricity production units. In most MARKAL models, CAPUNIT = 31.536 converts GW capacity into PJ production.

Instead of a fixed and constant availability throughout the year, seasonal and time-of-day dependent values may be assumed by means of division-dependent AF(Z)(Y), for instance to reflect resource availability for renewable power plants (hydro, solar, wind):

$$ELC_{CON}(Z)(Y) \leq CAP_{CON} \text{ x CAPUNIT x } AF(Z)(Y) \text{ x } QHR(Z)(Y)$$

In this case, the production in each time division cannot exceed either of the levels given in the two formulas above. The actual level of production for certain plants is then established as part of the solution of the MARKAL model, subject to these constraints and the demand load pattern.

For a variety of reasons the user may wish to limit the load following characteristics of specific power plants, e.g. to avoid unrealistic operation patterns, such so-called eXternally Load Managed plants (forming set XLM) can be introduced. Their production in each time division is fixed by the user by means of annual (CF) or time-sliced (CF(Z)(Y)) capacity factor parameters, but not both:

$$ELC_{XLM}(Z)(Y) = CAP_{XLM} \text{ x CAPUNIT x CF x } QHR(Z)(Y)$$

or:

$$ELC_{XLM}(Z)(Y) = CAP_{XLM} \text{ x } CAPUNIT \text{ x } CF(Z)(Y) \text{ x } QHR(Z)(Y)$$

As a side benefit, XLM plants do not generate six production variables for the six time divisions in each period, as is the case for other plants, and are thus less demanding in terms of the model dimensions. Thus, each plant that is put in set XLM saves 6 variables per period. Of course, care must be taken to leave sufficient freedom for the operation of the entire set of power plants to respond to fluctuating demand levels.

1.2.5.3 Baseload Plants

Base load power plants (members of set BAS) are assumed to produce at the same rate during day and night of each season (Z). In addition, their aggregate production during the night cannot exceed a modeler-specified share (BASELOAD) of the total electricity production during the night in each season (Z). Figure 1-2 below shows the part of the load subject to base load generation as the bottom block of the “curve.” Examples of base load plants are nuclear plants, and coal fired plants.

1.2.5.4 Peak requirements

A user specified share (PEAK) of the installed capacity of each plant is assumed to contribute to the peaking requirements. The peak may be located in winter-day (WD) or the summer-day (SD). The minimum installed capacity is calculated by adding a capacity reserve to the total electricity demands in WD or SD. The reserve margin is chosen by the user for each grid, as a percentage ERESERV of the demand in W-D or S-D. Note that ERESERV is typically much larger than prevailing rule-of-thumb values by the electric utilities. Reason for this is that the reserve margin in MARKAL also encompasses the difference between the levelized WD (or SD) demand and the actual peak occurring on one moment in that same period when the demand is actually the highest. The construction of the peak and associated minimum installed capacity requirement is shown in Figure 1-2 below.

The contribution of demands to the electricity peak can be adjusted by specifying which share of the total demand (ELF) coincides with the peak (default = 100%). Note also that some individual demand devices can be operated as night storage devices (members of set NST), with the result that some demand is shifted to the night hours. These NST plants do thus not contribute to the peak.

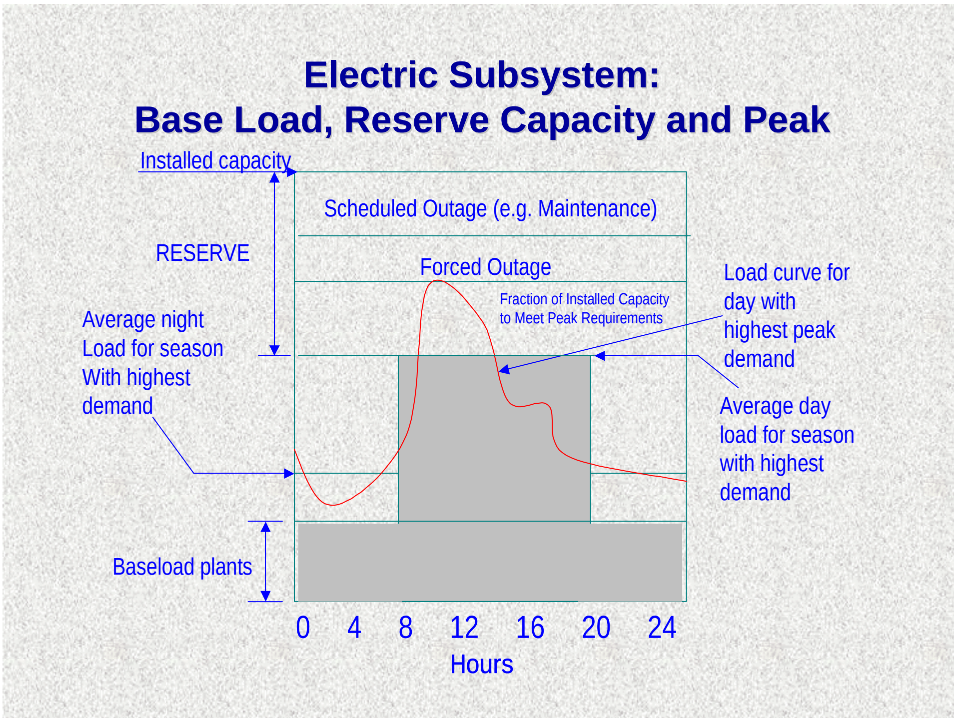

Figure 1-2. Base load, reserve capacity and peak

1.2.5.5 Transmission and Distribution – Losses and Costs

MARKAL distinguishes two kinds of power plants: centralized (CEN) and decentralized (DCN). Costs and losses of electricity transport and distribution are modeled through the following entries:

ETRANINV, investment cost per unit of new transport line capacity
ETRANOM,
EDISTINV, investment cost per unit of new distribution capacity
EDISTOM, annual fixed O&M cost per unit of installed distribution capacity, and
TE, transmission efficiency.

Distribution costs introduced at this grid level apply to all electricity consumed. Sector and/or application specific distribution costs are to be handled by assigning delivery costs (DELIV) to individual processes or end-use devices.

CEN plants are charged with both transport and distribution costs, while DCN plants only face distribution costs. Moreover, losses in the electricity grid occur at the transport level only, so DCN plants are not associated with any grid losses. These properties should be kept in mind when assigning power production plants to either of the two sets. As to the use of the investment cost parameters, they are added to the investment costs of the power plants themselves.

Moreover, it can be questioned to what extent investments in grid expansion are governed by investments in generating capacity, e.g. new plants can be hooked up to existing grids if they replace older units on the same site at little or no extra cost. The location of new capacity relative to the existing grid and to major consumption areas is typically the decisive factor, but also the extent to which grid capacity in place is utilized to full capacity.

Figure 1-3 depicts the electric grid network that can be constructed with MARKAL, where link (LNK) technologies server as grid exchange connection points. Each grid can be characterized by the various cost and efficiency parameters just discussed, and is tracked with separate balance, peak and base load constraints. When multiple grids are used, each power plant and consumer of electricity is associated with a specific grid.

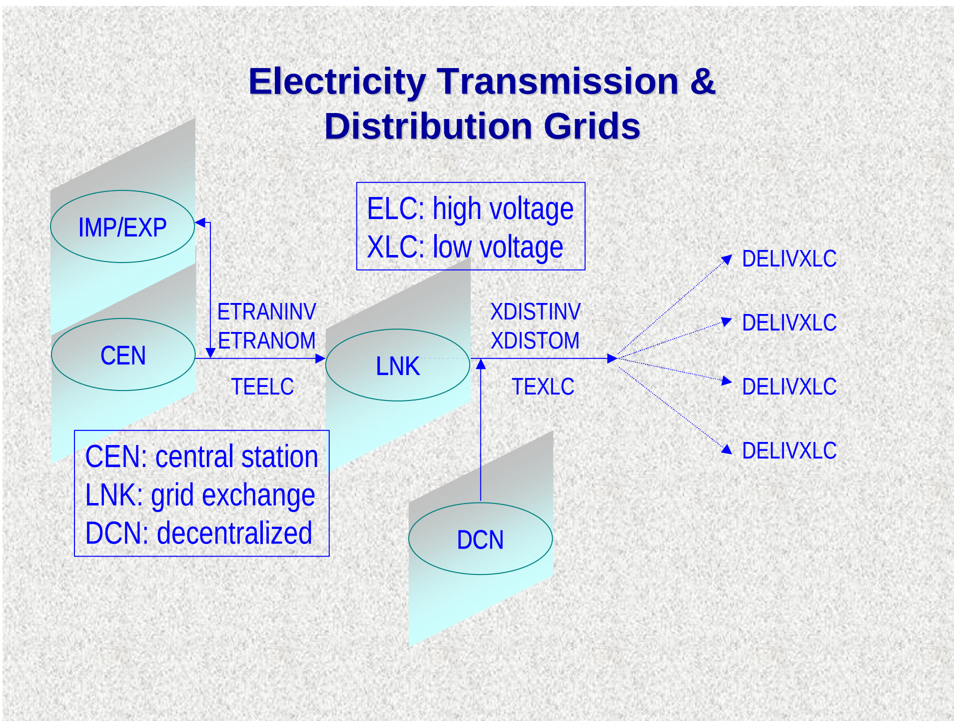

Figure 1-3. Electricity transmission and distribution grids

1.3 Economic rationale of the MARKAL model

This section provides a detailed economic interpretation of the MARKAL and other partial equilibrium models. These models have one common feature – they simultaneously configure the production and consumption of commodities (i.e. fuels, materials, and energy services) and their prices. The price of producing a commodity affects the demand for that commodity, while at the same time the demand affects the commodity's price. A market is said to have reached an equilibrium at price p^* when no consumer wishes to purchase any less and no producer wishes to produce any more of each commodity at that price. As will be explained further below, when all markets are in equilibrium the total economic surplus is maximized.

The concept of total surplus maximization extends the cost minimization approach upon which earlier bottom-up energy system models were based. These simpler models have fixed energy service demands, and thus are content to minimize the cost of supplying these demands. In contrast, the MARKAL demands for energy services are themselves elastic to their own prices, thus allowing the model to compute a *bona fide* supply-demand equilibrium.

Section 1.3.1 provides a brief review of different types of energy models. Section 1.3.2 discusses the economic rationale of the MARKAL model with emphasis on the features that distinguish MARKAL from other partial equilibrium models (such as the earlier incarnations of MARKAL, see Fishbone and Abilock, 1981, Berger et al., 1992).

1.3.1 A brief discussion of energy models

Many energy models are in current use around the world, each designed to emphasize a particular facet of interest. Differences include economic rationale, level of disaggregation of decision variables, time horizon over which decisions are made, and geographic scope. One of the most significant differentiating features among energy models is the degree of detail with which commodities and technologies are represented.

1.3.1.1 Top-Down Models

At one end of the spectrum are aggregated *General Equilibrium* (GE) models. In these, each sector is represented by a production function designed to simulate the potential substitutions between the main factors of production (also highly aggregated into a few variables such as: energy, capital, and labor) in the production of each sector's output. In this model category are found a number of models of national or global energy systems. These models are usually called "Top-Down", because they represent an entire economy via a relatively small number of aggregate variables and equations. In these models, production function parameters are calculated for each sector such that inputs and outputs reproduce a single base historical year.[6] In policy runs, the mix of inputs[7] required to produce one unit of a sector's output is allowed to vary according to user-selected elasticities of substitution. Sectoral production functions most typically have the following general form:

$$\boldsymbol{X_s} = \boldsymbol{A}_0\left(\boldsymbol{B_K} \cdot \boldsymbol{K_s^{\rho}} + \boldsymbol{B_L} \cdot \boldsymbol{L_S^{\rho}} + \boldsymbol{B_E} \cdot \boldsymbol{E_S^{\rho}}\right)^{1/\rho} \qquad (1.3\text{-}1)$$

[6] These models assume that the relationships (as defined by the form of the production functions as well as the calculated parameters) between sector level inputs and outputs are in equilibrium in the base year.

[7] Most models use inputs such as labor, energy, and capital, but other input factors may conceivably be added, such as arable land, water, or even technical know-how. Similarly, labor may be further subdivided into several categories.

where X_S is the output of sector *S*,
K_S, L_S, *and* E_S are the inputs of capital, labor and energy needed to produce one unit of output in sector *S*
ρ is the elasticity of substitution parameter
A_0 and the ***B***'s are scaling coefficients.

The choice of ρ determines the ease or difficulty with which one production factor may be substituted for another: the smaller ρ is (but still greater than or equal to 1), the easier it is to substitute the factors to produce the same amount of output from sector *S*. Also note that the degree of factor substitutability does not vary among the factors of production — the ease with which capital can be substituted for labor is equal to the ease with which capital can be substituted for energy, while maintaining the same level of output. GE models may also use alternate forms of production function (1.3-1), but retain the basic idea of substitutability of production factors.

1.3.1.2 Bottom-Up Models

At the other end of the spectrum are the very detailed, *technology explicit* models that focus primarily on the energy sector of an economy. In these models each important energy-using technology is identified by a detailed description of its inputs, outputs, unit costs, and several other technical and economic characteristics. In these so-called "Bottom-Up" models, a sector is constituted by a (usually large) number of logically arranged technologies, linked together by their inputs and outputs (which may be energy forms or *carriers*, materials, emissions and/or demand services). Some bottom-up models compute a partial equilibrium via maximization of the total net [consumer and producer] surplus; while others simulate other types of behavior by economic agents, as will be discussed below. In bottom-up models, one unit of sectoral output (e.g., a billion vehicle kilometers of heavy truck service or a Petajoule of residential cooling service) is produced using a mix of individual technologies' outputs. Thus the production function of a sector is *implicitly* constructed, rather than explicitly specified as in more aggregated models. Such implicit production functions may be quite complex, depending on the complexity of the Reference Energy System (RES) of each sector.

1.3.1.3 Recent Modeling Advances

While the above dichotomy applied fairly well to earlier models, these distinctions now tend to be somewhat blurred by recent advances in both categories of model. In the case of aggregate top-down models, several general equilibrium models now include a fair amount of fuel and technology disaggregation in the key energy producing sectors (for instance: electricity production, oil and gas supply). This is the case with MERGE[8] and SGM[9], for instance. In the other direction, the more advanced bottom-up models are 'reaching up' to capture some of the effects of the entire economy on the energy system. For instance, the MARKAL model has end-use demands (including demands for industrial output) that are sensitive to their own prices, and thus capture the impact of

[8] Model for Evaluating Regional and Global Effects (Manne et al., 1995)
[9] Second Generation Model (Edmonds et al., 1991)

rising energy prices on economic output and *vice versa*. Recent incarnations of technology-rich models are multi-regional, and thus are able to consider the impacts of energy-related decisions on trade. It is worth noting that while the multi-regional top-down models have always represented trade, they have done so with a very limited set of traded commodities – typically one or two, whereas there may be quite a number of traded energy forms and materials in multi-regional bottom-up models. MARKAL-MACRO (Part II of this documentation) is a hybrid model combining the technological detail of MARKAL with a succinct representation of the macro-economy consisting of a single producing sector. Because of its succinct single-sector production function, MARKAL-MACRO is able to compute a general equilibrium in a single optimization step. The NEMS[10] model is another example of a full linkage between several technology rich modules of the various sectors and a set of macro-economic equations, although the linkage here is not as tight as in MARKAL-MACRO, and thus requires iterative resolution methods.

In spite of these advances, there remain important differences. Specifically:

- Top-down models encompass macroeconomic variables beyond the energy sector proper, such as wages, consumption, and interest rates; and
- Bottom-up models have a rich representation of the variety of technologies (existing and/or future) available to meet energy needs, and, they often have the capability to track a wide variety of traded commodities.

The Top-down vs. Bottom-up strategy is not the only relevant difference among energy models. Among Top-down models, the so-called Computable General Equilibrium models (CGE) differ markedly from the macro econometric models. The latter do not compute equilibrium solutions, but rather simulate the flows of capital and other monetary quantities between sectors. They use econometrically derived input-output coefficients to compute the impacts of these flows on the main sectoral indicators, including economic output (GDP) and other variables (labor, investments). The sectoral variables are then aggregated into national indicators of consumption, interest rate, GDP, labor, and wages.

Among technology explicit models, two main classes are usually distinguished: the first class is that of the partial equilibrium models such as MARKAL, that use optimization techniques to compute a least cost (or maximum surplus) path for the energy system. The second class is that of simulation models, where the emphasis is on representing a system not governed purely by profit or utility maximizing behavior. In these simulation models (e.g. CIMS[11]), investment decisions taken by a representative agent (firm or consumer) are only partially based on profit maximization, and technologies may capture a share of the market even though their life-cycle cost may be higher than that of other technologies. Simulation models use market-sharing formulas that preclude the easy computation of an equilibrium -- at least in a single pass. The SAGE (see PART III)

[10] National Energy Modeling System, a detailed integrated equilibrium model of an energy system linked to the economy at large, US Dept of Energy, Energy Information Administration (2000)
[11] Canadian Integrated Modeling System, Jaccard et al, 2003.

incarnation of the MARKAL model possesses a market sharing mechanism that allows it to reproduce certain behavioral characteristics of observed markets. SAGE requires two successive runs of the model to implement its market sharing formula.

The next subsection focuses on the Standard MARKAL model, perhaps the most typical representative of the class of partial equilibrium, technology explicit models. These models are also known as Optimization based equilibrium models, or Least Cost models, since the equilibrium is obtained by maximizing the net total consumer and producer surplus.

1.3.2 The MARKAL paradigm

Since certain portions of this and the next sections require an understanding of the concepts and terminology of Linear Programming, the reader requiring a brush-up on this topic may first read section 1.7.1, and then, if needed, some standard textbook on LP, such as Hillier and Lieberman (1990) or Chvatal (1983). The application of Linear Programming to microeconomic theory is covered in Gale (1960), and in Dorfman, Samuelson, and Solow (1958, and subsequent editions).

A brief description of MARKAL would indicate that it is a:

- *Technology explicit*;
- *Multi-regional*;
- *Partial equilibrium* model;

that assumes:

- *Price elastic demands;* and
- *Competitive markets:* with
- *Perfect foresight (*resulting in *Marginal value Pricing)*

We now proceed to flesh out each of these properties.

1.3.2.1 A technology explicit model

As already presented in section 1.2 (and described in much more detail in chapter 2), each technology in MARKAL is described by a number of technical and economic parameters. Thus, each technology is explicitly identified and distinguished from all others in the model. A mature MARKAL model may include several thousand technologies in all sectors of the energy system (energy procurement, conversion, processing, transmission, and end-uses), perhaps for multiple regions. Thus, MARKAL is not only technology explicit; it is technology rich as well. Furthermore, the number of technologies and their relative topology may be changed at will purely via data input specification, without the user ever having to modify the model's equations.

1.3.2.2 Multi-regional feature

Some existing MARKAL models include up to 15 regional modules, and the number of regions is limited only by the difficulty of solving LP's of very large size. The individual regional modules are linked by energy trading variables, and by emission permit trading variables if desired. The trade variables transform the set of regional modules into a single multiregional, possibly global energy model, where actions taken in one region may affect all other regions. This feature is of course essential when global as well as regional energy and emission policies are being simulated.

1.3.2.3 Partial equilibrium properties

MARKAL computes a partial equilibrium on energy markets. This means that the model computes both the *flows* of energy forms and materials as well as their *prices*, in such a way that, at the prices computed by the model, the suppliers of energy produce exactly the amounts that the consumers are willing to buy. This equilibrium feature is present at every stage of the energy system: primary energy forms, secondary energy forms, and energy services. Any supply-demand equilibrium model must have an economic rationale that drives the computation of the equilibrium. The underlying principles central to the MARKAL equilibrium are that:

- Outputs of a technology are linear functions of its inputs;
- Total economic surplus is maximized over the entire horizon, and
- Energy markets are competitive, with perfect foresight.

As a result of these assumptions the following properties hold:

- The market price of each commodity is exactly equal to its marginal value in the overall system, and
- Each economic agent maximizes its own profit (or utility).

We discuss each of these five features in the next subsections.

1.3.2.4 Linearity

A linear input-to-output relationship means that each represented technology may be implemented at any capacity, from zero to some upper limit, without economies or diseconomies of scale. In a real economy, a given technology is usually available in discrete sizes, rather than on a continuum. In particular, for some real life technologies, there may be a minimum size below which the technology cannot be implemented (or else at a prohibitive cost), as for instance a nuclear power plant, or a hydroelectric project. In such cases, because MARKAL assumes that all technologies may be implemented in any size, it may happen that the model's solution shows some technology's capacity at an unrealistically small size. However, in the MARKAL context, such a situation is relatively infrequent and often innocuous, since the scope of application is at the regional level, and thus large enough that small capacities are unlikely to occur.

On the other hand, there may be situations where plant size matters, for instance when the region being modeled is very small. In such cases, it is possible to enforce a rule by which certain capacities are allowed only in multiples of a given size, by introducing integer variables. This option is available in MARKAL and is discussed in section 1.9. This approach should, however, be used sparingly because it greatly increases solution time. Alternatively and more simply, a user may add user-defined constraints to force to zero any capacities that are clearly too small.

It is the linearity property that allows the MARKAL equilibrium to be computed using Linear Programming techniques. In the case where economies of scale or some other non-convex relationship is important to the problem being investigated, the optimization program would no longer be linear or even convex. We shall examine such cases in sections 1.9 and 1.10.

The fact that MARKAL's equations are linear, however, *does not mean that production functions behave in a linear fashion*. Indeed, the MARKAL production functions are usually highly non-linear (although convex), representing non-linear functions as a stepped sequence of linear functions. As a simple example, a supply of some resource may be represented as a sequence of segments, each with rising (but constant within its interval) unit cost. The modeler defines the 'width' of each interval so that the resulting supply curve may simulate any non-linear convex function.

1.3.2.5 Maximization of total surplus: Price equals Marginal value

The *total surplus* of an economy is the sum of the suppliers' and the consumers' surpluses. The term *supplier* designates any economic agent that produces (and sells) one or more commodities (i.e., in MARKAL, an energy form, an emission permit, and/or an energy service). A *consumer* is a buyer of one or more commodities. Some agents may be both suppliers and consumers, but not for the same commodity. Therefore, for a given commodity, the Reference Energy System defines a set of suppliers and a set of consumers.

It is customary to represent the set of suppliers of a commodity by their *inverse production function*, that plots the marginal production cost of the commodity (vertical axis) as a function of the quantity supplied (horizontal axis). In MARKAL, as in other linear optimization models, the supply curve of a commodity is not explicitly expressed as a function of factor inputs (such as aggregate capital, labor and energy in typical production functions used in the economic literature). However, it is a standard result of Linear Programming theory that the inverse supply function is step-wise constant and increasing in each factor (see Figure 1-4 for the case of a single commodity[12]). Each horizontal step of the inverse supply function indicates that the commodity is produced by a certain technology or set of technologies in a strictly linear fashion. As the quantity produced increases, one or more resources in the mix (either a technological potential or some resource's availability) is exhausted, and therefore the system must start using a different (more expensive) technology or set of technologies in order to produce additional units of the commodity, albeit at higher unit cost. Thus, each change in production mix generates one step of the staircase production function with a value

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[12] This is so because in Linear Programming the shadow price of a constraint remains constant over a certain interval, and then changes abruptly, giving rise to a stepwise constant functional shape.

higher than the preceding step. The width of any particular step depends upon the technological potential and/or resource availability associated with the set of technologies represented by that step.

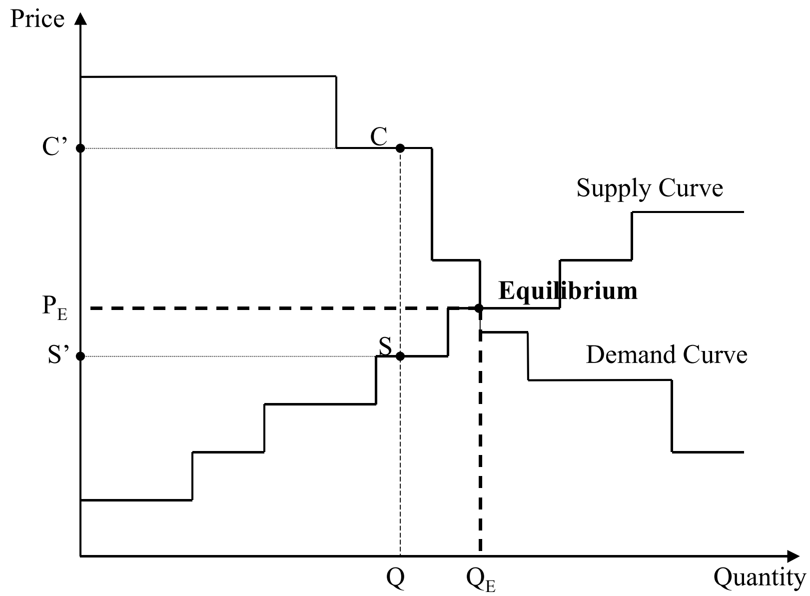

Figure 1-4. Equilibrium in the case of an energy form:
the model implicitly constructs both the supply and the demand curves

In an exactly symmetric manner, the set of consumers of a commodity have their implicit inverse demand function, which is a step-wise constant, decreasing function of the quantity demanded. As shown in Figure 1-4 the supply-demand equilibrium is at the intersection of the two functions, and corresponds to an equilibrium quantity Q_E and an equilibrium price P_E[13]. At price P_E, suppliers are willing to supply the quantity Q_E and consumers are willing to buy exactly that same quantity $Q_{E.}$ Of course, the MARKAL equilibrium concerns many commodities, and the equilibrium is a multi-dimensional analog of the above, where Q_E and P_E are now vectors rather than scalars.

The above description of the MARKAL equilibrium is valid for any energy form that is entirely endogenous to MARKAL, i.e. an energy carrier or a material. In the case of an energy service, MARKAL does not implicitly construct the demand function. Rather, the user *explicitly* defines the demand function by specifying its own price elasticity. Each energy service demand is assumed to have a constant own price elasticity function of the form (see Figure 1-5):

$$D/D_0 = (P/P_0)^E \qquad (3.3\text{-}1)$$

[13] As may be seen in figure 1.3, the equilibrium is not unique. In the case shown, any point on the vertical segment contain the equilibrium is also an equilibrium, with the same Q_E but a different P_E. In other cases, the multiple equilibria will have the same price and different quantities.

Where $\{D_0, P_0\}$ is a reference pair of demand and price values for that energy service over the forecast horizon, and *E* is the (negative) own price elasticity of that energy service demand, as chosen by the user (note that though not shown, this price elasticity also has a time dimension). The pair $\{D_0, P_0\}$ is obtained by solving MARKAL for a reference (base case) scenario. More precisely, D_0 is the demand projection estimated by the user in the reference case based upon explicitly defined relationships to economic and demographic drivers, and P_0 is the shadow price of that energy service demand obtained by running a reference case scenario of MARKAL.

Using Figure 1-4 as an example, the definition of the suppliers' surplus corresponding to a certain point S on the inverse supply curve is the difference between the total revenue and the total cost of supplying a commodity, i.e. the gross profit. In Figure 1-4, the surplus is thus the area between the horizontal segment SS' and the inverse supply curve. Similarly, the consumers' surplus for a point C on the inverse demand curve, is defined as the area between the segment CC' and the inverse demand curve. This area is a consumer's analog to a producer's profit, more precisely it is the cumulative opportunity gain of all consumers who purchase the commodity at a price lower than the price they would have been willing to pay. For a given quantity Q, the total surplus (suppliers' plus consumers') is simply the area between the two inverse curves situated at the left of Q. It should be clear from Figure 1-4 that the total surplus is maximized exactly when Q is equal to the equilibrium quantity Q_E. Therefore, we may state (in the single commodity case) the following Equivalence Principle:

"The equilibrium is reached when the total surplus is maximized"

In the multi-dimensional case, the proof of the above statement is less obvious, and requires a certain qualifying property (called the integrability property) to hold (Samuelson, 1952, Takayama and Judge, 1972). A sufficient condition for the integrability property to be satisfied is realized when the cross-price elasticities of any two energy forms are equal (i.e. $\partial \boldsymbol{P_j} / \partial \boldsymbol{Q_i} = \partial \boldsymbol{P_i} / \partial \boldsymbol{Q_j}$ for all $\boldsymbol{i}, \boldsymbol{j}$).

In the case of commodities that are energy services, these conditions are satisfied in MARKAL because we have assumed zero cross price elasticities. In the case of an energy form, where the demand curve is implicitly derived, it is also possible to show that the integrability property is always satisfied[14] and thus the equivalence principle is valid.

[14] This results from the fact that in MARKAL each price $\boldsymbol{P_i}$ is the shadow price of a balance constraint (see section 1.8), and may thus be (loosely) expressed as the derivative of the objective function $\boldsymbol{F}$ with respect to the right-hand-side of a balance constraint, i.e. $\partial \boldsymbol{F} / \partial \boldsymbol{Q_i}$. When that price is further differentiated with respect to another quantity $\boldsymbol{Q_j}$, one gets $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_i} \bullet \partial \boldsymbol{Q_j}$, which, under mild conditions is always equal to $\partial^2 \boldsymbol{F} / \partial \boldsymbol{Q_j} \bullet \partial \boldsymbol{Q_i}$, as desired.

In summary, the equivalence principle guarantees that the MARKAL supply-demand equilibrium maximizes total surplus. And the total surplus concept has long been a mainstay of social welfare economics because it takes into account both the surpluses of consumers and of producers.[15]

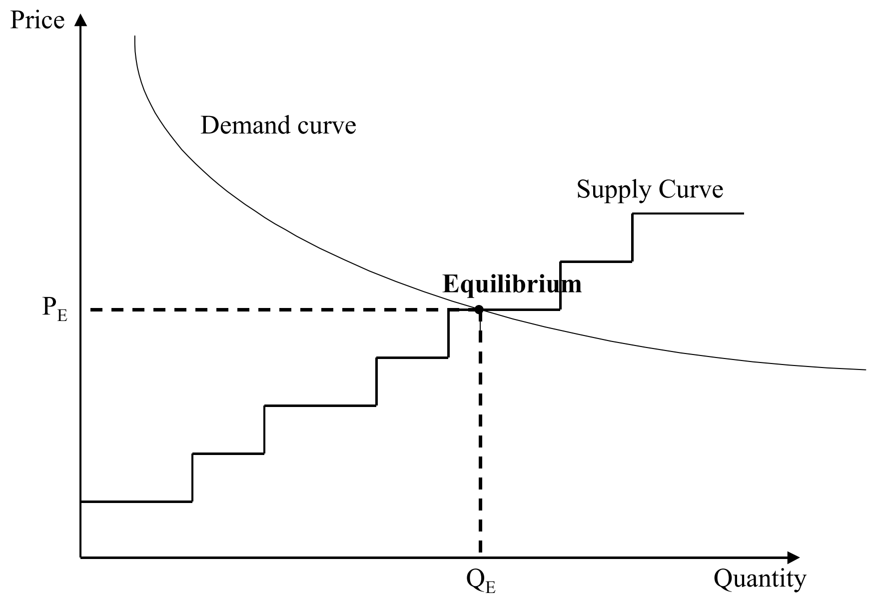

Figure 1-5. Equilibrium in the case of an energy service:
the user, usually with a simple functional form, explicitly provides the demand curve.

In older versions of MARKAL, and in several other least cost bottom-up models, energy service demands are completely exogenously specified by the modeler, and only the cost of supplying these energy services is minimized. Such a case is illustrated in Figure 1-6 where the "inverse demand curve" is a vertical line. The objective of such models was simply the minimization of the total cost of meeting exogenously specified levels of energy service.

[15] See e.g. Samuelson, P., and W. Nordhaus (1977)

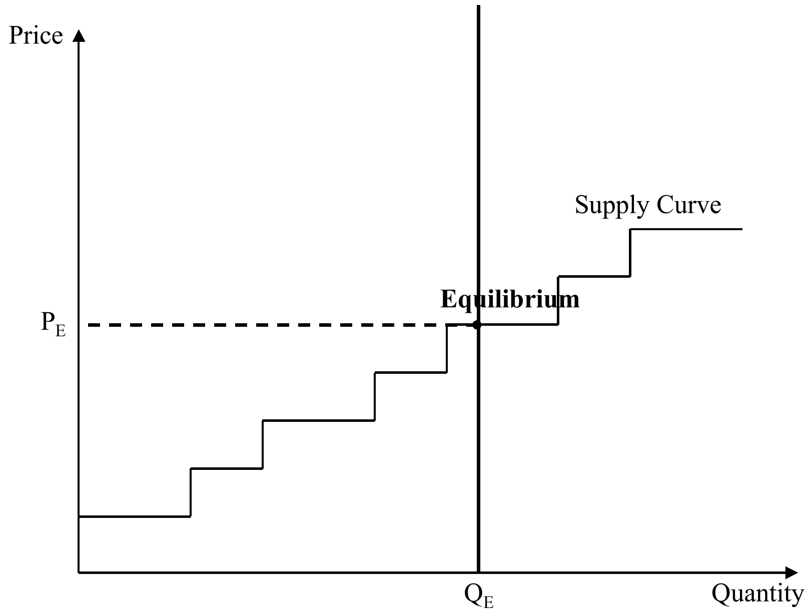

Figure 1-6. Equilibrium when an energy service demand is fixed

1.3.2.6 Competitive energy markets with perfect foresight

Perfectly competitive energy markets are characterized by perfect information and an atomization of the economic agents, which together preclude any of them from exercising market power. That is, neither the level any individual producer supplies, nor the level any individual consumer demands, affects the equilibrium market price (because there are many other buyers and sellers to replace them). It is a standard result of microeconomic theory that the assumption of competitive markets entails that the market price of a commodity is equal to its marginal value in the economy. This is of course also verified in the MARKAL economy, as discussed in the next subsection.

In Standard MARKAL, the perfect information assumption extends to the entire planning horizon, so that each agent has perfect foresight, i.e. complete knowledge of the market's parameters, present and future. Hence, the equilibrium is computed by maximizing total surplus in one pass for the entire set of periods. Such a farsighted equilibrium is also called an *inter-temporal, dynamic* equilibrium.

Note that there are at least two ways in which the perfect foresight assumption may be relaxed: In one variant, agents are assumed to have foresight over a limited portion of the horizon, say one period. Such an assumption of limited foresight is embodied in the SAGE variant of MARKAL, presented in Part III. In another variant, foresight is assumed to be imperfect, meaning that agents may only assume probabilities for certain key future events. This assumption is at the basis of the Stochastic Programming option in Standard MARKAL, which will be discussed in section 1.11.

1.3.2.7 Marginal value pricing

We have seen in the preceding subsections that the MARKAL equilibrium occurs at the intersection of the inverse supply and demand curves, and thus that the equilibrium prices are equal to the marginal system values of the various commodities. From a different angle, the duality theory of Linear Programming (section 1.7.) indicates that for each constraint of the MARKAL linear program there is a dual variable. This dual variable (when an optimal solution is reached) is also called the *shadow price*[16], and is equal to the marginal change of the objective function per unit increase of the constraint's right-hand-side. For instance (section 1.7.3), the shadow price of the balance constraint of a commodity (whether it be an energy form, material, a service demand, or an emission) represents the competitive market price of the commodity.

The fact that the price of a commodity is equal to its marginal value is an important feature of competitive markets. Duality theory does not necessarily indicate that the marginal value of a commodity is equal to the marginal cost of *producing* that commodity. For instance, in the equilibrium shown in Figure 1-7, the equilibrium price does not correspond to *any* marginal supply cost, since it is situated at a discontinuity of the inverse supply curve. In this case, the price is determined by demand rather than by supply, and the term *marginal cost pricing* (so often used in the context of optimizing models) is incorrect. The term *marginal value pricing* is a more generally appropriate term to use.

It is important to note that marginal value pricing *does not imply that suppliers have zero profit*. Profit is exactly equal to the suppliers' surplus, and Figure 1-4 through Figure 1-7 show that it is generally positive. Only the last few units produced may have zero profit, if, and when, their production cost equals the equilibrium price, and even in this case zero profit is not automatic as exemplified in Figure 1-6.

In MARKAL, the shadow prices of commodities play a very important diagnostic role. If some shadow price is clearly out of line (i.e. if it seems much too small or too large compared to the anticipated market prices), this indicates that the model's RES may contain some errors. The examination of shadow prices is just as important as the analysis of the quantities produced and consumed of each commodity and of the technological investments.

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[16] The term Shadow Price is often used in the mathematical economics literature, whenever the price is derived from the marginal value of a commodity. The qualifier 'shadow' is used to distinguish the competitive market price from the price observed in the real world, which may be different, as is the case in regulated industries or in sectors where either consumers or producers exercise market power. When the equilibrium is computed using LP optimization, as is the case for MARKAL, the shadow price of each commodity is computed as the dual variable of that commodity's balance constraint, as will be further developed in section 3.4.

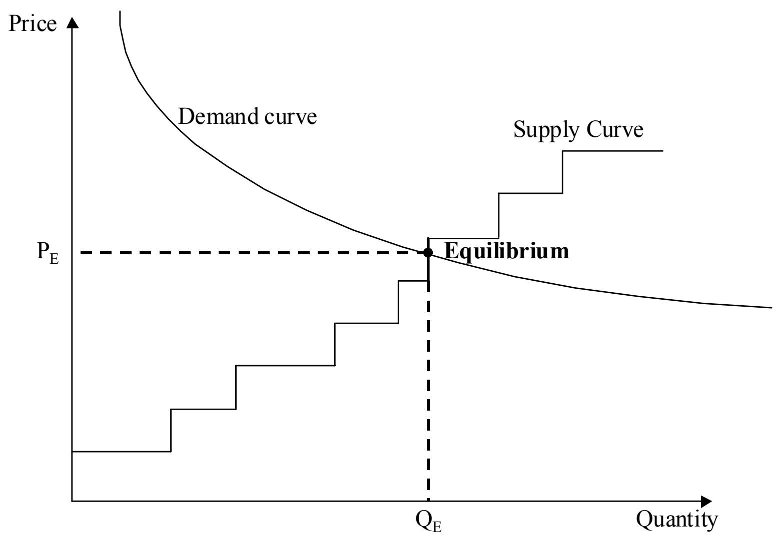

Figure 1-7. Case where the equilibrium price is not equal to any marginal supply cost

1.3.2.8 Profit maximization: the Invisible Hand

An interesting property may be derived from the assumptions of competitiveness. While the initial objective of the MARKAL model is to maximize the overall surplus, it is also true that each economic agent in MARKAL maximizes its own 'profit'. This property is akin to the famous 'invisible hand' property of competitive markets, and may be established rigorously by the following theorem that we state in an informal manner:

Theorem: Let (P,Q*) be the pair of equilibrium vectors. If we now replace the original MARKAL linear program by one where the commodity prices are fixed at value P*, and we let each agent maximize its own profit, there exists a vector of optimal quantities produced or purchased by the agents that is equal to Q*[17].*

This property is very important because it provides an alternative justification for the class of equilibria based on the maximization of total surplus. It is now possible to shift the model's rationale from a global, societal one (surplus maximization) to a local, decentralized one (individual utility maximization). Of course, the equivalence suggested by the theorem is valid only insofar as the marginal value pricing mechanism is strictly enforced—that is, neither individual producers' or individual consumers' behavior affect market prices—both groups are price takers. Clearly, many markets are not competitive in the sense the term has been used here. For example, the behavior of a few, state-owned oil producers has a dramatic affect on world oil prices, that then depart from their

[17] However, the resulting Linear Program has multiple optimal solutions. Therefore, although Q* is an optimal solution, it is not necessarily the one found when the modified LP is solved.

marginal system value. Market power[18] may also exist in cases where a few consumers dominate a market. The entire annual crop of a given region's supply of coffee beans may, for example, be purchased by a handful of purchasers who may then greatly influence prices.

1.4 A simplified description of the MARKAL Optimization Program

This and the next two sections describe the mathematical implementation of the economic concepts described in section 1.3. The computation of the MARKAL partial equilibrium is equivalent to the optimization of a suitably constructed mathematical program. A mathematical optimization program is defined as the minimization (or maximization) of an objective function, subject to constraints. If all the mathematical expressions representing the objective function and the constraints are linear, the problem becomes a Linear Program (LP), which may be solved via powerful standard Linear Programming optimizers.

In this section, we describe conceptually the MARKAL objective and constraints. In the next section, we provide a streamlined mathematical formulation of the MARKAL Linear Program. Section 1.5.3 gives additional details on Linear Programming concepts, and section 1.6 focuses on the computation of the equilibrium.

1.4.1 MARKAL objective function: total system cost

The MARKAL objective is to minimize the total cost of the system, adequately discounted over the planning horizon. Each year, the total cost includes the following elements:

- *Annualized investments* in technologies (see below)*;*
- Fixed and variable annual *Operation and Maintenance (O&M)* costs of technologies;
- Cost of exogenous energy and material *imports* and domestic resource *production* (e.g., mining);
- Revenue from exogenous energy and material *exports;*
- Fuel and material *delivery* costs;
- *Welfare loss* resulting from reduced end-use demands*.* Section 1.6 presents the mathematical derivation of this quantity.
- *Taxes* and *subsidies* associated with energy sources, technologies, and emissions.

In each period, the investment costs are first annualized, as detailed in the next section, before being added to the other costs (which are all annual costs) to obtain the annual cost in each period. MARKAL then computes a total net present value of all annual costs,

[18] An agent has market power if its decisions have an impact on the market price. Monopolies and Oligolpolies are example of markets where one or several agents have market power.

discounted to a user selected reference year, as explained in the next section. This quantity is the one that is minimized by the model to compute the equilibrium.

Important remark: the maximization of total surplus

The minimization of total cost described above is fully equivalent to the maximization of the total surplus defined as the sum of producers' and consumers' surpluses. This equivalence was discussed in detail in section 1.3, and will be further elaborated in section 1.6.

1.4.2 Constraints

While minimizing total discounted cost, the MARKAL model must obey a large number of constraints (the so-called *equations* of the model) which express the physical and logical relationships that must be satisfied in order to properly depict the associated energy system. MARKAL constraints are of several kinds. We list and briefly discuss the main groups of constraints. A full description is postponed until chapter 2. If any constraint is not satisfied the model is said to be *infeasible*, a condition caused by a data error or an over-specification of some requirement.

1.4.2.1 Satisfaction of Energy Service Demands:

Reference (or base) energy service demand projections are developed by the modeler for the entire forecast horizon based on exogenous regional economic and demographic projections (drivers), assumptions regarding each service demand's sensitivity to changes in the assumed driver, and calibration factors to account for planned or assumed structural changes in the energy system. In the reference case, the model must satisfy these demands in each time period, by using the existing capacity and/or by implementing new capacity for end-use technologies (set DMD). These demands are set only for the reference case, but are endogenously determined in alternate scenarios where the prices of energy services vary from the reference case prices (section 1.6). For example, a scenario causing the price of oil to rise would increase the cost of auto travel relative to the reference case and, *ceteris paribus* auto service demand would decline relative to the reference case. An increase in the price of oil relative to the reference case would also affect investment decisions. Over time, as the stock of equipment turns over, more efficient autos may be chosen, tending to lower the cost of auto travel service, thereby increasing auto service demand.[19]

1.4.2.2 Capacity Transfer (conservation of investments):

Investing in a particular technology increases its installed capacity for the duration of the physical life of the technology. At the end of that life, the total capacity for this technology is decreased by the same amount (unless some other investment is decided by

[19] The model can also be run with some or all demands assumed inelastic.

the model at that time). When computing the available capacity in some time period, the model takes into account the capacity resulting from all investments up to that period, some of which may have been made prior to the initial period but are still in operating condition (embodied by the residual capacity of the technology), and others that have been decided by the model at, or after, the initial period, but prior to the period in question.

1.4.2.3 Use of capacity:

In each time period, the model may use some or all of the installed capacity in that period according to the Availability Factor (AF) of that technology. Note that the model may decide to use *less* than the available capacity during certain time-slices, or even throughout one or more whole periods. This will of course occur only if such a decision contributes to minimizing the overall cost. Optionally, there is a provision for the modeler to force specific technologies to use their capacity to their full potential.

1.4.2.4 Balance for Commodities (except electricity and low temperature heat):

In each time period, the production plus import (from other regions) of an energy carrier must be at least as much as the amount consumed and exported (to other regions). In the case of materials, the constraint ensures that the quantity produced and imported be exactly equal to that consumed and exported.

1.4.2.5 Electricity & Heat Balance:

These two types of commodity are defined in each appropriate time-slice and therefore the balance constraint must be satisfied at each time-slice. Therefore, in each time period, (each region,) each season and time-of-day, electricity produced plus electricity imported (from other regions) must be at least as much as electricity consumed, plus electricity exported (to other regions), plus grid losses. A similar balance exists for low temperature heat, although only tracked by season.

1.4.2.6 Peaking Reserve Constraint (electricity and heat sectors only):

This constraint requires that in each time period (and for each region) total available capacity of electricity generating technologies (set ELE) exceeds the average load of the peaking time-slice by a certain percentage. This percentage is the Peak Reserve Factor (ERESERV) and is chosen to insure against possible electricity shortfalls due to uncertainties regarding electricity supply that may decrease capacity in an unpredictable way (e.g. water availability in a reservoir, or unplanned equipment down time). For instance, in a typical cold country, the peaking time-slice will be Winter-Day and the total electric plant generating (EPG) capacity must exceed the Winter-Day demand load by (say) 35%. In a warm country, the peaking time-slice may be Summer-Day.

1.4.2.7 *Base Load (electricity generation only):*

The user may identify which technologies should be considered as base load technologies by MARKAL; i.e. those whose operation must not fluctuate from day to night in a given season. The user may also specify the maximum fraction of night production that may be supplied from all base load technologies. Typically, nuclear plants and solid fuel plants are included in the Base load set, since they require considerable delays to be shut down or restarted.

1.4.2.8 *Seasonal availability factors (electricity and heat sectors only):*

The user may specify seasonal and even day-night limitations on the use of the installed capacity of some electricity or heat generation technologies. This is especially needed when the operation of the equipment depends on the availability of a resource that cannot be stored, such as Wind and Sun, or that can be only partially stored, such as water in a reservoir.

1.4.2.9 *Emission constraint(s):*

The user may impose upper limits on emissions of one or more pollutants (in a region). The limits may be set for each time period separately, so as to simulate a particular emission profile (also called emission target), or in a cumulative fashion. By suitably naming emissions, the user may also separately constrain emissions from specific sectors. Furthermore, the user may also impose global emission constraints that apply to several regions taken together.

1.5 A simplified Mathematical Formulation of the MARKAL Linear Program

The description of the objective function and constraints of the previous section may be translated into a formal set of mathematical expressions. In this section, we present a streamlined formulation of the *equations*[20], which ignores exceptions and some complexities that are not essential to a basic understanding of the principles of the model. The formal mathematical description is contained in chapter 2. The notation used in chapter 2 differs from the simplified one used here.

An optimization problem formulation consists of three types of entities:

- *decision variables:* i.e. the unknowns, to be determined by the optimization,
- *objective function*: expressing the criterion to minimize or maximize, and
- *constraints*: equations or inequations involving the decision variables, that must be satisfied by the optimal solution.

[20] This rather improper term includes equality as well as inequality relationships between mathematical expressions.

The model variables and equations use the following indexes:

r,r': indicates the region (omitted when a single region is modeled);
t: time period;
k: technology;
s: time-slice;
c: commodity (energy or material);
l: price level (used only for multiple sources of the same commodity distinguished only by their unit cost)

1.5.1 Decision Variables

The decision variables represent the choices made by the model. The various kinds of decision variables in a MARKAL model are elaborated here.

INV(r,t,k)**:** new capacity addition for technology ***k***, in period ***t***, in region ***r***. Typical units are PJ/year for most energy technologies, Million tonnes per year (for steel, aluminum, and paper industries), Billion vehicle-kilometers per year (B-vkms/year) for road vehicles and GW for electricity equipment (1GW=31.536 PJ/year). Note that an investment made in period ***t*** is assumed to occur at the beginning of that period, and remains available until the end of its lifetime. Note that the life of a technology may be a fractional multiple of the period length.

CAP(r,t,k): installed capacity of technology ***k***, in period ***t***, in region ***r.*** Typical units: same as for investments.

ACT(r,t,k,s): activity level of technology ***k***, in period ***t***, in region ***r***, during time-slice ***s.*** Typical units: PJ per year for all energy technologies. ***ACT*** variables are not defined for end-use technologies, for which it is assumed that activity is always equal to available capacity. With the exception of the conversion technologies, only annual activity is tracked (and the ***s*** index dropped).

MINING(r,t,c,l): quantity of commodity ***c*** (PJ per year) extracted in region ***r*** at price level ***l*** in period ***t***; the coefficient in the objective function is the unit cost of extracting the commodity, as provided by the user. These are domestic production resources, including physical renewables (such as biomass and municipal solid waste).

IMPORT(r,t,c,l), EXPORT(r,t,c,l): quantity of commodity ***c***, price level ***l,*** (PJ per year) exogenously imported or exported by region ***r*** in period ***t***. It is important to note that the model does not automatically balance the quantities exported and imported. In a global model, these quantities must be controlled by the user, if need be. These variables are convenient whenever endogenous trade is not being contemplated. For example, a single region MARKAL model, or one comprising the 3 regions of North America, would have to treat the imports and exports of oil to North America as exogenous; the coefficient of the import or export variable in the objective function is the unit price of importing or exporting the commodity, provided by the user. Note carefully that these variables may become unnecessary in a global model where all commodities are endogenously traded and priced (see ***TRADE*** variables below).

TRADE(r,t,c,s,imp) and TRADE(r,t,c,s,exp):) quantity of commodity ***c*** (PJ per year) sold (***exp***) or purchased (***imp***) by region ***r*** to/from all other regions in period ***t,*** for time-slice ***s*** (for electricity). This variable represents *endogenous* trade between regions. Trade of any given commodity is balanced by the model in each period, i.e. the algebraic sum of trade variables (possibly multiplied by a loss factor) over all trading regions is equal to 0. Note also that endogenous trade is classified either as *global* or as *bi-lateral*. In the first case, a commodity that exists under the same name in some or all regions, is 'put on the market' and may be bought by any other region. This case is convenient for global commodities such as emission permits or crude oil. In the other case (bi-lateral trade), a pair of regions is first identified, and trade must be balanced between these two regions. In both cases, MARKAL allows the modeler to charge transaction costs (or transportation costs) to quantities exported and imported.

D(r,t,d): demand for end-use ***d*** in region ***r***, in period ***t.*** In non-reference runs, ***D(r,t,d)*** may differ from the reference case demand for ***d***, due to the responsiveness of demands to their own prices (based on each service demand's own-price elasticity).

ENV(r,t,p): Emission of pollutant ***p*** in period ***t*** in region ***r.***

Remark: Note that most commodities are not represented by formal variables in MARKAL (except imported or exported commodities, emissions, and demands, that have their own variables). Instead, the quantity of each commodity produced or consumed is represented in MARKAL as an expression involving the activities of technologies that produce or consume that commodity. This modeling design choice was made in order to keep the model as parsimonious as possible in the number of variables.

1.5.2 Objective function

As explained in the previous section, the objective function is the sum over all regions of the discounted present value of the stream of annual costs incurred in each year of the horizon. Therefore:

$$\boldsymbol{NPV} = \sum_{\boldsymbol{r}=1}^{\boldsymbol{R}} \sum_{\boldsymbol{t}=1}^{\boldsymbol{t}=\boldsymbol{NPER}} (1+\boldsymbol{d})^{\boldsymbol{NYRS}\bullet(1-\boldsymbol{t})} \bullet \boldsymbol{ANNCOST}(\boldsymbol{r},\boldsymbol{t}) \bullet \left(1+(1+\boldsymbol{d})^{-1}+(1+\boldsymbol{d})^{-2}+\cdots+(1+\boldsymbol{d})^{1-\boldsymbol{NYRS}}\right)$$

where:
NPV is the net present value of the total cost for all regions (the MARKAL objective function)
ANNCOST(r,t) is the annual cost in region *r* for period *t*, discussed below
d is the general discount rate
NPER is the number of periods in the planning horizon
NYRS is the number of years in each period *t*
R is the number or regions

Note: the last factor in the expression is the intra period discount factor

The total annual cost ***ANNCOST(r,t)*** is the sum over all technologies ***k*****,** all demand segments ***d*****,** all pollutants ***p***, and all input fuels ***f,*** of the various costs incurred, namely: annualized investments, annual operating costs (including fixed and variable technology costs, fuel delivery costs, costs of extracting and importing energy carriers), minus revenue from exported energy carriers, plus taxes on emissions, plus cost of demand losses.

Mathematically, ***ANNCOST(r,t)*** is expressed as follows:

$$
\begin{aligned}
ANNCOST(r,t) = \Sigma_k \{ & Annualized\_Invcost(r,t,k) * INV(r,t,k) \\
& + Fixom(r,t,k) * CAP(r,t,k) \\
& + Varom(r,t,k) * \Sigma_{s,s} ACT(r,t,k,s) \\
& + \Sigma_c [Delivcost(r,t,k,c) * Input(r,t,k,c) * \Sigma_s ACT(r,t,k,s)] \} \\
+ \Sigma_{c,s} \{ & Miningcost(r,t,c,l) * Mining(r,t,c,t) \\
& + Tradecost(r,t,c) * TRADE(r,t,c,s,i/e) \\
& + Importprice(r,t,c,l) * Import(r,t,c,l) \\
& - Exportprice(r,t,c,l) * Export(r,t,c,l) \} \\
+ \Sigma_c \; & \{ Tax(r,t,p) * ENV(r,t,p) \} \\
+ \Sigma_d \; & \{ DemandLoss(r,t,d) \} \qquad (1.4\text{-}1)
\end{aligned}
$$

where:

Annualized_Invcost(r,t,k) [21] is the annual equivalent of the lump sum unit investment cost, obtained by replacing this lump sum by a stream of equal annual payments over the life of the equipment, in such a way that the present value of the stream is exactly equal to the lump sum unit investment cost, for technology ***k***,

[21] The annualized unit investment cost is obtained from the lumpsum unit investment cost via the following formula:

$$ANNUALIZED\_INVCOST = INVCOST / \sum_{j=1}^{LIFE} (1+h)^{-j}$$

where:
INVCOST is the lumpsum unit investment cost of a technology
ANNUALIZED_INVCOST is the annualized equivalent of INVCOST
LIFE is the physical life of the technology
h is the discount rate used for that technology, also called hurdle rate. If the technology specific discount rate is not defined, the general discount rate d is used instead.

The hurdle rate **h** may be technology, sector and/or region specific, so as to reflect the financial characteristics the analyst deems appropriate for each investment decision. For instance, if the initial capital cost of a car is $20,000, and if its technical life is 10 years, the annualized value of the capital cost assuming an 8% discount rate must be such that a stream of 10 such annualized payments adds up, after discounting, to exactly $20,000. The equivalent annualized value is $3,255, as computed using the expression above.

in period ***t***. Note carefully that by stopping the summation over ***t*** at the end of the horizon, the objective function automatically accounts for the salvage value of all assets stranded at the end of the horizon.
Fixom(k,t,r), Varom(r,t,k), are unit costs of fixed and operational maintenance of technology ***k***, in region ***r*** and period ***t***;
Delivcost(r,t,k,c) is the delivery cost per unit of commodity ***c*** to technology ***k,*** in region ***r*** and period ***t***;
Input(r,t,k,c) is the amount of commodity ***c*** required to operate one unit of technology ***k,*** in region ***r*** and period ***t***;
Miningcost(r,t,c,l) is the cost of mining commodity ***c*** at price level ***l,*** in region ***r*** and period ***t***;
Tradecost(r,t,c) is the unit transport or transaction cost for commodity ***c*** exported or imported by region ***r*** in period ***t***;
Importprice(r,t,c,l) is the (exogenous) import price of commodity ***c,*** in region ***r*** and period ***t***; this price is used only for exogenous trade, see below;
Exportprice(r,t,c,l) is the (exogenous) export price of commodity ***c,*** in region ***r*** and period ***t***; this price is used only for exogenous trade, see below;
Tax(r,t,p) is the tax on emission ***p,*** in region ***r*** and period ***t***; and
DemandLoss(r,t,d) represents the welfare loss (in non reference scenarios) incurred by consumers when a service demand ***d,*** in region ***r*** and period ***t,*** is less than its value in the reference case. This quantity will be derived in subsection 1.6.2.2.

Note that the ***TRADE(r,t,c,s, imp) and TRADE(r,t,c,s,exp)*** variables have no objective function coefficients, other than possibly some transport or transaction charges, since the revenue from export by the exporting region is exactly cancelled by the cost of import by the importing region. This is because the quantity as well as the unit cost of an endogenously traded commodity are both determined by the model.

1.5.3 Constraints (equations)

In what follows, we use bold italics for variable names and italics for parameters and indexes.

1.5.3.1 EQ_DEM(r,t,d) - Satisfaction of Demands

For each time period *t*, region *r*, demand *d*, the total activity of end-use technologies servicing that demand must be at least equal to the specified demand. Hence:

Sum *{over all end-use technologies k, such that k supplies service d}* *of* ***CAP(r,t,k)*** $\geq$ ***D(r,t,d)*** **(1.4-2)**

For example, car travel expressed in billion vehicle kilometers per year may be satisfied by a combination of several types of cars (gasoline, diesel, etc.).

1.5.3.2 EQ_CPT(r,t,k) - Capacity transfer

For each technology *k*, region *r*, period *t,* the available capacity in period *t* is equal to the sum of investments made by the model at past and current periods, and whose physical life has not ended yet, plus capacity in place prior to the modeling horizon and still in place.

> ***CAP(r,t,k)*** *=* ***Sum*** *{over t and all periods t' preceding t and such that t-t'<LIFE(k)} of* ***INV(r,t',k)*** *+ RESID(r,t,k)* **(1.4-3)**
>
> where *RESID(r,t,k)* is the capacity of technology *k* due to investments that were made prior to the initial model period and still exist in region *r* at time *t*.

Example: *Gasoline cars in period t have a total capacity limited by the investments in periods t-1 and t, if the assumed life duration of a car is equal to 2 periods (10 years). If the assumed life is 3 periods, then the capacity in period t is limited by the investments in t-2, t-1 and t. Furthermore, if the life were 12 years instead of 10, then 40% of the capacity would be available in third period after which the investment was made.*

1.5.3.3 EQ_UTL(r,t,k,s) - Use of capacity

For each technology *k*, period *t*, region *r*, and time-slice *s*, the activity of the technology may not exceed its available capacity, as specified by a user defined availability factor

> ***ACT (r,t,k,s)*** *≤ AF(r,t,k,s)* CAPUNIT** ***CAP(r,t,k)*** **(1.4-4)**

Example: *a coal fired power plant's activity in any time-slice is bounded above by 80% of its capacity, i.e.* ***ACT (r,t,k,s)*** *≤ 0 .8*31.536 ** ***CAP(r,t,k),*** *where 31.536 is the unit conversion between units of capacity (Gw) and activity (PJ/year).*

Note that this constraint is not written for end-use technologies, because an activity variable is not defined for them. Activity for end-use technologies is always assumed to be directly proportional to their installed capacities. For non-conversion technologies only annual availability is tracked, so the *s* index is dropped.

1.5.3.4 EQ_BAL(r,t,c,s) - Energy Balance

For each commodity ***c,*** time period ***t***, region ***r***, (and time-slice ***s*** in the case of electricity and low-temperature heat), this constraint requires that the disposition of each commodity may not exceed its supply. The disposition includes consumption in the region plus exports; the supply includes production in the region plus imports.

> ***Sum*** *{over all* ***k****} of: Output(r,t,k,c)*****ACT(r,t,k,s)*** *+* **(1.4-5)**
> ***Sum*** *{over all* ***l****} of:* ***MINING(r,t,c,l)*** *+*
> ***Sum*** *{over all* ***l****} of: FR(s)*****IMP(r,t,c,l)**** *+*

XCVT(c,i) **TRADE(r,t,c,s,i)***

$\geq$ ***or*** $=$

XCVT(c,o) ***TRADE(r,t,c,s,e)** +*
***Sum** {over all **l**} of: FR(s)* ***EXP(r,t,c,l)** +*
Sum** {over all **k**} of: Input(r,t,k,c)*ACT(r,t,k,c,s)***

where:
Input(r,t,k,c) is the amount of commodity ***c*** required to operate one unit of technology ***k**,* in region ***r*** and period ***t**;*
Output(r,t,k,c) is the amount of commodity ***c*** produced per unit of technology ***k**,* and
FR(s) is the fraction of the year covered by time-slice ***s*** (equal to 1 for non-seasonal commodities).
XCVT(c,i) and XCVT(c,o) are transaction or transport costs of importing or exporting one unit of commodity *c.* The constraint is $\geq$ for energy forms and $=$ for materials.

***<u>Example</u>**: Gasoline consumed by vehicles plus gasoline exported to other regions must not exceed gasoline produced from refineries plus gasoline imported from other regions.*

1.5.3.5 EQ_EPK/HPK(r,t,c,s) - Electricity and heat Peak Reserve Constraint

For each time period ***t*** and for region ***r***, there must be enough installed capacity to exceed the required capacity in the season with largest electricity (heat) commodity ***c*** demanded by a safety factor *E* called the *peak reserve factor*.

***Sum** {over all **k**} of CAPUNIT * Peak(r,t,k,c) * FR(s)* ***CAP(r,t,k)** +*
*XCVT(c,i) ** **TRADE(r,t,c,s,i)** + FR(s) ** **IMPORT(r,t,c)***
$\geq$ **(1.4-6)**
*[1+ERESERVE(r,t,c)] * [* ***Sum** {over all **k**} of Input(r,t,k,c) **
*FR(s) ** **ACT(r,t,k,s)** +XCVT(c,o) ** **TRADE(r,t,c,s,e)** + FR(s) **
***EXPORT(r,t,c)**]*

where:
ERESERVE(r,t,c) is the region-specific reserve coefficient, which allows for unexpected down time of equipment, for demand at peak, and for uncertain hydroelectric, solar, or wind availability.
Peak(r,t,k,c) (never larger than 1) specifies the fraction of technology *k*'s capacity in a region *r* for a period *t* and commodity *c* (electricity or heat only) that is allowed to contribute to the peak load. Many types of generating equipment are predictably available during peak load and thus have a peak coefficient equal to unity, whereas others such as wind turbines or solar plants

are attributed a peak coefficient less than 1 since they are on average only fractionally available at peak.
Note that in the peak equation (1.4-6), it is assumed that imports of electricity are contributing to the peak of the importing region (exports are of the *firm power* type).

Example: *A wind turbine typically has a peak coefficient of .25 or .3, whereas a hydroelectric plant, a gas plant, or a nuclear plant typically has a peak coefficient equal to 1.*

1.5.3.6 EQ_ENV(r,t,p) - Emission constraint or tax (optional)

In each region *r*, for each time period ***t,*** this constraint ensures that the total emission of pollutant ***p*** will not be greater than a user-selected upper bound, if such is provided. In MARKAL, pollutants may be emitted when a technology is active, but also when it is inactive (for example a hydro reservoir may emit methane even if no electricity is being produced). Emissions may also occur at the time of construction of the technology, in some instances. In each of these three cases, the emission coefficient is applied to the activity variable, to the capacity variable, or to the investment variable, respectively. This flexibility allows the accurate representation of various kinds of emissions. Technologies may also sequester or otherwise remove emissions as well via the use of a negative emission coefficient.

ENV(r,t,p) = Sum [**over all technologies k of** { **Eminv(r,t,p,k)*** INV(r,t,k)
+ **Emcap(r,t,k,p)***CAP(r,t,k)
+ **Emact(r,t,,k,p)*** Sum **{over s of** ACT(r,t,k,s) }]

and

ENV(r,t,p) ≤ **ENV_Limit(r,t,p)** *(1.4-7)*

where:
Eminv, Emcap, Emact are emission coefficients for pollutant *p* (possibly negative) linked respectively to the construction, the capacity, and the operation of a technology, and
ENV_LIMIT(r,t,p) is the upper limit set by the user on the total emission of pollutant *p* in region *r* at period *t.*

Instead of an emission limit, the user may specify an emission tax *Etax(r,t,p)*. If so, the quantity *ENV(r,t,p) * E tax(r,t,p)* is added to the ***ANNCOST*** expression, penalizing emissions at a constant rate.

Note that emission caps may be set globally for all regions, or for a group of regions, or by sector, etc. It is also possible to set a cumulative emission cap (for a group of time periods).

1.5.3.7 EQ_BAS(r,t,c) - Electricity Baseload constraint

For electricity *c*, in region *r* and period *t,* electricity generating technologies that are labeled as Baseload must produce the same amount of electricity at night as in the day. They may, however, vary their production from season to season. Therefore, for Base load technologies there are only three *ACT* variables (one per season) instead of 6 for other electric generation technologies. The baseload constraint then ensures that only a maximum percentage of the total night-time demand for electricity is met by such plants.

Sum { **over all technologies k consuming electricity c at night of: input(r,t,k,c)* Baseload(r,t,c)***ACT(r,t,k,'N') }
≥ *(1.4-8)*
Sum { **over all baseload technologies k producing electricity c at night of: Output(r,t,k,c)***ACT(r,t,k,'N') }

where:
Baseload(r,t,c) is the maximum share of the night demand for electricity *c* in region *r* and period *t.*

1.5.3.8 EQ_UDC(r,t,u) - User-defined constraints

In addition to the standard MARKAL constraints discussed above, the user interested in developing reference case projections of energy market behavior typically introduces many additional linear constraints to express special conditions.

User defined constraints may serve many functions in MARKAL. Their general purpose is to constrain the optimization problem in some way to account for factors based either on policy or on market behavior that affect investment decisions. For example, there may a user defined constraint limiting investment in new nuclear capacity (regardless of the type of reactor), or dictating that a certain percentage of new electricity generation capacity must be powered by renewable energy sources.

In order to facilitate the creation of a new user constraint, MARKAL provides a *template* for indicating a) the set of variables involved in the constraint, and b) the user-defined coefficients needed in the constraint.

1.5.4 Representation of oil refining in MARKAL

Two alternative approaches are available in MARKAL to represent oil refining. Under the simplified approach that is adopted in the majority of MARKAL models, the refinery is treated as a set of one or more standard MARKAL technologies. But a more sophisticated approach is available where the modeler wishes to specify *bona fide* quality requirement constraints for each refined product, such as: Octane Rating, Sulfur Content, Flash Index, Density, Cetane Number, Viscosity, Reid Vapor Pressure, etc. This approach is embodied in the special oil refining module of MARKAL – whose main features are outlined in this section – and requires additional parameters, variables, and constraints.

1.5.4.1 New sets and parameters

First, in order to properly apply the specific constraints to this sector, the model requires that several sets and other parameters that are unique to this sector be defined, as follows:

- Constant **REFUNIT**: specifies in what units the refinery streams are defined (volume, weight, or energy)
- Set **REF**: contains the list of refining processes (a subset of **PRC**)
- Set **OPR**: contains the intermediate energy carriers (refinery streams) that are produced by the members of **REF**, plus the available crude oils. **OPR** is a subset of **EFS ∪ SYN**. Each stream will enter the production of one or more RPP. The members of **OPR** are expressed in units specified by REFUNIT (volume, weight, or energy).
- Table **CONVERT**: contains the density and energy content (by weight or by volume) of each blending stream. These parameters are used as coefficients of the blending equations to convert them to correct units (se below).
- Set **SPE:** contains the names of the specifications that must be imposed on refined petroleum products (RPP's), such as octane rating, sulfur content, etc.
- Blending tables **BLEND(ENC)** and **BL(ENC)(SPE)**: each **BLEND** or **BL** table is attached to one quality specification, and describes the minimum or maximum quality requirement of one final product **ENC,** as well as the quality content of each stream entering the production of this product.
- Sets **BLENDENC and BLENCSPE**: contains the list of all blending tables. Set **BLENDENC** is for tables with coefficients that are time-invariant, and set **BLENCSPE** is for tables with time-dependent coefficients (not needed unless the quality requirements vary with time).

1.5.4.2 New variables

Once these constants, sets and tables are defined by the user, the model automatically creates blending variables as follows:

$\boldsymbol{BLND_{t,enc,opr}}$: is the amount of blending stock OPR entering the production of the refined product ***enc*** at time period ***t***.

1.5.4.3 New blending constraints

These variables and input parameters are finally used to express the following two types of blending constraints:

Blending by volume (e.g. octane rating)

$$\sum_{opr \in OPR} (\boldsymbol{oct}_{opr} \cdot \boldsymbol{BLND}_{t,enc,opr}) \le \boldsymbol{oct}_{enc} \cdot \sum_{opr \in OPR} \boldsymbol{BLND}_{t,enc,opr}$$

where

- $\boldsymbol{oct}_{opr}$ is the octane content by volume of one unit of stream ***opr*** (itself expressed in units **REFUNIT**. If **REFUNIT** is not equal to *'volume'*, some conversion coefficients (specified in table **CONVERT**) must be applied to the variables of the equation.
- $\boldsymbol{oct}_{enc}$ is the minimum required volume octane rating of the refined product ***enc***
- ***BLND*** variables are expressed in volume units.

<u>Blending by weight (e.g. sulfur content)</u>

$$\sum_{opr \in OPR} (\boldsymbol{sulf}_{opr} \cdot \boldsymbol{BLND}_{t,enc,opr}) \le \boldsymbol{sulf}_{enc} \cdot \sum_{opr \in OPR} \boldsymbol{BLND}_{t,enc,opr}$$

where

- $\boldsymbol{sulf}_{opr}$ is the sulfur content by weight of one unit of stream ***opr*** (itself expressed in units **REFUNIT**. If **REFUNIT** is not equal to *'weight'*, then conversion coefficients (specified in table **CONVERT**) must be applied to the variables of the equation.
- $\boldsymbol{sulf}_{enc}$ is the maximum allowed weight sulfur content of the refined product ***enc***
- ***BLND*** variables are expressed in weight units.

1.6 Elastic demands and the computation of the supply-demand equilibrium

In the preceding section, we have seen that MARKAL does more than minimize the cost of supplying energy services. Instead, it computes a supply-demand equilibrium where both the supply options and the energy service demands are computed by the model. The equilibrium is driven by the user-defined specification of demand functions, which determine how each energy service demand varies as a function of the market price of that energy service. The MARKAL code assumes that each demand has constant own-price elasticity in a given time period, and that cross price elasticities are zero[22]. Economic theory establishes that the equilibrium thus computed corresponds to the maximization of the net total surplus, defined as the sum of the suppliers and of the consumers' surpluses (Samuelson, 1952, Takayama and Judge, 1972). The total net surplus has been often considered a valid metric of societal welfare in microeconomic literature, and this fact confers strong validity to the equilibrium computed by MARKAL.

The MARKAL model is normally run in two contrasted modes: first to simulate some reference case, and then to simulate alternate scenarios, each of which departs in some way from the reference case assumptions and parameters. For instance, an alternate scenario may

[22] The MARKAL-MICRO variant allows non zero cross elasticities.

make different assumptions on the availability or the cost of some new technologies. Or, it may assume that certain energy or environmental policies are being implemented (e.g. emission taxes, or portfolio standards, or efficiency improvements). Or again, a scenario may assume that a certain goal must be reached (such as a cap on emissions), leaving the model free to achieve that goal at least cost. In almost any such alternate scenario, some strain is put on some sectors, resulting in increases in the marginal values of at least some energy services (for example, severe emission reductions may increase the price of auto transportation). In MARKAL, demands self-adjust in reaction to changes (relative to the reference case) of their own price, and therefore the model goes beyond the optimization of the energy sector only. Although MARKAL falls short of computing a general equilibrium, it does capture a major element[23] of the feedback effects not previously accounted for in bottom-up energy models.

In this section, we explain how Linear Programming computes the equilibrium. Additional technical details may be found in Tosato (1980) and in Loulou and Lavigne (1995). One of the first large scale application of these methods was realized in the Project Independence Energy System (PIES, Hogan, 1975), although in the context of demands for final energy rather than for energy services as in MARKAL. It should also be mentioned that the MARKAL-MACRO variant of the model was developed to compute a General Equilibrium, by the adjunction of several macro-economic equations (see PART II)

1.6.1 Theoretical considerations: the Equivalence Theorem

The computational method is based on the equivalence theorem presented in section 1.6.5, which we restate here:

"A supply/demand economic equilibrium is reached when the sum of the producers and the consumers surpluses is maximized"

Figure 1-5 provides a graphical illustration of this theorem in a case where only one commodity is considered.

1.6.2 Mathematics of the MARKAL equilibrium

1.6.2.1 Defining demand functions

For each demand category, define a demand curve, i.e. a function determining demand as a function of price. In MARKAL, a constant elasticity relationship is used, represented as:

[23] It has been argued, based on strong circumstantial evidence, that the change in demands for energy services is indeed the main feedback economic effect of energy system policies (Loulou and Kanudia, 2002)

$$\boldsymbol{DM_i(p\ ) = K_i \bullet p_i^{E_i}} \qquad (1.6-1)$$

where DM_i is the i^{th} demand, p_i is its price, taken to be the marginal cost of procuring the i^{th} commodity, and E_i is the own price elasticity of that demand. Note that although the region and time indexes *r, t* have been omitted in this notation, all quantities in Equation (1.6-1), including the elasticities are region (if appropriate) and time dependent. Constant K_i may be obtained if one point $(p^0{}_i, DM^0{}_i)$ of the curve is known (the reference case). Thus Equation (1.6-1) may be rewritten as:

$$\boldsymbol{DM_i / DM_i^0 = (p_i / p_i^0)^{E_i}} \qquad (1.6-2)$$

Or its inverse:

$$\boldsymbol{p_i = p_i^0 \cdot (DM_i / DM_i^0)^{1/E_i}}$$

where the superscript '0' indicates the reference case, and the elasticity E_i is negative.

1.6.2.2 Formulating the MARKAL equilibrium

With inelastic demands, the MARKAL model may be written as the following Linear Program

$$\begin{array}{lll} \boldsymbol{Min} & \boldsymbol{c \cdot X} & (1.6-3) \\ \boldsymbol{s.t.} & \sum_{\boldsymbol{k}} \boldsymbol{CAP_{k,i}(t) \geq DM_i(t)} \qquad \boldsymbol{i} = 1,2,..,\boldsymbol{I};\ \ \boldsymbol{t} = 1,..,\boldsymbol{T} & (1.6-4) \\ \boldsymbol{and} & \boldsymbol{B \cdot X \geq b} & (1.6-5) \end{array}$$

where $\boldsymbol{X}$ is the vector of all variables and $\boldsymbol{I}$ is the number of demand categories. In words:

- (1.6-3) expresses the total discounted cost to be minimized.
- (1.6-4) is the set of demand satisfaction constraints (where the ***CAP*** variables are the capacities of end-use technologies, and the ***DM*** right-hand-sides are the exogenous demands to satisfy).
- (1.6-5) is the set of all other constraints.

With elastic demands, the role of MARKAL is to compute a supply/demand equilibrium among equations (1.6-3) through (1.6-5) where both the supply side and the demand side adjust to changes in prices, and the prices charged by the supply side are the marginal costs of the demand categories, (i.e. p_i is the marginal cost of producing demand DM_i.) A priori, this seems to be a difficult task, because the prices used on the demand side are computed as part of the solution to equations (1.6-3), (1.6-4), and (1.6-5). The Equivalence Theorem, however, states that such an equilibrium is reached as the solution of the following mathematical program, where the objective is to maximize the net total surplus:

$$\boldsymbol{Max} \quad \sum_i \sum_t \left(\boldsymbol{p}_i^0(\boldsymbol{t}) \bullet \left[\boldsymbol{DM}_i^0(\boldsymbol{t})\right]^{-1/\boldsymbol{E}_i} \bullet \int_a^{\boldsymbol{DM}_i(\boldsymbol{t})} \boldsymbol{q}^{1/\boldsymbol{E}_i} \cdot \boldsymbol{dq} \right) - \boldsymbol{c} \cdot \boldsymbol{X} \qquad (1.6-6)$$

$$\boldsymbol{s.t.} \quad \sum_k \boldsymbol{CAP}_{k,i}(\boldsymbol{t}) - \boldsymbol{DM}_i(\boldsymbol{t}) \geq 0 \qquad \boldsymbol{i} = 1,..,\boldsymbol{I};\ \boldsymbol{t} = 1,..,\boldsymbol{T} \qquad (1.6-7)$$

$$\boldsymbol{and} \quad \boldsymbol{B} \cdot \boldsymbol{X} \geq \boldsymbol{b} \qquad (1.6-8)$$

Where ***X*** is the vector of all MARKAL variables with associated cost vector ***c***, (1.6-6) expresses the total net surplus, and ***DM*** is now a vector of *variables* in (1.6-7), rather than fixed demands.

The integral in (1.6-6) is easily computed, yielding the following maximization program:

$$\boldsymbol{Max} \quad \sum_i \sum_t \left(\boldsymbol{p}_i^0(\boldsymbol{t}) \bullet \left[\boldsymbol{DM}_i^0(\boldsymbol{t})\right]^{-1/\boldsymbol{E}_i} \bullet \boldsymbol{DM}_i(\boldsymbol{t})^{1+1/\boldsymbol{E}_i} /(1+1/\boldsymbol{E}_i) \right) - \boldsymbol{c} \cdot \boldsymbol{X} \qquad (1.6-6)'$$

$$\boldsymbol{s.t.} \quad \sum_k \boldsymbol{CAP}_{k,i}(\boldsymbol{t}) \geq \boldsymbol{DM}_i(\boldsymbol{t}) \qquad \boldsymbol{i} = 1,..,\boldsymbol{I};\ \boldsymbol{t} = 1,..,\boldsymbol{T} \qquad (1.6-7)'$$

$$\boldsymbol{B} \cdot \boldsymbol{X} \geq \boldsymbol{b} \qquad (1.6-8)'$$

1.6.2.3 Linearization of the Mathematical Program

The Mathematical Program embodied in (1.6-6)', (1.6-7)' and (1.6-8)' has a non-linear objective function. Because the latter is separable (i.e. does not include cross terms) and concave in the DM_i variables, each of its terms is easily linearized by piece-wise linear functions which approximate the integrals in (1.6-6). This is the same as saying that the inverse demand curves are approximated by stair-case functions, as illustrated in Figure 1-8. By so doing, the resulting optimization problem becomes linear again. The linearization proceeds as follows.

a) For each demand category ***i***, the user selects a range within which it is estimated that the demand value ***$DM_i(t)$*** will always remain, even after adjustment for price effects (for instance the range could be equal to the reference demand ***$DM^o_i(t)$*** plus or minus 50%). The smallest range value is denoted ***$DM(t)_{\min}$***.
b) Select a grid that divides each range into a number n of equal width intervals. Let $ß_i(t)$ be the resulting common width of the grid, $ß_i(t)= R_i(t)/n$. See Figure 1-8 for a sketch of the non-linear expression and of its step-wise constant approximation. The number of steps, n, should be chosen so that the step-wise constant approximation remains close to the exact value of the function.
c) For each demand segment ***$DM_i(t)$*** define *n* step-variables (one per grid interval), denoted $s_{1,i}(t)$, $s_{2,i}(t)$, …, $s_{n,i}(t)$. Each *s* variable is bounded below by 0 and above by $\beta_i(t)$. One may now replace in equations (1.6-6) and (1.6-7) each ***$DM_i(t)$*** variable by the sum of the *n*-step variables, and each non-linear term in the objective function by a weighted sum of the *n* step-variables, as follows:

$$DM_i(t) = DM(t)_{\min} + \sum_{j=1}^{n} s_{j,i}(t) \qquad 1.6-9$$

and

$$DM_i(t)^{1+1/E_i} \cong DM(t)_{\min}^{1+1/E_i} + \sum_{j=1}^{n} A_{j,s,i}(t) \bullet s_{j,i}(t) / \beta_i(t) \qquad 1.6-10$$

The resulting Mathematical Program is now fully linearized.

Remark: instead of maximizing the linearized objective function, MARKAL minimizes its negative, which then has the dimension of a cost. The portion of that cost representing the negative of the consumer surplus is akin to a *welfare loss*, a term already used in subsection 1.4.2

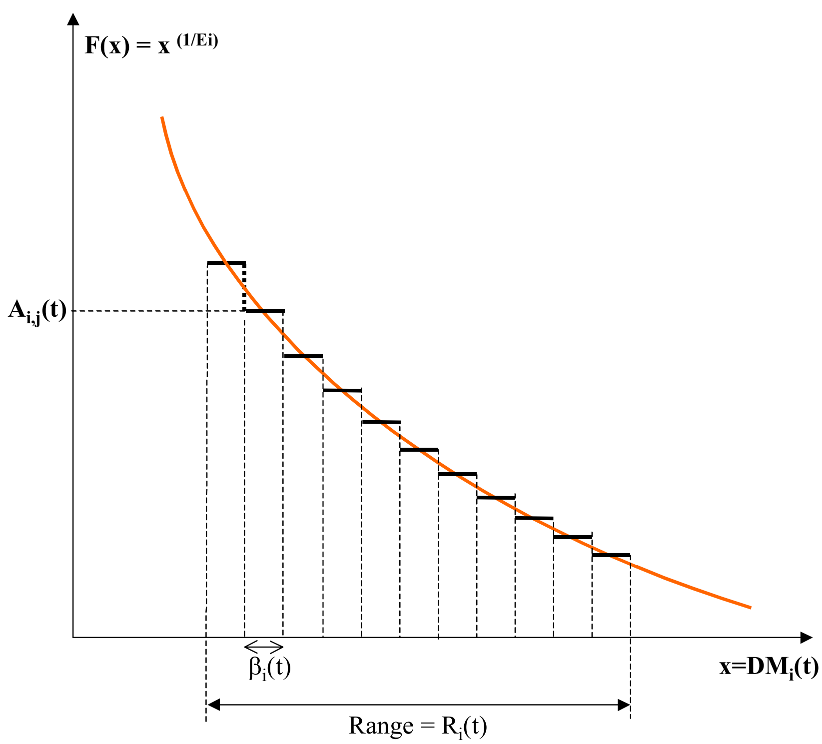

Figure 1-8. Step-wise approximation of the non-linear terms in the objective function

1.6.2.4 Calibration of the demand functions

Besides selecting elasticities for the various demand categories, the user must evaluate each constant $K_i(t)$. To do so, we have seen that one needs to know one point on each demand function in each time period, $\{ p^0_i(t), DM^0_i(t) \}$. To determine such a point, we perform a single preliminary run of the inelastic MARKAL model (with exogenous $DM^0_i(t)$), and use the resulting shadow prices $p^0_i(t)$ for all demand constraints, in all time periods for each region.

1.6.2.5 Computational considerations

Each demand segment that is elastic to its own price requires the definition of as many variables as there are steps in the discrete representation of the demand curve, for each period and region. Each such variable has an upper bound, but is otherwise involved in no new constraint. Therefore, the linear program is augmented by a number of variables, but does not have more constraints than the initial inelastic LP (with the exception of the upper bounds). It is well known that with the modern LP codes, the number of variables has little or no impact on computational time in Linear Programming, whether the variables are upper bounded or not. Therefore, the inclusion in MARKAL of elastic demands has a very minor impact on computational time or on the tractability of the resulting LP. This is an important observation in view of the very large LP's that result from representing multi-regional and global models in MARKAL.

1.6.3 Two additional options for modeling elastic demands

1.6.3.1 The MARKAL-MICRO alternative formulation of the elastic demand curves

In this variant of the partial equilibrium formulation, the partial equilibrium program embodied by equations (1-6.6') to (1-6.8') is kept in its non-linear form (it is not linearized). The MARKAL equilibrium must then be solved by a Non-Linear programming code. This variant is implemented for single-region models, and is fully described in chapter 2. [24]

1.6.3.2 Income elasticity of demands

In Standard MARKAL (with linearized demand curves), it is possible to further refine the demand curves by allowing the demands to depend not only on prices, but also on national income. In this variant, equation (1—6.2) is replaced by (1–6.11) below:

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[24] One advantage of MARKAL-MICRO would be that it could accept non-zero cross-price elasticities. However, this additonal capability is not yet present in the GAMS implementation. This feature would be particularly useful in the transport sector. For instance, considering freight transport, it may be assumed that some endogenous substitution will occur between truck and rail modes. This is accomplished in MARKAL-MICRO via a constant elasticity of substitution (CES) demand function for the aggregate freight transport and also for passenger transport. In the case of freight transport, a relationship enables some substitution between two MARKAL demands, road transport and rail transport as in the following equation

$$\boldsymbol{Q} = \left(\beta_1^{\frac{1}{\sigma}}(\boldsymbol{Q}_1 \cdot \boldsymbol{e}^{\gamma t})^{\frac{\sigma-1}{\sigma}} + \beta_2^{\frac{1}{\sigma}}(\boldsymbol{Q}_2 \cdot \boldsymbol{e}^{\gamma t})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where Q1 and Q2 are the demands for truck and rail transport respectively, and the sigma parameter is the elasticity of substitution

$$DM_i / DM_i^0 = (Y_t / Y_0)^{\alpha_i} \bullet (p_i / p_i^0)^{E_i} \qquad (1.6-11)$$

where
Y_t is the national income at time t, and
α_i is the (positive) income elasticity of the i^th demand

It should be carefully noted that Y_t *is not a MARKAL variable*, and therefore has to be estimated outside the model and injected as an input parameter.

1.6.4 Interpreting MARKAL costs, surplus, and prices

It is important to note that, instead of maximizing the net total surplus, MARKAL minimizes its negative (plus a constant), obtained by changing the signs in expression (1.6-6). For this and other reasons, it is inappropriate to pay too much attention to the meaning of the *absolute* objective function values. Rather, examining the difference between the objective function values of two scenarios is a far more useful exercise. That difference is of course, the negative of the difference between the net total surpluses of the two scenario runs.

Note again that the popular interpretation of shadow prices as the *marginal cost* of model constraints is only valid if the model has inelastic demands. When demands are elastic, the shadow price of a constraint is, by definition, the incremental value of the objective function per unit of that constraint's right hand side (RHS). The interpretation is that of a *surplus loss* per unit of the constraint's RHS. The difference is subtle but nevertheless important. For instance, the shadow price of some eenrgy service (say the production of steel) is not necessarily the marginal cost of producing one tonne of steel. Indeed, when the RHS of the steel demand balance constraint is increased by one unit, one of two things may occur: either the system produces one more unit of steel, or else the steel demand is adjusted downward by one tonne. It is therefore correct to speak of shadow prices as the marginal *system value* of a commodity, rather than the marginal *cost* of procuring that commodity. A modeler interested in obtaining the marginal costs of the various demand categories, could re-run the model with the demands fixed at their optimal levels in the elastic run.

1.7 Additional economic aspects of the MARKAL equilibrium

This section is not strictly needed for a basic understanding of the MARKAL model and may be skipped on a first reading. However, it does provide additional insights into the microeconomics of the MARKAL equilibrium. In particular, it contains a review of the theoretical foundation of Linear Programming and Duality Theory. This knowledge may help the user to better understand the role shadow prices and reduced costs play in the economics of the MARKAL model. More complete treatments of Linear Programming and Duality Theory may be found in several standard textbooks such as Chvátal (1983) or Hillier and Lieberman (1990 and subsequent edtions). Samuelson and Nordhaus (1977) contains a treatment of micro-economics based on mathematical programming.

1.7.1 A brief primer on Linear Programming and Duality Theory

1.7.1.1 Basic definitions

In this section, the superscript t following a vector or matrix represents the transpose of that vector or matrix. A Linear Program may always be represented as the following *Primal Problem* in canonical form:

$$\text{Max } c^t x \qquad \textbf{(1.7-1)}$$

$$\text{s.t.} \quad Ax \le b \qquad \textbf{(1.7-2)}$$

$$x \ge 0 \qquad \textbf{(1.7-3)}$$

where x is a vector of *decision variables*, $c^t x$ is a linear function representing the *objective* to maximize, and $Ax \le b$ is a set of inequality *constraints*. Assume that the LP has a finite optimal solution, x^*.

Then each decision variable, x^*_j falls into one of three categories. x^*_j may be:

- equal to its lower bound (as defined in a constraint), or
- equal to its upper bound, or
- strictly between the two bounds.

In the last case, the variable x^*_j is called *basic*. Otherwise it is *non-basic*.

For each primal problem, there corresponds a *Dual problem* derived as follows:

$$\text{Min } b^t y \qquad \textbf{(1.7-4)}$$

$$\text{s.t.} \quad A^t y \ge c \qquad \textbf{(1.7-5)}$$

$$y \ge 0 \qquad \textbf{(1.7-6)}$$

Note that the number of dual variables equals the number of constraints in the primal problem. In fact, each dual variable y_i may be assigned to its corresponding primal constraint, which we represent as: $A_i x \le b_i$, where A_i is the i^{th} row of matrix A.

1.7.1.2 Duality Theory

Duality theory consists mainly of three theorems[25]: weak duality, strong duality, and complimentary slackness.

Weak Duality Theorem

[25] Their proofs may be found in most textbooks on Linear Programming, such as Chvatal (1983) or Hillier and Lieberman (1990).

If x is any feasible solution to the primal problem and y is any feasible solution to the dual, then the following inequality holds:

$$c^t x \le b^t y \qquad \textbf{(1.7-7)}$$

The weak duality theorem states that the value of a feasible dual objective is never smaller than the value of a feasible primal objective. The difference between the two is called the *duality gap* for the pair of feasible primal and dual solutions *(x,y)*.

Strong duality theorem

If the primal problem has a *finite, optimal* solution x^*, then so does the dual problem (y^*), and both problems have the same optimal objective value (their duality gap is zero):

$$c^t x^* = b^t y^* \qquad \textbf{(1.7-8)}$$

Note that the optimal values of the dual variables are also called the *shadow prices* of the primal constraints.

Complementary Slackness theorem

At an optimal solution to an LP problem:

- If y^*_i is > 0 then the corresponding primal constraint is satisfied at equality (i.e. $A_i x^* = b_i$ and the i^{th} primal constraint is called *tight*. Conversely, if the i^{th} primal constraint is *slack* (not tight), then $y^*_i = 0$,
- If x^*_j is basic, then the corresponding dual constraint is satisfied at equality, (i.e. $A^t_j{}^* y = c_j$, where A^t_j is the j^{th} row of A^t, i.e. the j^{th} column of A. Conversely, if the j^{th} dual constraint is slack, then x^*_j is equal to one of its bounds.

1.7.2 Sensitivity analysis and the economic interpretation of dual variables

It may be shown that if the j^{th} RHS b_j of the primal is changed by an infinitesimal amount d, and if the primal LP is solved again, then its new optimal objective value is equal to the old optimal value plus the quantity $y_j^* \bullet d$, where y_j^* is the optimal dual variable value.

Loosely speaking[26], one may say that *the partial derivative of the optimal primal objective function's value with respect to the RHS of the* i^{th} *primal constraint is equal to the optimal shadow price of that constraint.*

1.7.2.1 Economic Interpretation of the Dual Variables

[26] Strictly speaking, the partial derivative may not exist for some values of the RHS, and may then be replaced by a directional derivative.

If the primal problem consists of maximizing the surplus (objective function $c^t x$), by choosing an activity vector x, subject to upper limits on several resources (the b vector) then:

- Each a_{ij} coefficient of the dual problem matrix, A, then represents the consumption of resource b_j by activity x_i;
- The optimal dual variable value y^*_j is the unit price of resource j, and
- The total optimal surplus derived from the optimal activity vector, x^*, is equal to the total value of all resources, b, priced at the optimal dual values y^* (strong duality theorem).

Furthermore, each dual constraint $A^t_j{}^*y \geq c_j$ has an important economic interpretation. Based on the Complementary Slackness theorem, if an LP solution x^* is optimal, then for each x^*_j that is not equal to its upper or lower bound (i.e. each basic variable x^*_j), there corresponds a *tight* dual constraint $y^*A'_j = c_j$, which means that the revenue coefficient c_j must be exactly equal to the cost of purchasing the resources needed to produce one unit of x_j. In economists' terms, *marginal cost equals marginal revenu, and both are equal to the market price of* $\mathbf{x^*}_j$. If a variable is not basic, then by definition it is equal to its lower bound or to its upper bound. In both cases, the unit revenue c_j need not be equal to the cost of the required resources. The technology is then either non-competitive (if is is at its lower bound) or it is super competitive and makes a surplus (if it is at its upper bound).

1.7.2.2 Reduced Surplus and Reduced Cost

In a maximization problem, the difference $y^*A'_j - c_j$ is called the *reduced surplus* of technology j, and is available from the solution of a MARKAL problem. It is a useful indicator of the competitiveness of a technology, as follows:

- if x^*_j is at its lower bound, its unit revenue c_j is *less* than the resource cost (i.e. its reduced surplus is positive). The technology is not competitive (and stays at its lower bound in the equilibrium);
- if x^*_j is at its upper bound, revenue c_j is *larger* than the cost of resources (i.e. its reduced surplus is negative). The technology is super competitive and produces a surplus; and
- if x^*_j is basic, its reduced surplus is equal to 0. The technology is competitive but does not produce a surplus

We now restate the above summary in the case of a Linear program that minimize cost subject to constraints:

$$\begin{aligned} & Min\ c^t x \\ s.t. \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

In a minimization problem (such as the usual formulation of MARKAL), the difference $c_j - y^*A'_j$ is called the *reduced cost* of technology j. The following holds:

- if x^*_j is at its lower bound, its unit cost c_j is *larger* than the value created (i.e. its reduced cost is positive). The technology is not competitive (and stays at its lower bound in the equilibrium);
- if x^*_j is at its upper bound, its cost c_j is *less* than the value created (i.e. its reduced cost is negative). The technology is super competitive and produces a profit; and
- if x^*_j is basic, its reduced cost is equal to 0. The technology is competitive but does not produce a profit

The reduced costs/surpluses may thus be used to rank all technologies, *including those that are not selected by the model.*

1.7.3 MARKAL Dual constraints (price formation equations)

In MARKAL, the objective function is the negative of a surplus (i.e. a cost) to minimize; therefore, the dual variables have the dimension of a price, and the price formation equations are expressed in monetary value per unit of the corresponding constraint. The reader may consult the MARKAL LP matrix provided as a separate spreadsheet to verify that the equations below are indeed the correct dual constraints.

1.7.3.1 Dual constraint corresponding to an INVestment variable [INV(r,t,k)]

A glance at the MARKAL LP matrix shows that, in the absence of emission constraints, for a given technology *k* in region *r and* period *t*, the only non zero matrix coefficients in the *INV(r,t,k)* column have value 1, and appear at the intersections of the column with the capacity transfer constraints *EQ_CPT(r,t',k), with t'= t, t-1, t-2, ..., t-LIFE+1*. Therefore, the dual constraint reads:

INVCOST(r,t,k) ≥ Sum $_{\text{over } t'=t\text{-}LIFE+1 \text{ to } t'=t}$ of discounted shadow prices of the EQ_CPT(k,t') constraint.

Recall that the shadow price of *EQ_CPT(k,t')* is the value of one additional unit of capacity for period *t'*. The Complementary Slackness theorem indicates that at an LP optimal solution, the above constraint must hold with equality whenever the model invests in technology *k* at an intermediate level (basic). Hence, in this case, the dual constraint says that the sum (over the equipment life) of its discounted unit system values of capacity is equal to the unit investment cost, i.e. the reduced cost is equal to zero.

1.7.3.2 Economic balance of running a technology [ACT(r,t,k)]

The *ACT(r,t,k)* column of the matrix has coefficients with the balance constraint and emission constraints (if any). Therefore, the dual constraint of variable *ACT(r,t,k)* is:

*Sum {over all c} of: output(r,t,k,c)*Shadow Price of Commodity(r,t,c) -*
*Sum {over all c'} of: input(r,t,k,c')*Shadow Price of Commodity(r,t, c') -*

*Sum {over all c} of: Delivcost(r,t,k,c) *Input(r,t,k,c) –*
*Sum {over all p} of: emact(r,t,k,p)*Shadow Price of Pollutants p ≤ Varom(r,t,k)*

At optimality, the above constraint is tight (i.e. it must hold with equality) whenever the technology is indeed active at a basic level, and therefore the net value of the outputs minus the value of the inputs of the technology is equal to the variable cost of running the technology.

1.7.3.3 Economic profitability of a mining, an import or an export node [SRC(r,t,c,l)]

The following are the dual constraints of a mining, an import, and an export variable, respectively (assuming no transaction cost).

Shadow Price of Commodity (r,c,t) ≤ Miningcost(r,t,c,l)
Shadow Price of Commodity (r,c,t) ≤ Importprice(r,t,c,l)
Shadow Price of Commodity (r,c,t) ≥ Exportprice(r,t,c,l)

At optimal, these dual constraints are tight whenever the mining, export, or import variable is basic. This ensures that the system mines, imports, or exports commodities whose system value is equal to the procurement price (reduced cost =0). In all cases, if the dual constraint is not tight (e.g. if the system value of an import is less than its price), the Complementary Slackness condition guarantees that the activity (mining, import, or export) is at its lower bound (upper for exports).

1.8 Damage Costs

This MARKAL option is intended for modelers who wish to evaluate the environmental externalities created by an energy system. For instance, emissions of toxic or environmentally harmful pollutants from the energy system create social costs linked to impacts of the pollution on human health and the environment. In another example, in global studies of GHG emissions, it may be of interest to evaluate the impact of GHG emissions on concentrations and ultimately on damages created by climate change induced by increased concentration of GHG's.

Until recently, in most studies involving bottom-up models emission externalities have been modeled in one of two ways: either by introducing an emission tax, or by imposing emission caps. In the first case, the tax is (ideally) supposed to represent the external cost created by one unit of emission. However, using a tax assumes that the cost is a linear function of emissions. In the second approach, it is assumed that such a cost is unknown but that exogenous studies (or regulations, treaties, etc.) have defined a level of acceptable emissions that should not be exceeded. However, using this approach is akin to making the implicit assumption that emissions in excess of the cap have an infinite external cost. Both of these approaches have merit and have been successfully applied to many MARKAL studies.

It is however possible to extend these two approaches by introducing an option to better model the cost of damages created by emissions. Thus, the option discussed in this section extends the concept of an emission tax by modeling more accurately the assumed cost of damages due to emissions of a pollutant.

Two approaches are modeled in Standard MARKAL:

1. the environmental damages are computed ex-post, without feedback into the optimization process, and
2. the environmental damages are part of the objective function and therefore taken into account in the optimization process.

In both approaches, a number of assumptions are made:

- damages in a region are due to emissions in the same region (no trans-boundary pollution);
- damages in a given time period are linked to emissions in that same period only (damages are not delayed, nor are they cumulative); and
- damages due to several pollutants are the sum of damages due to each pollutant (no cross impacts).

In a given time period, and for a given pollutant, the damage cost is modelled as follows:

$$\boldsymbol{DAM}(\boldsymbol{EM}) = \alpha \bullet \boldsymbol{EM}^{\beta} \qquad \textbf{(1.8-1)}$$

where:

EM is the emission in the current period;
DAM is the damage cost in the current period;
β >1 is a shape parameter for the damage cost function, selected by the user; and
α is a calibrating parameter, which may be obtained from dose-response studies that allow the computation of the marginal damage cost per unit of emission in period 1. If we denote this marginal cost by $\boldsymbol{MC_1}$, the following holds:

$$\boldsymbol{MC}_1 = \alpha \cdot \beta \cdot \boldsymbol{EM}_1^{\beta-1}$$

where $\boldsymbol{EM_1}$ is the emission in period ***1***

and therefore expression (**1.8-1**) may be re-written as:

$$\boldsymbol{DAM}(\boldsymbol{EM}) = \boldsymbol{MC}_1 \cdot (\boldsymbol{EM}_1^{1-\beta} / \beta) \cdot \boldsymbol{EM}^{\beta} \qquad \textbf{(1.8-2)}$$

The approach to damage costs described in this section applies more particularly to local pollutants. Extension to global emissions such GHG emissions requires the use of a global MARKAL model and a reinterpretation of the equations discussed above.

See chapter 2 for details on the implementation, switches, data and model variables and equations.

Important remark: The modeling of damage costs via equation (**1.8-2)** introduces a non-linear term in the objective function if the $\boldsymbol{\beta}$ parameter is strictly larger than unity. This in turn requires that the model be solved via a Non-Linear Programming (NLP) algorithm rather than a LP algorithm. However, this departure from linearity is much less consequential than the one observed in the Lumpy investment case (section 1.9), because the resulting Non-Linear Program remains convex as long as the shape parameter is equal to or larger than unity. For additional details on convex programming, see Nemhauser et al (1989) . In particular, NLP algorithms are efficient (particularly when all non-linearity is strictly in the objective function) and the duality theory is entirely preserved. If linearity must be preserved (for instance if problem instances are very large), it would be rather easy to approximate expression (1.8-2) by a sequence of linear segments with increasing slopes, and thus to obtain a Linear Program, just as was done in the linearization of the demand curves described in section 1.6.

1.9 The Lumpy Investment option

In some cases, the linearity property of the MARKAL model may become a drawback for the accurate modeling of certain investment decisions. Consider for example a MARKAL model for a relatively small community such as a city. For such a scope, the *granularity* of some investments may have to be taken into account. For instance, the size of an electricity generation plant proposed by the model would have to conform to an implementable minimum size (it would make no sense to decide to construct a 50 MW nuclear or coal fired plant). Another example for multi-region modeling might be whether or not to build cross-region electric grid(s) or gas pipeline(s) in discrete size increments.

For other types of investments, size does not matter: for instance the model may decide to purchase 10,950.52 electric cars, which is easily rounded to 10,950 without any serious inconvenience. The situation is similar for a number of residential or commercial heating devices, or for the capacity of wind turbines or industrial boilers, or for any technologies with relatively small minimum feasible sizes. Such technologies would not be candidates for treatment as "lumpy" investments.

It is the user's responsibility to decide that certain technologies should (or should not) respect the minimum size constraint, weighing the pros and cons of so doing. This section explains how the MARKAL LP may be transformed into a Mixed Integer Program (MIP) to accommodate minimum or multiple size constraints, and states the consequences of so doing on computational time and on the interpretation of duality results.

1.9.1 Options for modeling lumpy investments

Three options for modeling lumpy investments are available, each of which activated by specifying certain parameters (see chapter 2 for details on each parameter's name and format). Each option corresponds to the representation of a particular real life situation.

Option A: Investment in technology ***k*** is either zero (no investment) or a multiple of some fixed number ***BLOCK(k)***. This option is suitable for any standard, replicable technology such as a nuclear plant, a fossil fuel plant, etc.

Option B: Investment in technology ***k*** is either zero (no investment) or is equal to some fixed number ***BLOCK(k)***. This option is suitable for standard technologies for which no more than one unit is possible at any given period. Note that this option is the same as combining Option A with an upper bound (equal to ***BLOCK(k)***) on the investment variable.

Option C: Investment in technology ***k*** is either zero (not at all) or is equal to some fixed number ***BLOCK(k)*** and is restricted to occur at most once over the whole horizon. This option is suitable for single, non replicable projects such as a specific hydroelectric project or gas pipeline.

Each of these three options requires the introduction of integer variables in the formulation. The optimization problem resulting from the introduction of integer variables into a Linear Program is called a Mixed Integer Program (MIP).

1.9.2 Formulation and Solution of the Mixed Integer Linear Program

Typically, the modeling of a lumpy investment involves Integer Variables, i.e. variables whose values may only be non-negative integers (0, 1, 2, …). The mathematical formulations of the three options described above are presented below.

Option A may be written as follows:

$$\boldsymbol{INV}(\boldsymbol{k},\boldsymbol{t}) = \boldsymbol{BLOCK}(\boldsymbol{k}) \bullet \boldsymbol{Z}(\boldsymbol{k},\boldsymbol{t}) \qquad \boldsymbol{each}\ \boldsymbol{t} = 1,..,\boldsymbol{T}$$

with ***Z(k,t)*** = 0, 1, 2, …

Option B is expressed by:

$$INV(k,t) = BLOCK(k) \bullet Z(k,t) \qquad each\ t = 1,..,T$$

with ***Z(k,t)*** = 0 or 1

Option C is expressed as:

$$INV(k,t) = BLOCK(k) \bullet Z(k,t) \qquad each\ t = 1,..,T$$
$$and$$
$$\sum_{t=1}^{T} Z(k,t)$$

with ***Z(k,t)*** = 0 or 1

Although the formulation of lumpy investments *looks* simple, it has a profound effect on the resulting optimization program. Indeed, MIP problems are notoriously more difficult to solve than LPs, and in fact many of the properties of linear programs discussed in section 1.7 do not hold for MIPs, including duality theory, complementary slackness, etc. Note that the constraint that ***Z(k,t)*** should be integral departs from the *divisibility* property of linear programs. This means that the *feasibility domain* of integer variables (and therefore of some investment variables) is no longer contiguous, thus making it much more difficult to apply purely algebraic methods to solve MIPs. Therefore, practically all MIP solution algorithms make use (at least to some degree) of partial enumerative schemes, which tend to be time consuming and less reliable[27] than the algebraic methods used in pure LP.

The reader interested in more technical details on the solution of LPs and of MIPs is referred to references (Hillier and Lieberman, 1990, Nemhauser et al. 1989). In this section we shall be content to state one important remark on the interpretation of the dual results from MIP optimization.

1.9.3 Important remark on the MIP dual solution (shadow prices)

Using MIP rather than LP has an important impact on the interpretation of the MARKAL shadow prices. Once the optimal MIP solution has been found, it is customary for MIP solvers to fix all integer variables at their optimal (integer) values, and to perform one additional iteration of the LP algorithm, so as to obtain the dual solution (i.e. the shadow prices of all constraints). However, the interpretation of these prices is different from that of a LP. Consider for instance the shadow price of the natural gas balance constraint: in a pure LP, this value represents the price of natural gas. In MIP, this value represents the

[27] A MARKAL LP program for a given region tends to have fairly constant solution time, even if the database is modified. In contrast, a MARKAL MIP may show some erratic solution time. One may observe reasonable solution times (although longer than LP solution times) for most instances, with an occasional very long solution time for some MIP instances. This occasional phenomenon is predicted by the theory of complexity as applied to MIP, see Papadimitriou and Steiglitz (1982)

price of gas *conditional on having fixed the lumpy investments at their optimal integer values.* What does this mean? We shall attempt an explanation via one example: suppose that one lumpy investment was the investment in a gas pipeline; then, *the gas shadow price will not include the investment cost of the pipeline, since that investment was fixed when the dual solution was computed*.

In conclusion, when using MIP, only the primal solution is fully reliable. In spite of this major caveat, modeling lumpy investments may be of paramount importance in some instances, and may thus justify the extra computing time and the partial loss of dual information.

See chapter 2 for details on the implementation, switches, data and model variables and equations.

1.10 Endogenous Technological Learning (ETL)

In a long term dynamic model such as MARKAL, the characteristics of future technologies are almost inevitably changing over the sequence of future periods, due to *technological learning*.

In some cases, it is possible to forecast such changes in characteristics as a function of time, and thus to define a time-series of values for each parameter (e.g. unit investment cost, or efficiency). In such cases, technological learning is *exogenous* since it depends only on time elapsed and may thus be established outside the model.

In other cases, there is evidence that the pace at which some technological parameters change, itself depends on the *experience* acquired with this technology. Such experience is not solely a function of time elapsed, but typically depends on the cumulative investment (often global) in the technology. In such a situation, technological learning is *endogenous*, since the future values of the parameters are no longer a function of time elapsed alone, but depend on the cumulative investment decisions taken by the model (which are unknown). In other words, the evolution of technological parameters may no longer be established outside the model, since it depends on the model's results. ETL is also named *Learning-By-Doing* (LBD) by some authors.

Whereas exogenous technological learning does not require any additional modeling, endogenous technological learning (ETL) presents a tough challenge in terms of modeling ingenuity and of solution time. In MARKAL, there is a provision to represent the effects of endogenous learning on the unit investment cost of technologies. Other parameters (such as efficiency) are not treated, at this time.

1.10.1 The basic ETL challenge

Empirical studies of unit investment costs of several technologies have found an empirical relationship between the unit investment cost of a technology at time $\boldsymbol{t}$, $\boldsymbol{INVCOST_t}$, and the cumulative investment in that technology up to time $\boldsymbol{t}$,

$$\boldsymbol{C_t} = \sum_{j=-1}^{t} \boldsymbol{VAR\_INV_j} \ .$$

A typical relationship between unit investment cost and cumulative investments is of the form:

$$\boldsymbol{INVCOST_t} = \boldsymbol{a} \cdot \boldsymbol{C_t^{-b}} \qquad (1.11-1)$$

where $\boldsymbol{a}$ is the initial unit investment cost (when $\boldsymbol{C_t}$ is equal to 1) and $\boldsymbol{b}$ is the learning index, representing the speed of learning[28]. As experience builds up, the unit investment cost decreases, and thus may make investments in the technology more attractive. It should be clear that near-sighted investors will not be able to detect the advantage of investing early in learning technologies, since they will only observe the high initial investment cost and, being near-sighted, will not anticipate the future drop in investment cost resulting from early investments. In other words, tapping the full potential of technological learning requires far-sighted agents who accept making initially non profitable investments in order to later benefit from the investment cost reduction.

With regard to actual implementation, simply using (1.10-1) as the objective function coefficient of $\boldsymbol{VAR\_INV_t}$ will yield a non-linear, non-convex expression. Therefore, the resulting mathematical optimization is no longer linear, and requires special techniques for its solution. In MARKAL, a Mixed Integer Programming (MIP) formulation is used, that we now describe.

1.10.2 The MARKAL formulation of ETL

We follow the basic approach described in Barreto (2001). The first step of the formulation is to express the total investment cost, i.e. the quantity that should appear in the objective function for the investment cost of a learning technology in period $\boldsymbol{t}$. $\boldsymbol{TC_t}$ is obtained by integrating expression (1.10-1):

$$\boldsymbol{TC_t} = \int_0^{C_t} \boldsymbol{a} \cdot \boldsymbol{y}^{-\boldsymbol{b}} * \boldsymbol{dy} = \frac{\boldsymbol{a}}{1-\boldsymbol{b}} \cdot \boldsymbol{C_t}^{-\boldsymbol{b}+1} \qquad (1.10-2)$$

$\boldsymbol{TC_t}$ is a concave function of $\boldsymbol{C_t}$, with a shape as shown in Figure 1-9

[28] It is usual to define, instead of $\boldsymbol{b}$, another parameter, $\boldsymbol{pr}$ called the *progress ratio*, which is related to $\boldsymbol{b}$ via the following relationship:

$$pr = 2^{-b}$$

Hence, $\boldsymbol{1\text{-}pr}$ is the cost reduction incurred when cumulative investment is doubled. Typical observed $\boldsymbol{pr}$ values are in a range of .75 to .95

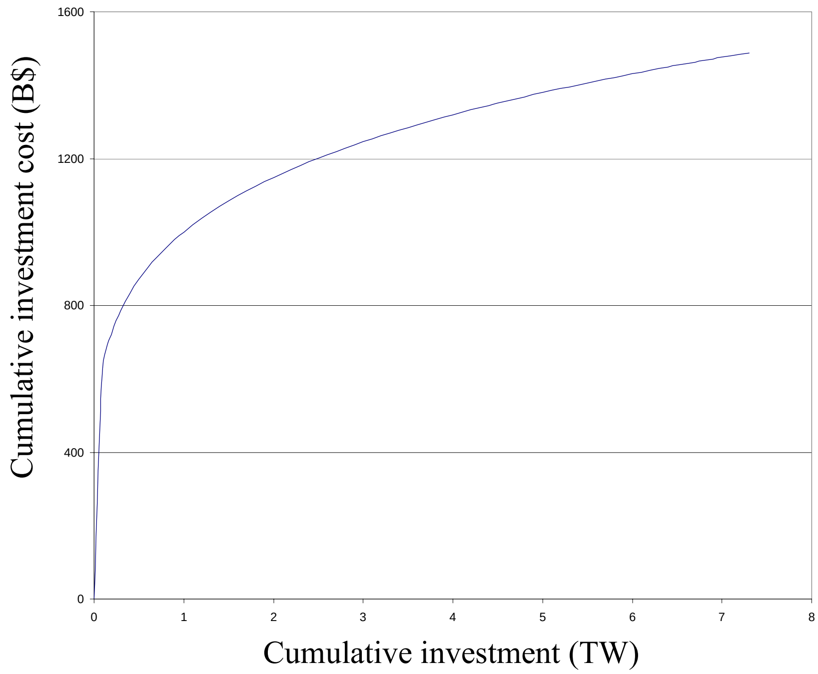

Figure 1-9. Example of a cumulative learning curve

With the Mixed Integer Programming approach implemented in MARKAL, the cumulative learning curve is approximated by linear segments, and binary variables are used to represent some logical conditions. Figure 1-10 shows a possible piecewise linear approximation of the curve of Figure 1-9. The choice of the number of steps and of their respective lengths is carefully made so as to provide a good approximation of the smooth cumulative learning curve. In particular, the steps must be smaller for small values than for larger values, since the curvature of the curve diminishes as total investment increases. The formulation of the ETL variables and constraints proceeds as follows (we omit the period, region, and technology indexes for notational clarity):

1. The user specifies the set of learning technologies (TEG).
2. For each learning technology, the user provides:
    a) The progress ratio pr (from which the learning index b may be inferred)
    b) One initial point on the learning curve, denoted (C_0, TC_0)
    c) The maximum allowed cumulative investment C_{max} (from which the maximum total investment cost TC_{max} may be inferred)
    d) The number N of segments for approximating the cumulative learning curve over the (C_0, C_{max}) interval (note that N may be different for different technologies).
3. The model automatically selects appropriate values for the N step lengths, and then proceeds to generate the required new variables and constraints, and the new objective function coefficients for each learning technology. The detailed formulae are shown and briefly commented on below.

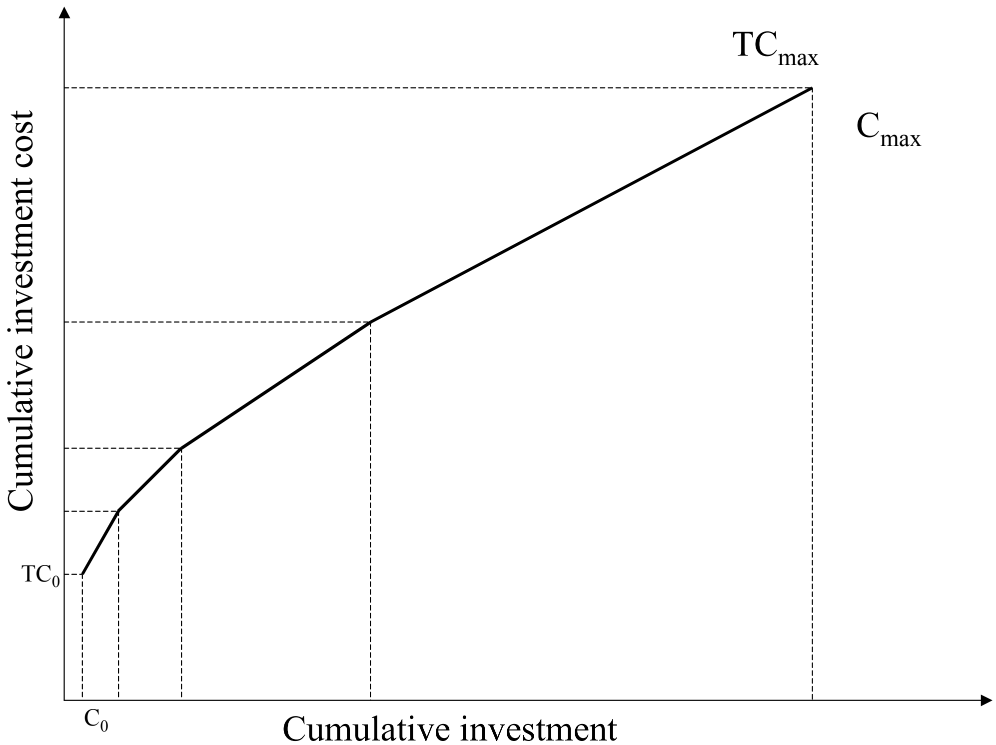

Figure 1-10. Example of a 4-segment approximation of the cumulative cost curve

1.10.2.1 Calculation of break points and segment lengths

The successive interval lengths on the vertical axis are chosen to be in geometric progression, each interval being twice at wide as the preceding one. In this fashion, the intervals near the low values of the curve are smaller so as to better approximate the curve in its high curvature zone. Let $\{TC_{i-1}, TC_i\}$ be the i^{th} interval on the vertical axis, for $i = 1, \ldots, N-1$. Then:

$$TC_i = TC_{i-1} + 2^{i-N-1}(TC_{\max} - TC_o)/(1-0.5^N) \ , \qquad i = 1,2,\cdots,N$$

Note that TC_{max} is equal to TC_N.

The break points on the horizontal axis are obtained by plugging the TC_i 's into expression (1.10-2), yielding:

$$C_i = \left(\frac{(1-b)}{a}(TC_i)\right)^{\frac{1}{1-b}} \ , \qquad i = 1, 2, \ldots, N$$

1.10.2.2 New variables

Once intervals are chosen, standard approaches are available to represent a concave function by means of 0-1 variables. We describe the approach used in MARKAL. First, we define *N* continuous variables x_i, $i = 1,\dots,N$. Each x_i represents the portion of cumulative investments lying in the ith interval. Therefore, the following holds:

$$C = \sum_{i=1}^{N} x_i \qquad 1.10-3$$

We now define N integer 0-1 variables z_i that serve as indicators of whether or not the value of C lies in the i^{th} interval. We may now write the expression for TC, as follows:

$$TC = \sum_{i=1}^{N} a_i z_i + b_i x_i \qquad 1.10-4$$

where b_i is the slope of the i^{th} line segment, and a_i is the value of the intercept of that segment with the vertical axis, as shown in Figure 1-11. The precise expressions for a_i and b_i are:

$$b_i = \frac{TC_i - TC_{i-1}}{C_i - C_{i-1}} \qquad i = 1,2,\dots,N$$

$$a_i = TC_{i-1} - b_i \cdot C_{i-1} \qquad i = 1,2,\dots,N \qquad 1.10-5$$

1.10.2.3 New constraints

For (1.10-4) to be valid, we must make sure that exactly one z_i is equal to 1, and the others equal to 0. This is done (recalling that the z_i variables are 0-1) via:

$$\sum_{i=1}^{N} z_i = 1$$

We also need to make sure that each x_i lies within the i^{th} interval whenever z_i is equal to 1 and is equal to 0 otherwise. This is done via two constraints:

$$C_{i-1} \cdot z_i \le x_i \le C_i \cdot z_i$$

1.10.2.4 Objective function terms

Re-establishing the period index, we see that the objective function term at period t, for a learning technology is thus equal to $TC_t - TC_{t-1}$, which needs to be discounted like all other investment costs.

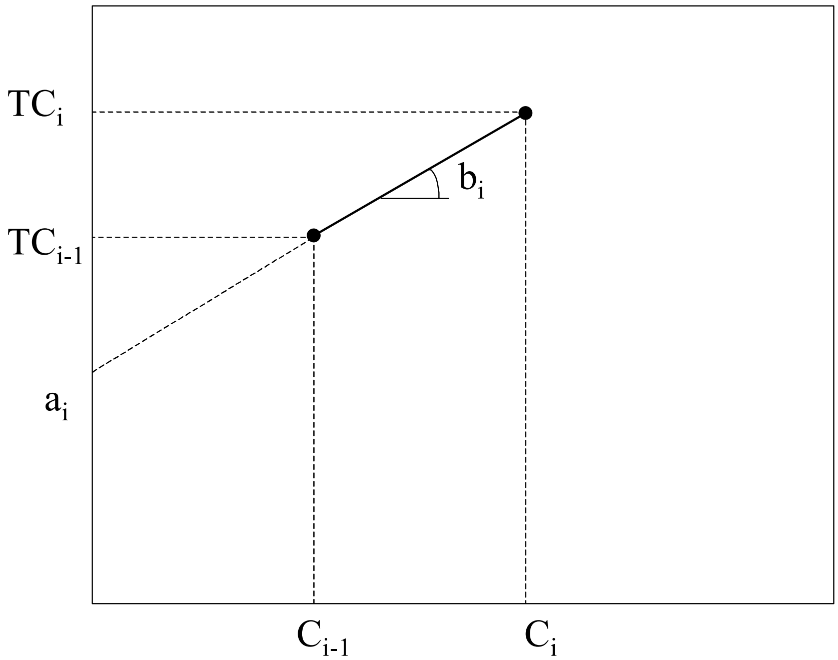

Figure 1-11. The i^{th} segment of the step-wise approximation

1.10.2.5 Additional (optional) constraints

Solving integer programming problems is facilitated if the domain of feasibility of the integer variables is reduced. This may be done via additional constraints, that are not strictly needed, but that are guaranteed to hold. In our application, we know that experience (i.e. cumulative investment) is always increasing as time goes on. Therefore, if the cumulative investment in period t lies in segment i, it is certain that it will not lie in segments i-1, i-2, .., 1 in time period t+1. This leads to two new constraints (re-establishing the period index ***t*** for the ***z*** variables):

$$\sum_{j=1}^{i} z_{j,t} \geq \sum_{j=1}^{i} z_{j,t+1}$$

$$i = 1,2,...,N-1, \; t = 1,2,...,T-1$$

$$\sum_{j=i}^{N} z_{j,t} \leq \sum_{j=i}^{N} z_{j,t+1}$$

Summarizing the above formulation, we observe that each learning technology requires the introduction of ***N*T*** integer 0-1 variables. For example, if the model has 10 periods and a 5-segment approximation is selected, 50 integer 0-1 variables are created for that learning technology, assuming that the technology is available in the first period of the model. Thus, the formulation may become very onerous in terms of solution time, if many learning technologies are envisioned, and if the model is of large size to begin with. In section 1.10.5, we provide some comments on ETL, as well as a word of warning.

1.10.3 Clustered learning

An interesting variation of ETL is also available in MARKAL, which treats the case where several technologies use the same key technology component, itself subject to learning. For instance,
Table 1-3 lists 11 technologies using the key Gas Turbine technology. As experience builds up for gas the turbine, each of the 11 technologies in the cluster benefits. The phenomenon of clustered learning is modeled in MARKAL via the following modification of the formulation of the previous section.

Let ***k*** designate the key technology and let ***l* = 1, 2, …, *L*** designate the set of clustered technologies attached to ***k***. The approach consists of three steps:

a) Step 1: designate ***k*** as a learning technology, and write for it the formulation of the previous section;
b) Step 2: subtract from each $\boldsymbol{INVCOST_l}$ the initial investment cost of technology ***k*** (this will avoid double counting the investment cost of ***k***);
c) Step 3: add the following constraint to the program, in each time period. This ensures that learning on ***k*** spreads to all members of its cluster:

$$VAR\_INV_k - \sum_{l=1}^{L} VAR\_INV_l = 0$$

Table 1-3. Cluster of gas turbine technologies
(from A. Sebregts and K. Smekens, unpublished report, 2002)

Description
Integrated Coal gasification power plant
Integrated Coal Gasification Fuel Cell plant
Gas turbine peaking plant
Existing gas Combined Cycle power plant
New gas Combined Cycle power plant
Combined cycle Fuel Cell power plant
Existing gas turbine CHP plant
Existing Combined Cycle CHP plant
Biomass gasification: small industrial cog.
Biomass gasification: Combined Cycle power plant
Biomass gasification: ISTIG+reheat

1.10.4 Learning in a Multiregional MARKAL Model

Technological learning may be acquired via global or local experience, depending on the technology considered. There are examples of techniques that were developed and perfected in certain regions of the World, but have tended to remain regional, never fully spreading globally. Examples are found in land management, irrigation, and in household heating and cooking devices. Other technologies are truly global in the sense that the same (or close to the same) technology becomes rather rapidly commercially available globally. In the latter case, global experience benefits users of the technology world wide. Learning is said to *spillover* globally. Examples are found in large electricity plants, in steel production, and many other sectors.

The first and obvious implication of these observations is that the appropriate model scope must be used to study either type of technology learning.

The formulation described in the previous sections is adequate in two cases: a) learning in a single region model, and b) regional learning in a multiregional model. It does *not* directly apply to global learning in a multiregional global model, where the cumulative investment variable must represent the sum of all cumulative investments in all regions together. We now describe an approach to model this case.

The first step in modeling multiregional ETL (MRETL) is to create one additional region, region 0, that will play the role of the Manufacturing Region. This region's RES consists only of the set of (global) LT's. Each such LT has the following specifications:

a) The LT has no commodity inputs
b) The LT has only one output, a dummy commodity ***c*** representing the 'learning'. This output is precisely equal to the investment level in the LT in each period.
c) Commodity ***c*** may be exported to all other regions
d) Finally, in each 'real' region, the LT is represented with all its attributes *except the INVCOST*. In addition, the construction of one unit of the LT requires an input of one unit of the learning commodity ***c***. This ensures that the sum of all investments in the LT in the real regions is equal to the investment in the LT in region 0, as desired.

1.10.5 Endogenous vs. Exogenous Learning

In this section, we state a few comments and warnings that may be useful to potential users of the ETL feature. We start by stating a very important caveat to the formulation described earlier: if a model is run with such a formulation, it is likely that the model will select some technologies, and *will invest massively at some early period* in these technologies unless it is prevented from doing so by additional constraints. Why this is likely to happen may be qualitatively explained by the fact that once a learning technology is selected for investing, two opposing forces are at playin deciding the optimal timing of the investments. On the one hand, the discounting provides an incentive for postponing investments. On the other hand, investing early allows the unit cost to drop immediately, and thus allows much cheaper investments in the learning technologies in all periods (current and future). Given the considerable cost reduction that may be provoked by learning, the first factor (discounting) is highly unlikely to predominate, and hence the model will tend to invest massively and early in such technologies, or not at all. Of course, what we mean by "massively" depends on the other constraints of the problem (such as whether or not there is a use for the commodity produced by the learning technology, the presence of existing technologies that compete with them, etc.). However, there is a clear danger that we may observe investment results in some learning technologies that are clearly unrealistic.

ETL modelers are well aware of this phenomenon, and use additional constraints to control the penetration trajectory of learning technologies. These constraints may take the form of upper bounds on the capacity of or the investment in of the learning technologies in each time period, reflecting what is considered by the user to be realistic penetrations. These upper bounds play a determining role in the solution of the problem, and it is most often observed that the capacity of a learning technology is either equal to 0 or to the upper bound. This last observation indicates that the selection of upper bounds by the modeler is the predominant factor in controlling the penetration of successful learning technologies. Hence the question: is it worthwhile for the modeler to go to the trouble of modeling e*ndogenous* learning (with all the attendant computational burdens) when the results are in great part conditioned by *exogenous* upper bounds? We do not have a clear and unambiguous answer to this question; that is left for each modeler to evaluate.

Given the above caveat, a possible alternative to ETL would consist in using exogenous learning trajectories. To do so, the same sequence of 'realistic' upper bounds on capacity would be selected by the modeler, and the values of the unit investment costs (INVCOST) would be externally computed by plugging these upper bounds into the learning formula (1.10-1). This approach makes use of the same exogenous upper bounds as the ETL approach, but avoids the MIP computational burden of ETL. Of course, the running of exogenous learning scenarios is not entirely rigorous, since it is not guaranteed that the capacity of a learning technology will turn out to be exactly equal to its exogenous upper bound. If that were not the case, a modified scenario would have to be run, with upper bounds adjusted downward. Conversely, if an upper bounded variable reaches its upper bound and shows a high reduced cost, this is a signal that ways to increase that bound should be sought (still within the bounds of realism). This trial-and-error approach may seem awkward, but it should be remembered that it (or some other heuristic approach) may prove to be necessary in those cases where the number of leaning technologies and the model size are both large (thus making ETL computationally intractable).

1.11 Modeling of Uncertainty in Standard MARKAL

1.11.1 Introduction

The long term analysis of an energy system is fraught with uncertainties, be it the specification of demands and prices, or the availability and characteristics of future technologies, or the emission targets that should be adopted. Older versions of MARKAL, along with most least-cost bottom-up models assume perfect foresight, and thus a deterministic environment. This is also the case for traditional general equilibrium models, although there are important exceptions. In the absence of explicit modeling of uncertainties, model users resort to scenario analysis, i.e. accounting for multiple possible futures via contrasted scenarios of demands, technological development, and emission constraints. Although the multiple scenario approach is useful, it remains somewhat incomplete, and even problematic for the following reason: suppose that two scenarios are modeled and run by a technological model, and suppose further that one main (uncertain) event is going to occur, say 15 years from now. To fix ideas, let us consider an example where the event is the development of a key energy technology. It is quite likely that the two alternate scenario runs (*with* vs. *without* the technology) will produce very different recommendations on investments. If we focus our attention on investments in the initial 15 years (i.e. prior to the event resolution date), we may face two widely different investment recommendations from the model, *and we have no easy way of resolving the dilemma.*

An alternate approach to running multiple deterministic scenarios consists in building a single scenario, but one where the future event bifurcation is embedded. The resulting *stochastic model* will be quite different in nature from the initial model. The *Stochastic Programming* paradigm consists of representing multiple scenarios (usually called *states-of-the-world,* or *sow*), each having a possibility of occurring, within a single coherent formulation. The key to success is simply to define, for each particular decision to be considered, as many decision variables as there are states-of-the-world in that time period. For instance, in the example evoked above, there should be a single copy of investment and other variables for the 15 years prior to uncertainty resolution (the point in time when we know which outcome materializes), precisely because a single strategy *must* be followed during that period of time. On the contrary, for later periods, the outcome of the event is unknown, and therefore, there should be *two* sets of variables, each representing a particular decision *contingent on the state-of-the-world that might prevail*.

Stochastic programming (Dantzig, 1963) is easily generalized to any number of events, each with many possible outcomes. The resulting stochastic scenario is best represented by an *event tree*, such as the one depicted in Figure 1-12, showing 4 states-of-the-world.

This example concerns the Greenhouse Gas emissions from the Quebec energy system. The system is subjected to an upper bound on its cumulative GHG emissions over the entire horizon from 2000 to 2030. However, the upper bound is uncertain the outset, and it is estimated that it may take any of four values, each with a certain probability of occurrence. Table 1-4 describes the four sow's and their probabilities. It is further assumed that the outcome will be known in 2015. In this example, the event tree consists of two stages only, separated by the single resolution time (2015), as shown in Figure 12 (other examples may have several events, with different resolution times, but such *multi-stage* cases are currently not covered in Standard MARKAL)

Table 1-4. An example with four states of the world

	Cumulative emission bound (CO2 eq)	Probability of occurrence
Sow 1	3675 Mt	0.10
Sow 2	3337 Mt	0.50
Sow 3	2940 Mt	0.35
Sow 4	2515 Mt	0.05

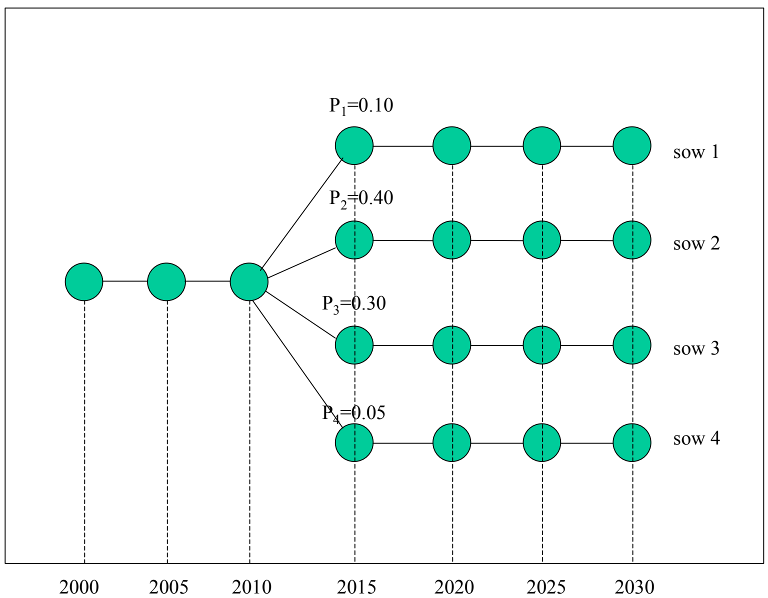

Figure 1-12. Event tree with four states-of-the-world and resolution time at period 2015

In the context of energy-environment systems, stochastic modeling has been extensively used to study restricted energy systems such optimizing the electricity generation process (e.g. Kunsch and Teghem, 1987; Gorenstin et al., 1993). Studies of socio-economic impacts of the uncertain outcomes of global warming have also used stochastic models (Fankhauser, 1994; Manne and Richels, 1995). In the case of integrated energy systems, a two-step model for robustness analysis in energy planning was implemented in Larsson and Wene (1993) and Larsson (1993). The method provided for assessing the efficiency and robustness of exogenously determined alternative strategies. Similar work using the MESSAGE model was reported by Gerking and Voss (1986). Birge and Rosa (1996) have included uncertainty in the return on investments in new technologies in the Global 2100 model. Stochastic programming has been used for energy-environment policy modeling recently, but mostly by the very aggregated global models like DICE (Nordhaus, 1993), MERGE (Manne et al., 1995), and CETA-R (Peck and Teisberg, 1995). Reports on formal inclusion of future uncertainties in bottom-up energy-environment modeling are scant. Fragnière and Haurie (1996) have taken an approach similar to MARKAL on this problem, with an original optimization algorithm based on LP decomposition and interior point methods.

One important issue raised by uncertain outcomes is the choice of the criterion to use for the optimization. The frequently used *expected cost* criterion computes a weighted average cost, where the cost under each outcome is weighted by its probability of occurring. While expected cost is the most often discussed and used criterion, it presupposes first that probabilities of event outcomes are available and second that the decision maker is risk neutral. Both of these assumptions are debatable in long term strategic policy analysis, where certain events are truly uncertain (i.e. there is no consensus on the likelihoods of their outcomes, e.g. the probability of a 3^{o}C rise in global temperature), and where certain outcomes would carry such a huge cost, that they would not be acceptable, however low their probability (high risk aversion).

Both concerns may be addressed by replacing the expected cost criterion by an *expected utility* criterion, where the utility is defined so as to reflect the policy maker's biases with respect to certain cost outcomes (e.g. various degrees of risk aversion). A special case consists in defining an objective that does not make use of probabilities. For instance, the Savage criterion (Raiffa, 1968) consists of choosing the strategy that minimizes the largest *regret* that could be experienced by the policy maker (the regret being defined as the cost difference between the most favorable outcome and the one incurred with the chosen strategy). Loulou and Kanudia (1999) describes an implementation of the Minimax Regret criterion in a particular version of the MARKAL model. Subsection 1.11.4 describes an intermediate situation where probablilities are used, but the criterion to be minimized includes a correction term for risk aversion.

One practical limitation of the stochastic programming approach is the model size, which may reach gigantic proportions when complex event trees are modeled. This drawback is much less acute now than in the past, thanks to the rapid progress of computing power and of algorithms. Care must nevertheless be exercised to model only the major uncertainties, so as to keep the event tree reasonably parsimonious.

1.11.2 Two-Stage Stochastic MARKAL Formulation (Minimizing Expected Cost)

The stochastic programming option available in Standard MARKAL is based on the two-stage Stochastic Programming paradigm described in the previous section (Dantzig, 1963), in which all uncertainties are resolved at a single future stage, the same for all events. Other MARKAL versions are more general and allow Multi-stage Stochastic Programming with various criteria (Kanudia and Loulou, 1996; Loulou and Kanudia, 1999), but are not directly transferable to Standard MARKAL as they are written in a different programming language.

Stochastic MARKAL uses the concept of an *event tree* described earlier, where each scenario is represented by a *path* from beginning to end of horizon, and each path has a probability of occurrence. Figure 1-12 shows the event tree of the example described in the section. The period when all uncertainties are resolved is called the *resolution time* (2015 in Figure 1-12). The general mathematical formulation of the two-stage problem derives from the following three principles:

1. In each period, there are as many replications of the MARKAL variables as there are different outcomes (sow's) in that period. Thus prior to the resolution of uncertainty there is a single sow. In those periods when there *are* multiple realizations (i.e. at and after the resolution time), each variable set should be considered as a set of *conditional variables*, i.e. variables representing *contingent actions* that will be taken only if the corresponding realization occurs. Note that this requires that each MARKAL variable, attribute, and constraint have a new, state-of-the-world index, denoted ***w***.
2. Each set of variables corresponding to a possible scenario must satisfy all constraints of MARKAL. Therefore, whatever scenario eventually occurs, the corresponding set of variables (decisions) is feasible. The multi-period constraints, such as capacity transfer, cumulative emission and cumulative resource usage, are replicated, one for each path of the event tree (however, see the Important remark below). The single period constraints are repeated as many times as there are different realizations in that period, and that number may differ with the period.
3. The objective function (expected cost) is equal to the weighted sum of the scenarios' objective functions (costs), each weighted by the scenario's probability of occurring.

Using these three principles, the following Linear Programming problem is formulated:

$$\textit{Minimize} \quad \boldsymbol{Z} = \sum_{\boldsymbol{w} \in \boldsymbol{W(t)}} \sum_{\boldsymbol{t} \in \boldsymbol{T}} \boldsymbol{C}_{t,w} \bullet \boldsymbol{X}_{t,w} \bullet \boldsymbol{p}_{t,w}$$

$$\textit{subject to:} \quad \boldsymbol{A}_{t,w} \bullet \boldsymbol{X}_{t,w} \geq \boldsymbol{b}_{t,w} \quad , \qquad \forall \boldsymbol{t} \in \boldsymbol{T}, \forall \boldsymbol{w} \in \boldsymbol{W}(\boldsymbol{t})$$

where:

t = time period
T = set of time periods
*t** = resolution time
w = outcome index (sow)
W(t) = set of outcome indices for time period *t*. For all *t* prior to resolution time
*t**, *W(t)* has a single element (stage one). For $t \geq t^*$, *W(t)* has multiple
elements (stage two);

$X_{t,w}$ = the column vector of decision variables in period t, under scenario w

$C_{t,w}$ = the cost row vector in time t under scenario w;

$p_{t,w}$ = probability of scenario w in period t; $p_{t,w}$ is equal to 1 for all t prior to t^*, and

$$\sum_{w \in W(t)} p_{t,w} = 1 \quad \text{for all } t.$$

$A_{t,w}$ = the coefficient matrix (single period constraints) in time period t, under scenario w

$b_{t,w}$ = the right-hand-side column vector in time period t, under scenario w

The modeler must provide the information on: $W(t)$, $p_{t,w}$, $A_{t,w}$, $b_{t,w}$, $C_{t,w}$, for all t in the format described in chapter 2.

Important remark – the current implementation of 2-stage stochastics in MARKAL does not handle models that employ inter-temporal commodity flows that either delay the release of a commodity (LAG), or require that a commodity be produced in the previous period (LED).

1.11.3 Interpreting results from a stochastic run of MARKAL

1.11.3.1 Hedging strategy, and perfect foresight strategies

The results of a stochastic MARKAL run are of the same nature as those of a deterministic run. In this section, we make a few comments on the interpretation of such results, and we discuss a few useful additional results that are more specifically attached to uncertain situations.

First, let us make more precise the concept of *Optimal Hedging Strategy:* Stochastic Programming produces a *single strategy*. However, that strategy is composed of contingent (conditional) actions, that will diverge at periods later than the resolution dates. We will call it the *Optimal Hedging Strategy.* In contrast, the classical approach of using several alternate scenarios (each deterministic) leads to as many strategies as there are scenarios (4 in our example). These strategies will differ between themselves *even prior to the resolution time.* As noted earlier, this is unrealistic, and constitutes a major reason for using Stochastic Programming. We shall call these four deterministic strategies, the *Perfect Foresight* strategies, denoted PF_i where i designates the particular scenario being considered. The word Strategy in this context is somewhat misleading since the policy maker *believes* that a particular deterministic scenario will realize, but in actual fact, any one of the four possible scenario outcomes may yet occur! Therefore, *the Perfect Foresight strategies are not real* *strategies*. They are idealized, but still useful for comparison purposes, as we now proceed to show.

We illustrate with the example of the Québec energy system, already presented in section 1.11.1, where the random event is the cumulative emission cap, for which four possible outcomes are identified (table 1-4). All other parameters are the same for the four outcomes. The four emission targets are denoted sow_1, sow_2, sow_3, and sow_4. Figure 1-13 shows the annual GHG emission trajectories under five strategies: the optimal hedging strategy obtained via Stochastic MARKAL (solid lines) and the four PF_i strategies (dashed lines). Note carefully that, in Figure 1-13, a perfect foresight trajectory is the one that would prevail if the decision-maker were able to guess the correct state-of-the-world that will prevail. We repeat here that it is *not* a realistic strategy.

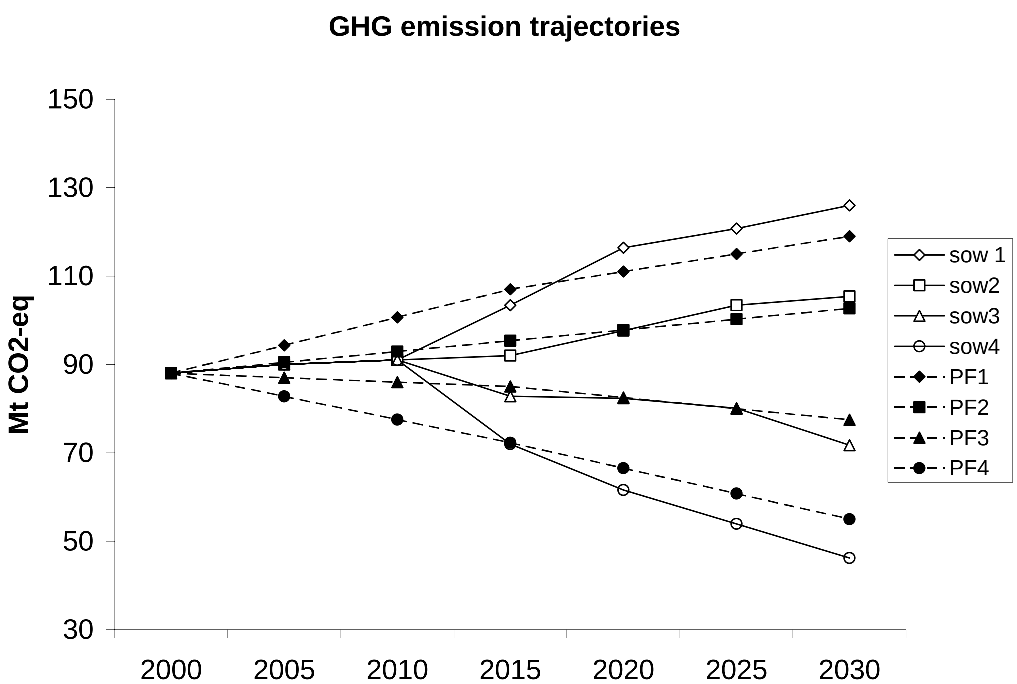

Figure 1-13. Emission trajectories under 5 strategies

As expected, in Figure 1-13, the optimal hedging strategy takes a middle-of-the-road path until uncertainty is resolved. Subsequently, the annual GHG emissions fall sharply in cases where severe mitigation realizes (scenarios 3 and 4), and turn upward in other cases (scenarios 1 and 2). We also note that the four *PF* trajectories differ quite substantially in 2005 and 2010, confirming their meaninglessness.

Similar figures could be obtained for other quantities, such as final and primary energy, capacities of technologies, etc. In particular, it is of great interest for the energy analyst to identify those technologies that are *robust* under uncertainty, i.e. those that are used in the hedging strategy in periods 2005 and 2010. It often happens that such technologies *are different from those selected by the model under any of the PF scenarios,* thus showing that the stochastic programming treatment of major uncertainties can yield insights that are beyond the scope of an analysis based on multiple deterministic scenarios.

1.11.3.2 Expected Value of Perfect Information (EVPI)

A useful concept when planning under risk, is the Expected Value of Perfect Information (***EVPI***, see Raiffa, 1968), i.e. the cost savings that would accrue if all uncertainties were resolved right now rather than at some future resolution time. Hence, ***EVPI*** is the cost saving that would accrue (relative to the cost of the optimal hedging strategy) if the policy maker knew today with certainty which of the outcomes will occur at resolution time (hence if the *PF* strategies were realistic). For our example, ***EVPI*** may be computed by solving five LP's: the Stochastic Programming problem, plus the four deterministic LPs corresponding to perfect knowledge of each outcome. We designate these costs by $COST_{hedge}$, $COST_{PF1}$, $COST_{PF2}$, $COST_{PF3}$, and $COST_{PF4}$, respectively. Then:

$$\boldsymbol{EVPI} = \boldsymbol{COST_{hedge}} - \sum_{\boldsymbol{i}=1}^{4} \boldsymbol{p_i} \bullet \boldsymbol{COST_{PFi}}$$

This quantity gives a measure of the cost handicap introduced by uncertainty. In the example discussed in this section, EVPI is equal to $4.86 billion, a significant amount. This is how much it would be worth to us to know the truth about the future right now.

1.11.3.3 The cost of guessing wrong

In order to evaluate the benefit of using stochastic programming (rather than traditional multiple deterministic scenarios), we construct another useful quantity as follows: we evaluate the expected loss incurred when the decision maker believes that a particular outcome (say scenario ***k***) will happen. In such a case, he would follow the corresponding PF_k path until just before resolution time. At resolution time, he would *know the true outcome* (which may be any of the scenarios) and would then re-optimize his path *from then on*. In our example with 4 outcomes, the decision maker would have to compute 4 re-optimized paths after having initially chosen some PF_k initial strategy. Denote by $\boldsymbol{COST_{j|k}}$, ***j=1,…,4*** the conditional cost of the re-optimized strategy that *assumes k* to be the outcome, when *j* is the true revealed outcome[29]. From these conditional costs, we may now build the conditional expected cost incurred for any strategy ***k***:

$$\boldsymbol{EC_k} = \sum_{\boldsymbol{j}=1}^{\boldsymbol{J}} \boldsymbol{p_j} \cdot \boldsymbol{COST_{j|k}}\ , \qquad \boldsymbol{each\ k} = 1,2,..,\boldsymbol{J}$$

[29] The cost $\boldsymbol{COST_{j|k}}$ is determined for each *k* through the following three-step process:

Step 1: Run the model with the parameters corresponding to the *assumed* perfect foresight scenario, to obtain strategy PF_k.

Step 2: Freeze Strategy PF_k. for all periods prior to resolution time *t** (i.e. freeze all decision variables in MARKAL).

Step 3: Run the model again *J-1* more times, with the parameters set to those of each of the other J-1 possible outcomes (*j=1,2,3,J, excepting j= k*), and with the initial decisions frozen as in *step 2*. These runs provide the values of $\boldsymbol{COST_{j|k}}$

where $\boldsymbol{J}$ is the number of possible outcomes (4 in our example). We may now compute the decision maker's expected loss if he has followed the k^{th} pure scenario, instead of the optimal hedging strategy:

$$\boldsymbol{EL_k} = \sum_{j=1}^{J} \boldsymbol{p_j} \cdot \boldsymbol{COST_{j|k}} - \boldsymbol{COST_{hedge}} \ , \quad \boldsymbol{each\ k} = 1,2,..,\boldsymbol{J}$$

For illustration, we report in
Table 1-5 the four expected losses for our example. As can be seen from this example, it may be very costly to follow a pure strategy $\boldsymbol{PF_k}$.

Table 1-5. Expected value of losses under assumed perfect foresight strategies ($B)

STRATEGY	**K=1**	**K=2**	**K=3**	**K=4**
$\boldsymbol{EL_K}$	3.27	0.58	10.42	12.50

1.11.4 Stochastic Programming with risk aversion

The preceding description of stochastic programming assumes that the policy maker accepts the expected cost as his optimizing criterion. This is equivalent to saying that he is risk neutral. In many situations, the assumption of risk neutrality is only an approximation of the true utility function of a decision maker. In MARKAL, there is a feature for taking into account that a decision maker may be risk averse, by defining a new utility function to replace the expected cost.

The approach is based on a variant of the classical E-V model (an abbreviation for Expected Value-Variance). In the E-V approach, it is assumed that the variance of the cost is an acceptable measure of the risk attached to a strategy in the presence of uncertainty. The variance of the cost of any fixed strategy k is computed as follows:

$$\boldsymbol{Var(C_k)} = \sum_{j} \boldsymbol{p_j} \bullet (\boldsymbol{Cost_{j|k}} - \boldsymbol{EC_k})^2$$

where $\boldsymbol{Cost_{j|k}}$ is the cost of strategy $\boldsymbol{k}$ when the $\boldsymbol{j^{th}}$ state of nature prevails, and $\boldsymbol{EC_k}$ is the expected cost of strategy $\boldsymbol{k}$ defined as usual by:

$$\boldsymbol{EC_k} = \sum_{j} \boldsymbol{p_j} \bullet \boldsymbol{Cost_{j|k}}$$

An E-V approach would thus replace the expected cost criterion by the following utility function to minimize:

$$U = EC + \lambda \cdot \sqrt{Var(C)}$$

where λ is a measure of the risk aversion of the decision maker. For λ=0, the usual expected cost criterion is obtained. Larger values of λ indicate increasing risk aversion.

In Stochastic MARKAL, a further refinement is introduced: the semi-variance is used rather than the variance. Indeed, it may be argued that the variance includes both "upward" (or bad) risk, and "downward" (or good) risk. The upward semi-variance is defined by:

$$UpVar(Cost_k) = \sum_{j \ni Cost_{j|k} \geq EC_k} p_j \bullet (Cost_{j|k} - EC_k)^2$$

If only upward risk is considered, the Stochastic MARKAL Utility function reads as follows:

$$U = EC + \lambda \cdot \sqrt{UpVar(C)}$$

Of course, taking risk aversion into account leads to a non-linear MARKAL model, with all its ensuing computational restrictions. The modeler should therefore assess the pros and cons of simulating risk aversion against the serious limitations this formulation introduces on model size.[30]

[30] Note that to avoid non-linearities, it is possible to replace the semi-variance by the Upper-absolute-deviation, defined by:

$$UpAbsDev(Cost_k) = \sum_{j \ni Cost_{j|k} \geq EC_k} p_j \bullet \left\{Cost_{j|k} - EC_k\right\}^+$$

where $y = \{x\}^+$ is defined by the following two *linear* constraints: $y \geq x$, and $y \geq 0$
and the Utility would now be the following linear expression:

$$U = EC + \lambda \cdot UpsAbsDev(C)$$

This linearized version of the risk aversion is not available in the current MARKAL code.

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2. Reference Guide for Standard MARKAL

The purpose of this Reference Guide is to lay out the full details of the Standard MARKAL[31] model, including data specification, internal data structures, and mathematical formulation of the model's Linear Program (LP) formulation, as well as the Mixed Integer Programming (MIP) and Non-Linear Programming (NLP) formulations required by some of its options. As such, it provides the MARKAL modeler/programmer with sufficiently detailed information to fully understand the syntax and purpose of the data components, model equations and variables, and links these to the relevant GAMS source code and to the Linear Programming Matrix shown in the separate spreadsheet file, where appropriate. A solid understanding of the material in this chapter is a necessary prerequisite for anyone considering making programming changes in the MARKAL source code.

This chapter is divided into11 sections, as follows:

- 2.1 – Notation, syntax, conventions and definitions: lays the groundwork for understanding the rest of the material in the Reference Guide;
- 2.2 – Sets: explains the meaning and role of various sets that identify how the model components are grouped according to their nature (e.g. demand devices, power plants, energy carriers, etc.) in a MARKAL model;
- 2.3 – Parameters: presents the user-provided numerical data as well as the internally constructed data structures used by the model generator (and report writer) to derive the coefficients of the LP matrix;
- 2.4 – Variables: defines each variable that may appear in the matrix, both explaining its nature and indicating how if fits into the matrix structure;
- 2.5 – Equations: states each equation in the model, both explaining its role, and providing its explicit mathematical formulation;
- 2.6 – The Lumpy Investment option
- 2.7 – The Damage Cost option
- 2.8 – The Endogenous Technological Learning option
- 2.9 – The Stochastic Programming option
- 2.10 – The MARKAL-MICRO version of the elastic demand mechanism.
- 2.11 – GAMS Modeling Environment for MARKAL.

Collectively, the information assembled in this Reference Guide provides a comprehensive specification of the MARKAL model and its implementation. This Reference Guide documents the multi-region MARKAL model formulation, but is also directly applicable to single-region MARKAL by simply ignoring the notation, text, and equations related to multi-regional issues. There are some slight (systematic) differences in how the variables and equations are named, and how the code runs, and these differences are mentioned in the various sections of the chapter.

[31] The USDOE-EIA SAGE model is a particular variant of the core MARKAL code. As such most of the references to Standard MARKAL apply to SAGE, with the notable exception of the Objective Function and the NLP model variants.

2.1 Notation, syntax, conventions and definitions

2.1.1 Basic notation and conventions

The next four sections describe the sets (section 2.2), parameters (section 2.3), variables (section 2.4), and equations (section 2.5) that define the MARKAL linear programming model. To assist the reader, the following conventions are employed consistently throughout this chapter:

- Sets, and their associated index names, are in lower case, usually within parentheses, e.g., (tch)[32];
- Literals, explicitly defined in the code, are in upper case within single quotes, e.g., 'UP';
- Parameters, and scalars (constants, i.e., un-indexed parameters) are in upper case, e.g., AF(Z)(Y)[33];
- Equations are in upper case with a prefix of MR_[34], e.g., MR_ADRAT1; and
- Variables are in upper case with a prefix of R_[35], e.g., R_ACT.

MARKAL tracks most activities (capacity, new investments, technology operation, non-electric/heat commodity flows) at an annual level. Each parameter, variable and equation thereby is related to a typical year within a period (MARKAL is typically run using 5-year periods). For electricity and low-temperature heat, however, sub-annual flows and technology operation are needed to track the seasonal/time-of-day dependent load curves resulting from various energy service demands. Collectively, the season/time-of-day splits are referred to as time-slices. MARKAL includes 6 pre-defined time-slices, combining 3 seasonal divisions (Winter (W), Intermediate (I), and Summer (S)), with 2 time-of-day divisions (Day (D), and Night (N)).

2.1.2 GAMS Modeling Language and MARKAL Implementation

MARKAL consists of generic variables and equations constructed from the specification of sets and parameter values depicting an energy system for each distinct region in a model. To construct a MARKAL model, a preprocessor first translates all data defined by the modeler into special internal data structures representing the coefficients of the MARKAL variables of section 2.4 in the equations of section 2.5. This step is called Matrix Generation. Once the model is solved (optimized) a Report Writer assembles the results of the run for analysis by the modeler. The matrix generation, report writer and

[32] For multi-region models _R is appended to each set name though this is not reflected in this document.
[33] For multi-region models _R is appended to each parameter name though this is not reflected in this document.
[34] For single-region models MR_ is replaced by EQ_. So EQ_ADRAT1 is the single-region constraint that corresponds to MR_ADRAT1.
[35] For single-region models the R_ prefix is dropped. So ACT is the single-region variable that corresponds to R_ACT.

control files are written in GAMS[36] (the General Algebraic Modeling System), a powerful high-level language specifically designed to facilitate the process of building large-scale optimization models. GAMS accomplishes this by relying heavily on the concepts of sets, compound indexed parameters, variables and equations. Thus the fit with the overall concept of the RES specification embodied in MARKAL is very strong, and GAMS is very well suited to the MARKAL paradigm.

Furthermore, GAMS is so designed that the GAMS code is very similar to the mathematical description of the equations provided in Section 2.5. Thus, the approach taken to implement the model is to "massage" the input data by means of a (rather complex) preprocessor that handles the necessary exceptions that need to be taken into consideration to construct the matrix coefficients in a form ready to be applied to the appropriate variables in the respective equations. GAMS also integrates seamlessly with a wide range of commercially available optimizers that are charged with the task of solving the MARKAL LP, MIP and NLP models. This step is called the Solve or Optimization step. CPLEX or XPRESS are the optimizers employed to solve the MARKAL LP and MIP formulations, and MINOS the NLP variants.

As already alluded to, MARKAL has several variants, such as Standard MARKAL, MARKAL-MACRO, MARKAL-MICRO, and SAGE. The Standard formulation also has optional features, such as lumpy investments, damage costs, endogenous technology learning and stochastic programming. To accommodate this situation most of the model is based upon the same core GAMS code.

2.2 Sets

A detailed understanding of the MARKAL variables and equations must start with the definition of all sets used in the model. The use of carefully defined sets enables MARKAL to succinctly define the elements of the MARKAL Reference Energy System (RES). The nature of an element of the RES is defined by its (possibly multiple) set memberships. For example, an energy carrier that is a fossil fuel belongs to two sets, ENC (the name given in MARKAL to the set of all energy carriers) and EFS (the name given in MARKAL to the set of all fossil energy carriers).

Set memberships determine the parameters or input data permitted for each component of a MARKAL RES. Thus if a technology is described as a resource supply option different parameters are used to describe it than those used for say demand devices. The set definitions ultimately serve as indices for MARKAL parameters, variables and equations.

The components of a MARKAL model are broken into five primary groups corresponding to:

[36] *GAMS A User's Guide*, A. Brooke, D. Kendrick, A. Meeraus, R. Raman, GAMS Development Corporation, December 1998.

- Demands for energy services (e.g., residential heating, commercial cooling, passenger transport);
- Emissions (e.g., CO_2, SOx, NOx);
- Energy carriers (e.g., coal, oil, gas, electricity);
- Materials (e.g., bauxite); and
- Technologies, that are split into resource supply or export options (e.g., oil imports, gas exports, coal mining, oil/gas extraction) and actual technologies (power plants and heat plants, process technologies that convert standard energy forms into other forms (e.g., a refinery, pipeline)), and end-use devices satisfying energy services (e.g., cars, air conditioners).

This split corresponds to the highest-level sets in MARKAL as noted in Table 2-1, below. Besides indicating how components fit into the underlying RES, membership in these groups also controls which other subset groupings are permitted (e.g., energy carriers are split between fossil, renewable, etc.; technologies into power plants, demand devices, etc.), as well as what actual data can be associated with a RES component (e.g., emission rates only for emissions, resource supply cost only for resources, investment cost only for technologies).

Table 2-1. Main RES Components Sets

RES Component	**Main Sets**	**Key Subsets**	**Description**
Demand Services	dm		All demand sectors.
Emissions	env	gwp	All emissions or other environmental indicators associated with the energy system, grouped by the modeler (for instance using a global warming potential or other multiplier when desired).
Energy Carriers/ Materials	ent	enc elc lth mat	All energy or material commodities (i.e. all commodities excluding emissions and demand services) present in the RES, broken out into energy carriers [electricity (elc), low-temperature heat (lth), and other energy carriers (enc)], and materials (mat)].
Technologies	srcencp	--	All resource supply options and export options.
	tch	con dmd prc	All (non-resource) technologies, broken out into Conversion Plants (con), Demand Devices (dmd), and Processes (prc).

Note that sets are qualitative rather than quantitative data. The complexity, scope, detail, and size of the model is determined by the number and types of elements included in the sets. In order to include technologies, fuels, emissions and energy service demands in the model they must be properly declared as members of the relevant sets.

Two types of sets are employed in the MARKAL modeling framework: user input sets and internal sets. The sets of the first group are the responsibility of the modeler, who assigns the RES components to the appropriate sets, such as the primary groupings just mentioned in Table 2-1 above. Membership in the internal sets is determined by the code for the matrix generator and report writer. Internal sets serve to both ensure proper exception handling (e.g., from when is a technology available, in which time-slices is a technology permitted to operate), as well as sometimes just to improve the performance or smooth the complexity of the actual model code. As noted earlier and elaborated in Table 2-2 of Section 2.2.2, in this documentation a one-letter index is used – instead of the actual set name employed in the code – when the sets, parameters, equations and variables are presented in the various Tables and mathematical formulas.

When running multi-region MARKAL the sets and parameters have a suffix ("_R") appended to their names, and reg as the first index. However, this suffix and the index are not indicated in the Tables of this chapter.

The two distinct groups of sets in MARKAL, User Input Sets and Internal (source code generated) Sets, are discussed in the following subsections.

2.2.1 Overview of User Input Sets

As noted in the previous section, sets are defined according to their role in the RES as:

- Demands;
- Emissions;
- Energy Carriers;
- Materials; and
- Technologies.

In the subsequent sub-sections (2.2.1.1 through 2.2.1.6) each of the user input sets is defined in detail, including an explanation of the implications for the model when an item is assigned to each set.

2.2.1.1 Demand Sets

The elements of the master demand set (dm) define the different categories (or end-use sectors) consuming energy services included in the model. As well as being assigned to set (dm), each demand service must also be assigned to one of six main end-use demand categories, depicted in Figure 2-1, where:

- **AGR** is the set of energy services associated with the agricultural sector;
- **COM** is the set of energy services associated with the commercial sector;
- **IND** is the set of energy services associated with the industrial sector;
- **NON** is the set of energy services associated with the non-energy sector;
- **RES** is the set of energy services associated with the residential sector, and
- **TRN** is the set of energy services associated with the transportation sector.

Within each of these sectors there may be any number of sub-sectors representing specific demand for energy services. For example, in the commercial sector there may space conditioning (both heating and cooling), lighting, hot water, etc. By convention a sub-sector name begins with the first character of the demand sector it belongs to, e.g., a commercial sub-sector name begins with "C"; and the names of devices that service a sub-sector begin with the sub-sector name, e.g., if the commercial cooking sub-sector is named "CCK" then by convention devices that service this sub-sector have names beginning with "CCK". Note that while MARKAL does not *require* such naming conventions, the adoption of carefully thought out naming conventions greatly facilitates the task of working with a large MARKAL model.

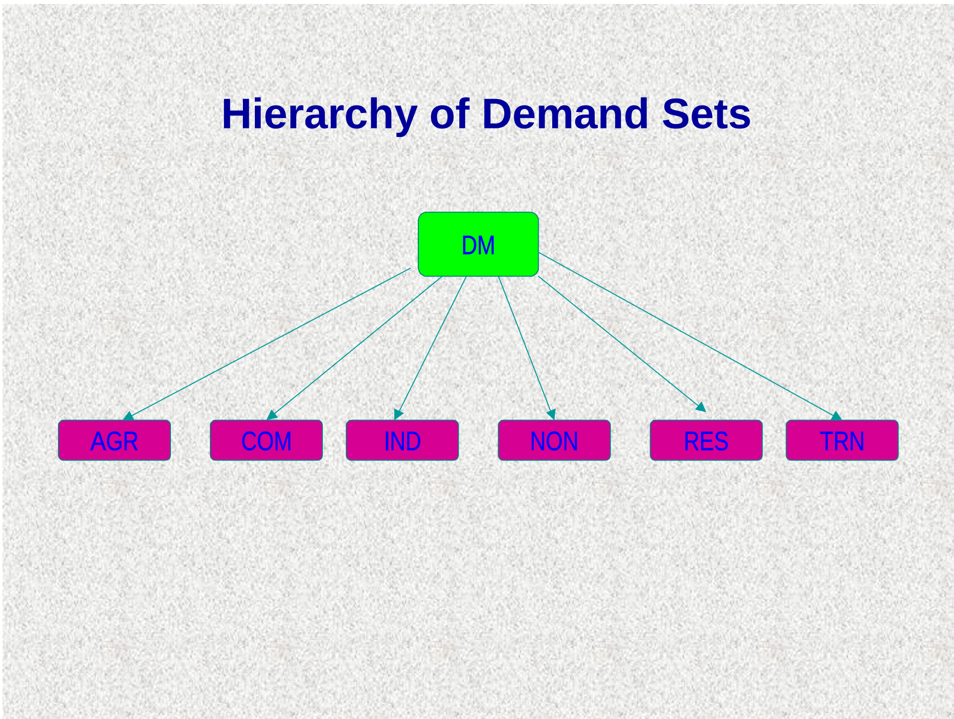

Figure 2-1. Hierarchy of demand sector sets

There is one other aspect of characterizing demands that are serviced by devices consuming electricity or low-temperature heat: this is indicating whether or not the shape of the load curve for the demand is uniformly spread over the year or demand specific. In the former case (specified by set **unifdist**) the default regional load shape (specified by parameter QHR(Z)(Y)) is used, and in the latter case the user provides the demand specific shape by means of the demand load fractions (FR(Z)(Y) for the non-unified distribution).

2.2.1.2 Emission Sets

The elements of Emission (or Environmental Indicator) sets (env) identify the environmental indicators that are to be tracked throughout the RES by the model according to the activity of, capacity of, or new investments in technologies, sources, and sinks. The modeler may also use the Emission Indicator to monitor any other entity that can be directly associated with components of the RES (e.g., land-use).

Besides the complete list, a subset may be defined that represents greenhouse gas emissions that are subject to global warming potential estimates. The individual emissions can then be associated with an aggregate indicator by means of an emission-to-emission mapping parameter (ENV_GWP). The same technique may be used to compile sector or total emissions by providing an ENV_GWP entry of units between two indicators.

2.2.1.3 Energy Carrier Sets

Energy carriers flow between the various technologies and other entities in the Reference Energy System establishing the inter-connectivity between the technologies.

Energy carriers are organized into sets according to the hierarchy tree shown in Figure 2-2 below. See Table 2-3 for details.

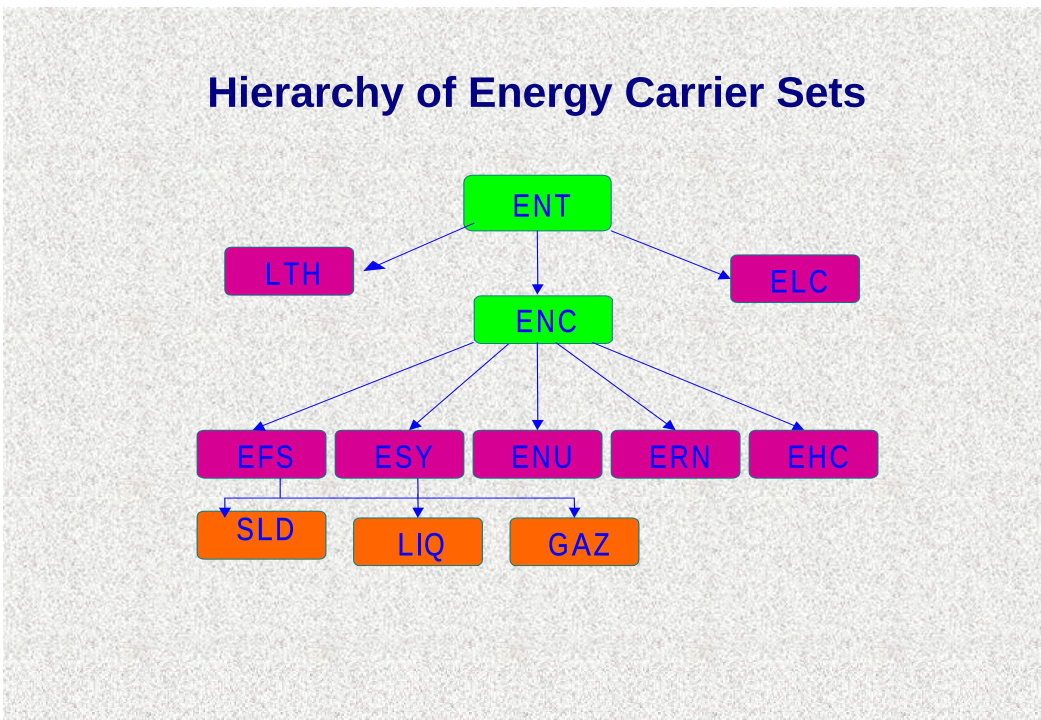

Figure 2-2. Hierarchy of energy carrier sets[37]

As part of describing an energy carrier to the model the modeler must assign it to one of several types according to the nature of the energy carrier, e.g., fossil (efs), synthetic (esy), nuclear (enu), renewable (ern), high-temperature heat (ehc), and in the case of fossil or synthetic energy carriers the form (solid, liquid or gas). The modeler needs to be

[37] Though not depicted in the schematic, in the GAMS code the set ENT actually encompasses all primary flow commodities (other than emissions and demand services) and thus includes both energy and materials (the latter not shown).

aware of the consequence of including an energy carrier in certain sets, most notably renewables (ern). Where energy carriers are characterized as renewable and no physical resource option is provided via srcencp (e.g., solar, wind) it is assumed that they are unlimited and they are tracked in a non-binding balance equation (MR_BAL_N), as opposed to a greater than or equal to constraint (MR_BAL_G) or slack balance constraint (MR_BAL_S) employed for the other energy carriers (including those renewables for which a physical resource limit is provided). Thus under normal circumstances the production of an energy carrier is expected to equal or exceed the consumption of that energy carrier.

As has already been suggested in the context of demand sub-sectors and demand devices, the adoption of carefully thought out naming conventions for energy carriers greatly facilitates the task of working with a large MARKAL model. In particular, naming conventions are suggested so as to better impart the nature of the energy carrier by indicating the kind of energy (e.g., coal, oil, gas, electricity) and the sector in which this particular instance of the energy carrier appears (e.g., upstream, "feeding" residential or commercial, etc). So adopting COMDSL, COMGAS, COMELC, COMCOA, etc, would identify the particular instance of diesel, gas, electricity, coal and other energy carriers that directly feed the commercial sector.

2.2.1.4 Material Sets

Materials are organized into sets according to the hierarchy tree shown in Figure 2-3 below, divided by the way their material is measured (by weight – mwt, or by volume – mvo).

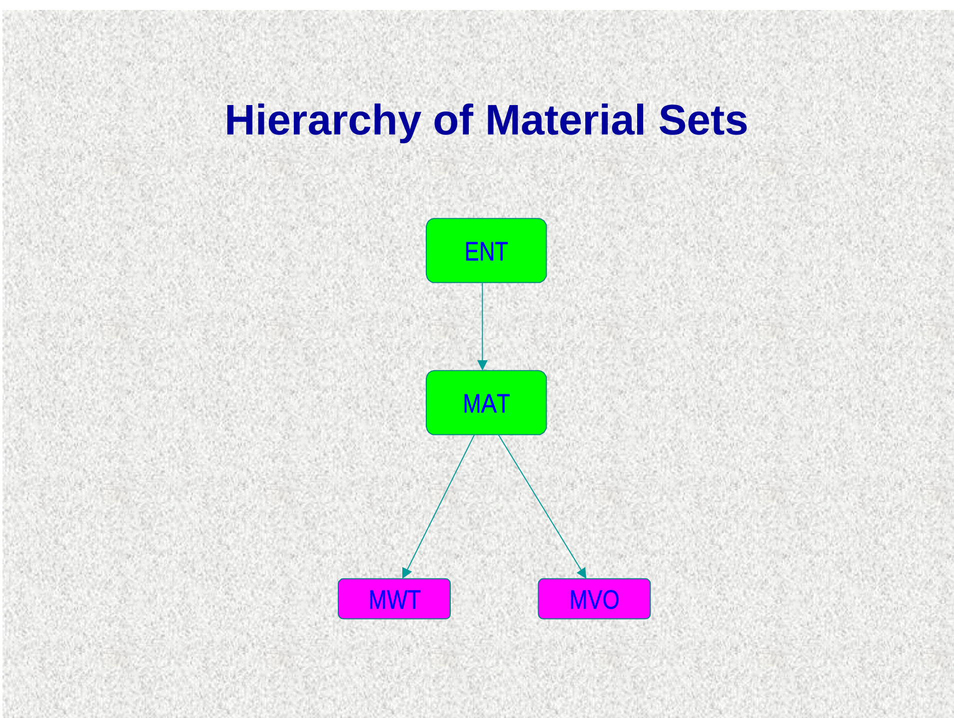

Figure 2-3. Hierarchy of material sets

The modeler needs to be aware of the consequence of describing a commodity as a material. All materials are subject to a material balance *equality* constraint (MR_BAL_E). Thus, the production of a material is forced to equal the consumption of that material[38]. As noted previously, in the GAMS code the set (ent) actually encompasses energy carriers as well as materials.

2.2.1.5 Technology Sets

Technologies are central to the definition of a RES and most of the user-provided input data is associated with them. While there are quite a large number of technology sets in MARKAL, there are just four main technology groups: resources, processes, conversion plants, and demand devices. The primary nature of a technology is determined by the sets to which it is assigned, although in some cases data parameter instances further refine some aspect of a technology (e.g., peaking only devices, or decentralized power plants). As a result there is a more complex hierarchy and inter-relationship between the technology sets than between the commodity sets.

The basic topological principle of the MARKAL RES is that commodities flow between technologies. For the most part, technologies consume one or more commodities and produce other, different commodities. More precisely, no technology, other than storage plants (stg), is allowed to have the same commodity as an input and an output, at least not using the same commodity name. This is what enables the RES connectivity to be unambiguously defined by the model code without the need to explicitly define actual inter-connections (links and nodes) embodied in the RES network. For certain classes of technologies noted below commodities may only be produced or consumed, but not both.

As previously noted, technologies are divided into four main groups, listed and depicted (in Figure 2-4) below, then elaborated upon in the rest of this section.

- Resources (srcencp) – are the extraction, import or export of a commodity that enable resources to enter/leave the RES. Resources may also consume commodities (e.g., electricity for mining). The resource technologies are treated separately and distinctly from the rest of the technologies, and have fewer parameters.
- Processes (prc) – are technologies that convert input commodities into output commodities, where the output commodities exclude electricity, heat, and demand services. Typical processes include refineries, pipelines, boilers, and emission

[38] The modeler can made good use of this feature. As an example, consider the case of a gaseous effluent that may either be processed by a depolluting device, or be vented into the atmosphere and then be subjected to a tax. If the balance constraint for this commodity were an inequality, the model would certainly choose to make that effluent "disappear" by allowing some slack in the balance constraint; in this case, the effluent would neither be taxed nor processed. This undesirable behaviour of the model is impossible when the balance constraint is an equality.

reduction technologies. They may also be introduced to the RES to facilitate (sub-sector) tracking of commodities and/or emissions.

- Conversion plants (con) – are technologies that produce electricity and/or low-temperature heat. These technologies require special handling, as their activity is at a sub-annual or time-slice level.
- Demand devices (dmd) – are the only technologies that can meet the demands for energy services. As such they typically consume commodities but output one or more services rather than another commodity.

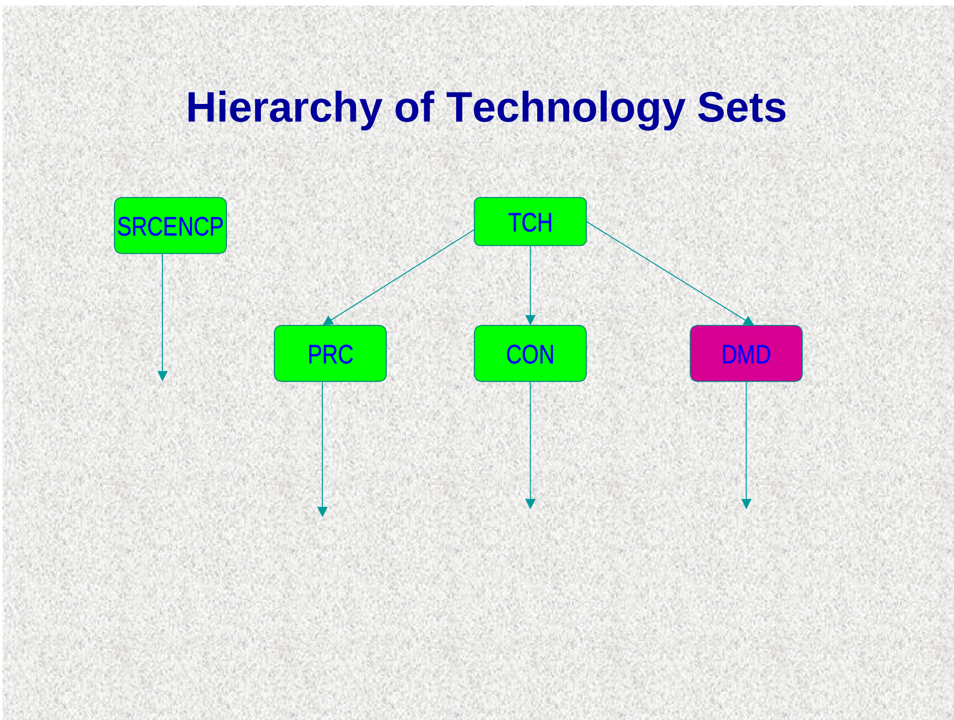

Figure 2-4. Main hierarchy of technology sets

The main technology sets are further sub-divided to indicate the specific nature of each technology. Many of these sub-sets affect the matrix generator while some are strictly used for grouping technologies for reports. In Section 2.2.3 each of the user sets is discussed, with Table 2-3 providing detailed descriptions of the purpose of the set and how membership influences the model. Each of the four main core technology groups is presented in the subsequent sub-sections.

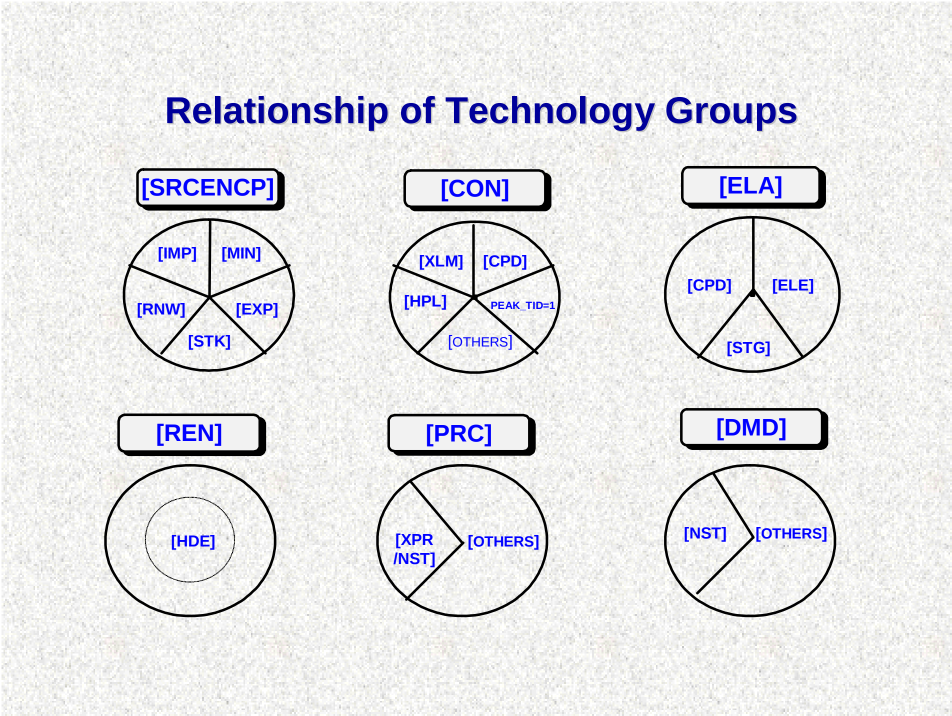

Figure 2-5. Relationship of technology groups

Resource Supply Sets (Sources and Sinks)
Resource supply options (srcencp) define the way commodities enter or leave the RES for each region. They are divided into five primary groups according to the nature of the resource supply activity as described here.

- **EXP**ort – enables a commodity to leave the area of study, or be exchanged between two regions by allowing the commodity to leave the current region. As part of the (regional) energy balance exports are consumption activities, with any associated emissions leaving the region with the commodity (if desired, by means of a negative emission coefficient).
- **IMP**ort – enables a commodity to enter the area of study, or be exchanged between two regions by allowing the commodity to enter the current region. In the (regional) energy balance imports are production activities, with any associated emissions charged to the region (if desired).
- **MIN**ing or extraction – is the domestic production of a resource such as mining of coal or the production of oil or gas.
- **R**e**N**e**W**able – identifies those sources producing physically renewable resources such as municipal solid waste, biomass, etc.
- **ST**oc**K**piling – is the movement into/out of a stockpile of a commodity. The commodity must first be supplied to the system by means of a resource activity or process. The stockpiled commodities are accumulated, passed from period to period and drawn down as necessary. Stockpiling is most often used for representing nuclear fuel cycles. Commodities remaining in a stockpile at the end of the modeling horizon are "credited back" in the objective function.

The modeler is strongly encouraged to follow the approach outlined below when naming the individual resource options:

- The first three characters of the resource name should correspond to one of the five primary group names (EXP, IMP, MIN, RNW, STK) just discussed.
- The next 3-6 characters should correspond to the primary output (or input in the case of exports) commodity associated with the resource activity. It is strongly recommended that these characters match the name of the commodity indicated by the OUT(ENT)r parameter (see section 2.3).
- The last character is a unique alphanumeric that serves as the supply step cost indicator, so that a number of similar resource supply options but with differing costs can be defined for the same commodity.

The reason for encouraging the modeler to follow this approach is that if it is *not* followed, and there is a need to look into the actual GAMS listing (.LST) file (usually only necessary if the model is infeasible) then there may be a potentially confusing difference between what appears in the GAMS .LST file and what might be expected based on the resource name.

There is one other aspect of naming resource supply options that the modeler needs to be aware of. For bi-lateral trade the cost step (last character) of the corresponding import/export options in the two regions **MUST** be the same, though the commodities traded are permitted to have different names in each region.

The hierarchy for the resource supply options is shown in Figure 2-6.

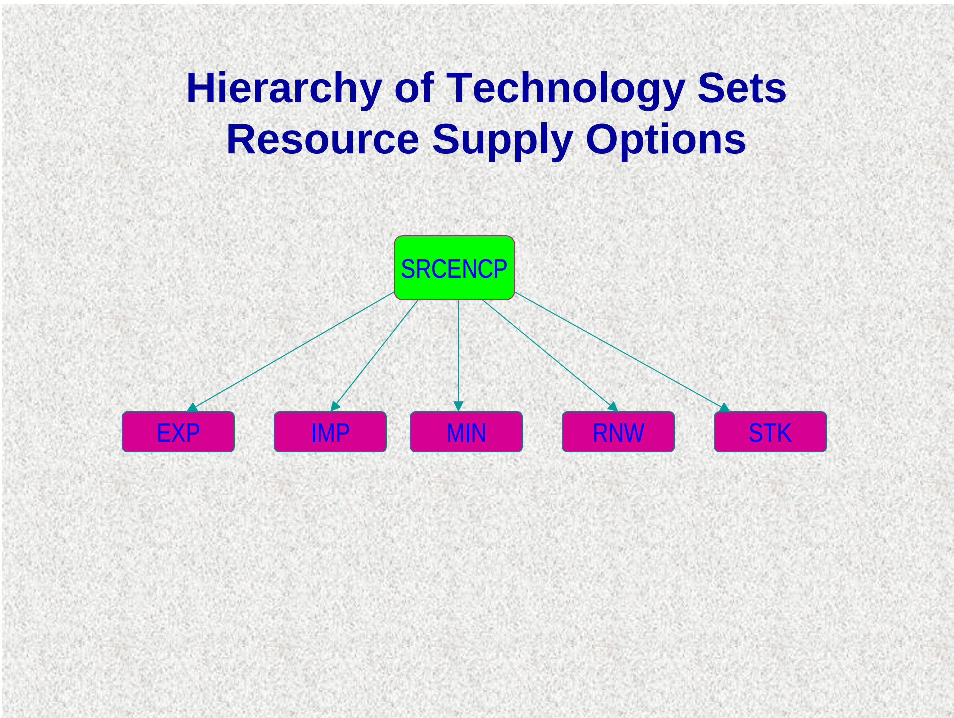

Figure 2-6. Hierarchy of resource supply sets

Process Sets

Processes are the most prevalent type of technology that can be depicted in MARKAL. They can consume any commodity except low-temperature heat, and output any other except electricity, low-temperature heat and demand services. They range from pipelines to gasification plants or refineries. They are also commonly used to model emissions reduction options, including sequestration.

Under most circumstances the inputs and outputs are in fixed proportions relative to the activity of the process, with the exception of flexible limit processes (identified by means of the LIMIT parameter) that permit variable levels (within a specified range) of the individual outputs from a process. Processes are divided into two primary groups according to whether they primarily handle energy or materials. That is not to say that a process cannot use or produce both energy and materials, but it is the modeler's responsibility to ensure that all units and relationships are consistently defined. The material processes are further split into those that are modeled in terms of volume versus weight.

Any process can be further qualified as to its operational nature by assigning it additional set characterizations if appropriate (e.g., night storage (nst) or externally load managed (xpr), and reporting type (renewable (rnt) and dummy (dum)). Night storage processes (nst) strictly consume off-peak electricity during the night to produce their annual output; an example might be hydrogen production. This hierarchy is shown in Figure 2-7 below. See Table 2-3, Definition of User Input Sets, for further elaboration as to the role of each of the sets and their implication on the model structure.

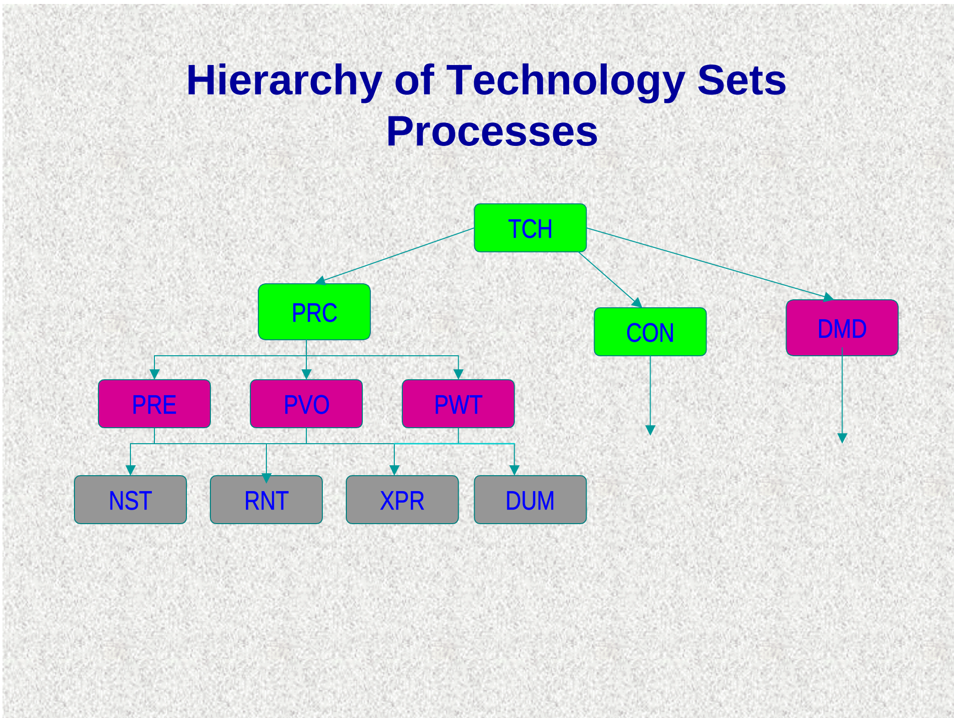

Figure 2-7. Hierarchy of process sets

Conversion Plant Sets

Conversion plants are the most elaborate and complicated technologies depicted in a MARKAL model. This is to be expected, as they are the technologies that handle the production of the time-sliced energy carriers electricity and low-temperature heat. But besides that, there are several types of conversion plants that can be modeled including ones that only produce electricity (ele and stg), those that can produce both electricity and heat (cpd), and heating only plants (hpl). Among these three major groups the power plants, except for storage plants, can be characterized as centralized (cen) or decentralized (dcn). In addition these facilities can be either base load (bas) or non-load management (nlm) plants, or neither of these. Both base load and non-load management plants can contribute to meeting the base load constraint, but the former are required to operate at the same level day and night. This is often the case for most nuclear and large coal-fired power plants. All of the plant types, other than coupled heat and power, may also be characterized as externally load managed (xlm). Such plants operate according to a fixed operational profile, rather than the flexible nature under which most plants operate. This feature is most often used for renewable technologies where the operation of the plant is dependent upon external factors (such as weather that affects solar and wind technologies). The conventional electricity only plants may also be characterized as hydroelectric (hde) that permits them to be subject to annual or seasonal reservoir availability, as will be seen later. Finally, electricity and heat grid interchange links (lnk and hlk respectively) can be identified. Such technologies accept electricity or heat from one grid and release it to another. This hierarchy is shown in Figure 2-8 below. Note that the FOS, NUC, and REN sets are defined by the model by examining the type of input energy carrier required by their members. For instance, if a power plant has some fossil fuel as input, it is automatically put in the FOS set.

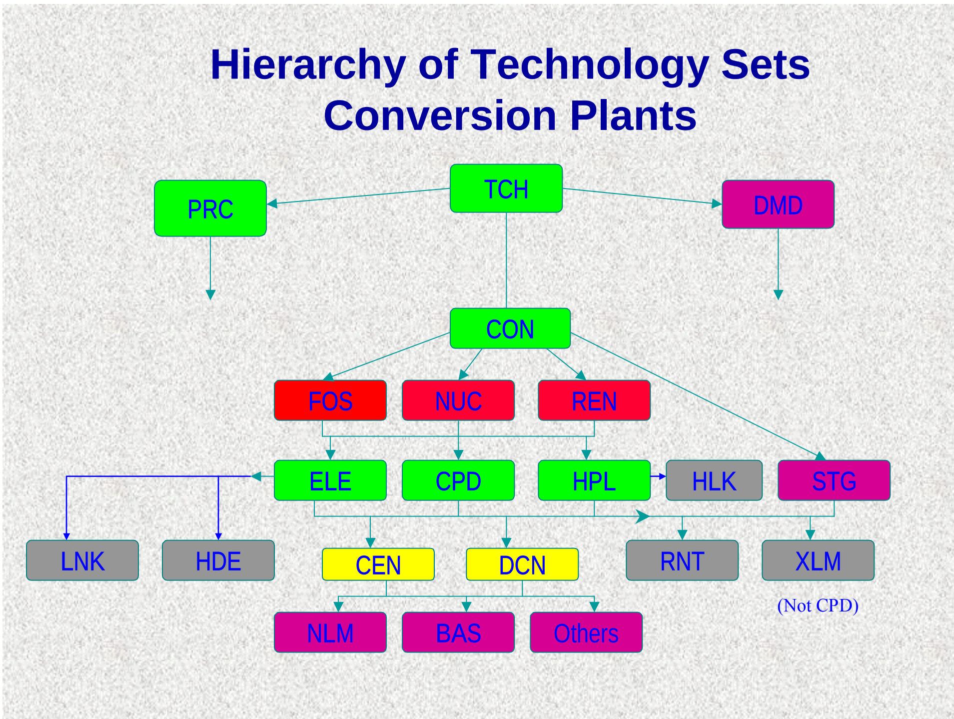

Figure 2-8. Hierarchy of conversion plant sets

Besides the sets shown in Figure 2-8 there are other characteristics that can be assigned to conversion plants: some plants may be designated as peaking only devices. Such plants (which cannot be coupled heat and power or externally load managed facilities) are identified by means of a parameter (PEAK_TID(CON)), discussed in section 2.3.

See Table 2-3, Definition of User Input Sets, for further elaboration as to the role of each of the sets and their implication on the model structure.

Demand Device Sets

Demand devices are modeled in a simpler manner than other technologies in that their level of activity is derived directly from the installed capacity in place based upon a fixed capacity utilization factor. In addition they can only accept commodities in fixed proportions. Typically they provide demand services to a single sector, although there is a provision for devices to service multiple demands (e.g. a furnace may feed hot water and heat). The only additional set characterization that can be associated with a demand device is to indicate that it is a renewable technology (rnt) or a night storage technology (nst). The latter indicates that the technology consumes off-peak (night-time) electricity, for example a battery or electric car that is charged in the evening. This hierarchy is shown in Figure 2-9. See Table 2-3, Definition of User Input Sets, for further elaboration as to the role of each of the sets and their implication on the model structure.

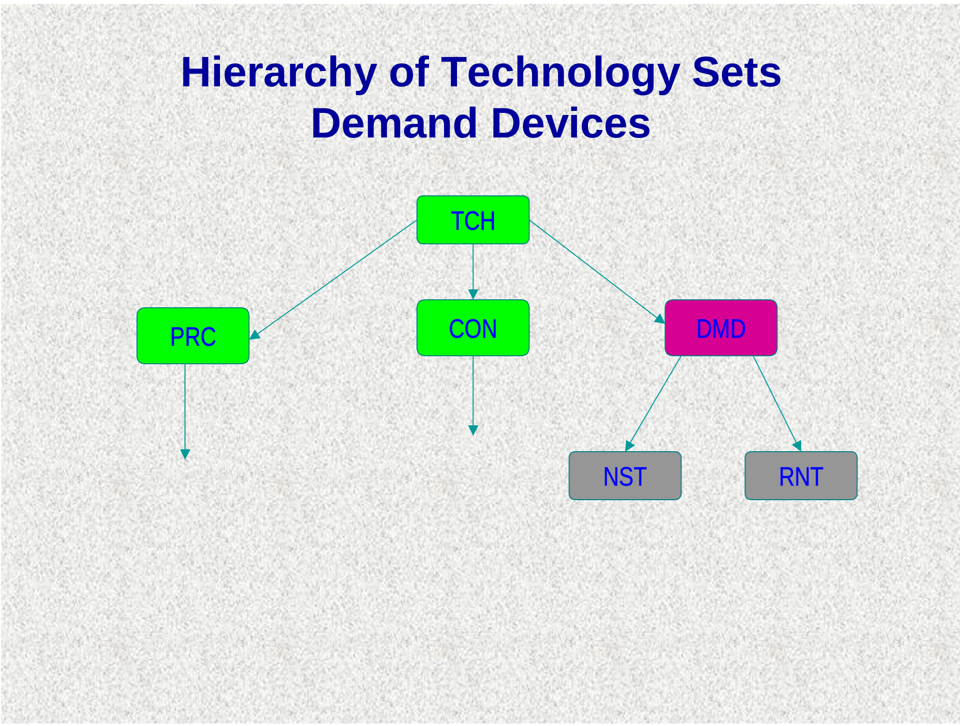

Figure 2-9. Hierarchy of demand device sets

2.2.1.6 Other User Input Sets

There are three other sets that may be supplied by the modeler.

The first is the list of user-defined *ad hoc* (or *user*) constraints (referred to in the GAMS code as the set (adratio)). These constraints are used to define special circumstances to the model that cannot be handled using the conventional constraints of the model. Some examples might include:

- Grouping competing technologies into market share groups to ensure that a particular set of technologies does not acquire more or less than a given market share;
- Establishing a renewable energy portfolio standard that requires a percent of total generation to be met by renewable technologies; and
- Tying the capacity of two technologies to one another (e.g., applying an emission reduction technology to existing power plants, using two separate demand devices to represent one physical device –e.g. a heat pump serving both heating and cooling demands).

The set of user-defined constraints (adratio) is simply the list of all such constraints. These specialized constraints are discussed in Section 2.5. In multi-region models there is also the ability to define cross-region user-defined constraints (xarat).

In order to employ the Endogenous Technology Learning mechanism (ETL for MARKAL), the modeler needs to provide the list of learning technologies as the set (teg). The technology learning capability is described in Section 2.8.

Another set that may be defined by the user is the list of commodity-based tax/subsidies (taxsub) that may be imposed on the energy system. The effect of a tax/subsidy is to increase or decrease the relative cost of a technology/commodity at various points in the system.

2.2.2 Sets and Indexes

As a reminder a short table of indexes is provided below. The process of constructing the parameters, variables and equations that control the actual intersections in the matrix is controlled by subsets of the referenced indexes, particular with regard to the technology references. For the most part these indexes correspond directly to user input sets discussed in section 2.2.3. The reader is referred to that section for a fuller description of each index, and for an indication of how entries in the sets influence the resulting model.

Table 2-2. Definition of indexes and their associated sets

Index	Input Set	Description
a	adratio,xarat	The user-defined **A**d hoc constraints that are used to guide market splits, group limits for technologies, etc., for each region (adratio) or across regions (xarat).
b	bd	**B**ounds or right-hand-side constraint types (‘LO’,‘FX’,‘UP’, and variations thereof (e.g., ‘MIN’,‘FIX’,‘MAX’)).
c	p	The **C**osts, or step indicators, for the resource supply options.
d	dm	The useful energy **D**emand sub-sectors.
e	ent/mat	The **E**nergy carriers and materials.
g	trd	The **G**lobally traded (as opposed to bi-lateral) commodities ((ent) or (env)).
p	tch	The technologies or **P**rocesses.
r	reg	The **R**egions for a multi-region model.
s	srcencp/sep	The resource **S**upply options, including those serving as the trade variables into/out of the regions.
t	year/tp	The **T**ime periods for the data (year) and requested for the current model run (tp).
txs	taxsub	The **T**a**X**es/**S**ubsidies on a technology/commodity combination.
u	step	The steps used to approximate (linearize) the elastic demand curves (consumer **U**tility).
v	env	The emissions or en**V**ironmental indicators.
w	td=z+y	The (pre-defined) season/time-of-day compound indexes (season (z) and time-of-day (y)) identifying the time-slices **W**hen an activity related to electricity or low-temperature heat occurs.

Index	Input Set	Description
x	ie	Export/import e**X**change indicator. Note: the definition of the set (ie) is internal to the model and fixed at 'EXP' for exports and 'IMP' for imports. But for any trade variables defined over (ie) there are only variables for the instances for which the modeler has explicitly provided data.
z	z	The (pre-defined) seasons for electricity and heat generation.

2.2.3 Synopsis of User Input Sets

In Table 2-3 the main sets seen by the modeler are listed. For each set a short description is provided, along with any Alias/Linked sets associated with the main set. The Alias/Linked sets are either directly mapped to the main set, or directly derived subsets of the main set. For the most part such subsets have compound names (e.g., basnuc) built from the various main sets from which they are derived, and are created as the intersection or union of the named sets, possibly subject to certain other conditions. These sets are used subsequently in the code to conditionally control execution of the matrix generator and/or report writer, or to improve the performance of the code for frequently used indexes and conditions. (The Linked sets are further defined in Table 2-4: Definition of Internal Sets.) The Description column has two purposes. It first provides a brief statement of what the set is all about, and then a series of bullet points indicates how the set impacts the model structure.

Table 2-3. Definition of user input sets

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
ADRATIO (a/m)	xarat mkt_id adr_xxx	The list of user-defined ad hoc constraints, or ADRATIOs, for a region (adratio) or across regions (xarat). For each user-defined constraint (not a cross-region constraint), a MR_ADRATn equation is created (as long as a RAT_RHS entry is provided). • For each cross-region user-defined constraint a MR_XARATn equation is created (as long as a XARATRHS entry is provided). • The adr_xxx sets indicate which technologies are involved in which user-defined constraints and using which variables (R_ACT, R_CAP, R_INV, R_TCZYH, R_TEZY, R_THZ, R_TSEP).

[39] The SetID column uses upper case for the input sets, breaking with the convention employed in the rest of this document, to distinguish them from the single-character indexes (in parentheses).
[40] The SetIDs are used in the GAMS code as indexes into other sets, the parameters and the matrix rows/columns. For those SetIDs that correspond to indexes used in this document the index character is given within parentheses after the SetID.
[41] The user input sets are often combined into the Alias/Linked related sets for use in the GAMS code. ONLY those users interested in gaining an understanding of the underlying GAMS code need to become familiar with the Alias/Linked sets.
[42] In the Description column the bulleted entries address the implications for the model of including an item in the Set.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
BAS	basnuc cenbas con cpdbas elabas elaxbas	Base load power plants. These are plants that tend to operate continuously (e.g., nuclear or large fossil-fired) and are thus forced to operate at the same levels day and night, within a season, in the model. • Contributes to the base load constraint (MR_BAS) that limits the fraction of highest night-time demand that can be met by such plants. • If there are no entries in BAS the constraint is not generated.
BD (b)		Bound type = 'LO', 'FX', 'UP', 'NON'. • Indicates the nature of a bound (or RHS if a user constraint) that is applied to a variable (or equation), as well as the direction of the demand elasticity specification. For example, the modeler may wish to disallow investment in new nuclear capacity. This can be accomplished by putting an upper bound of zero on the appropriate new investment variable (via the IBOND(BD) parameter).
BI_TRDELC		The 2-tuple of region/electricity involved in bi-lateral trade of electricity by season/time-of-day. For each form of the electricity energy carrier an associated import/export supply option (srcencp) must also be specified for the same cost step (c). Note that the trade is unidirectional, that is only from export to import region, and that it only occurs for the time-slices specified. • Controls the generation of the bi-lateral electricity trade constraint (MR_BITRDE) between two regions for the time-slices specified. • The time-sliced electricity trade variable (R_TSEPE) is set equal to the traditional trade variable (R_TSEP), that is then used in the objective function, emission constraints, etc.; the seasonal variable appears directly in all the electricity specific constraints (MR_BALE, MR_BAS, MR_EPK). • Only for multi-region models.
BI_TRDENT		The 2-tuple of region/commodity involved in bi-lateral trade. For each commodity an associated import/export supply option (srcencp) must also be specified for the same cost step (c). Note that the trade is unidirectional, that is only from export to import region. • Controls the generation of the bi-lateral commodity trade constraint (MR_BITRD) between two regions. • Only for multi-region models.
CEN	con cenbas cencpd elacen	Centralized electric (ela), heat (hpl) and coupled-production (cpd) power plants. These plants have associated investments in transmission and distribution infrastructure whenever new capacity is added, and the energy generated is subject to transmission losses. Centralized and decentralized (dcn) plants incur distribution costs as well. • The cost of expanding the centralized grid (ETRANINV for electricity, DTRANINV for heat) is added to the investment cost (INVCOST) associated with each unit of new capacity, along with the distribution cost for electricity (EDISTINV). • The cost of keeping the grid operational (ETRANOM for electricity, DTRANOM for heat) is added to the total cost according to the installed capacity of the grid.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
CON	pon tch	All conversion plants, that is those technologies producing electricity and/or low-temperature heat. • The technology is modeled at a finer operational level, season/time-of-day (td=z+y) for electricity plants (ela, R_TEZY/R_TCZYH) and season (z) for heating plants (hpl, R_THZ), unless externally load managed (xlm, or lifetime (LIFE) = 1 period and no investment cost (INVCOST) is specified).
CPD	cencpd con cpdbas dcncpd ela tpcpd	The subset of conversion plants that produce both electricity and low-temperature heat. These are sometimes referred to as coupled-production plants. • The technology output is modeled at the season/time-of-day level. • If the plant has a flexible pass-out turbine (CEH(Z)(Y)), then a separate variable is generated for the dynamic heat output (R_TCZYH) , otherwise the heat produced is calculated directly from the electricity production variable (R_TEZY).
CPTEL	$SET switch, added manually by the user to the GAMS command files	Switch indicating whether the capacity transfer equation (MR_CPT) for conversion plants (con) and processes (prc) is to be an equality (=E=, the default), or an inequality (=L=).
DM (d)		All useful energy demand sub-sectors. • An equation is created (MR_DEM) to ensure that the combined output of all demand devices (dmd) servicing the demand meets or exceeds the demand. • The reporting subsystem breaks the demands and demand devices (dm, dmd) out into sectors passed upon the first character of their name (Agriculture, Commercial, Industrial, Non-energy, Residential, Transportation).
DMD	tch dmd_dm dmd_dms dmdnst tpdmd	The demand devices. Demand devices are different from other technologies in that they usually do not produce an energy carrier (or material), but rather provide end-use demand services. • Only demand devices can deliver demand services, and thus satisfy the demand constraint. • Demand devices are assumed to operate if built. Therefore in the model their activity/output is determined directly from the level of installed capacity (R_CAP) and there is no separate variable for the activity (c.f. the situation with process (prc) and conversion (con) technologies). • The reporting subsystem breaks the demands and demand devices (dm, dmd) out into sectors passed upon the first character of their name (Agriculture, Commercial, Industrial, Non-energy, Residential, Transportation).
DUM		The reporting of the production and use of commodities by technologies in this set is suppressed.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
ECV	enc enc_n	Those energy carriers that are considered conservation. • Conservation energy carriers are tracked by a non-binding balance equation (MR_BAL_N) that simply monitors the total fossil equivalent (FEQ) consumption.
EFS	enc enc_g	Those energy carriers that are considered fossil energy carriers. • Controls the generation of the non-binding fossil (MR_FOSSIL/TFOSSIL) and non-renewable (MR_NONRNW/TNONRNW) accounting equations. • Used for some of the ANSWER report tables, and when building production/consumption Tables in VEDA by fuel types.
EHC	enc enc_g	Those energy carriers that are considered high-temperature process heat. • Used for standard MARKAL reports T03 and T05, and when building production/consumption Tables in VEDA.
ELC	e ent	Electricity energy carriers, where for regions with multi-grids there may be more than one. • For each electricity energy carrier there are a series of time-slice based electricity equations generated for the production/consumption balance (MR_BALE), the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
ELE	ela	The subset of conventional conversion plants that produce only electricity, excluding storage (stg, e.g., pumped hydro) facilities. • For all such power plants not subject to external load management (xlm), activity is modeled by season/time-of-day (R_TEZY) as a function of the installed capacity (R_CAP) and the associated availability factor (AF/AF(Z)(Y)) by means of a utilization equation (MR_TEZY). • Identifies which technologies contribute to the time-slice based electricity equations for the production portion of the balance equations (MR_BALE), the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
ENC	ent mat	All energy and material commodities, except electricity and heat.
ENT (e)	elc enc lth mat	All energy and material commodities, including electricity and heat.
ENU	enc enc_g	The nuclear energy carriers. • Used for standard MARKAL reports T03 and T05, and when building production/consumption Tables in VEDA.
ENV (v)	gwp	All emission indicators.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
ERN	enc ernxmac mac	Those energy carriers that are considered renewable. • Renewable energy carriers for which no resource supply option is provided are considered "free" (e.g., solar sunlight, the wind) and are tracked using a non-binding fossil equivalent (FEQ) accounting equation (MR_BAL_N). • Renewable energy carriers for which a resource option is provided (via RNWencp) are handled as all other energy carriers by means of a greater than or equal to balance constraint (MR_BAL_G). They are often tracked by a parallel fossil energy equivalent (FEQ) accounting renewable energy carrier.
ESY	enc enc_g	Those energy carriers that are considered synthetic or derived/transformed (e.g., hydrogen). The set has no explicit impact on the matrix or reports, except to complete the list of all standard energy carriers (enc).
FEQ		The set is a short name for Fossil Equivalent, and contains a single reserved element 'FEQ'. • The fossil equivalent value corresponds to the reciprocal of the average efficiency of the fossil fuel power plants (in a region), and is used for reporting the primary energy equivalent of renewable energy use.
GAZ	efs ent	Gaseous fossil energy carriers. • Used for standard MARKAL Tables T02-T05, and VEDA reporting.
G_TRADE (g)		Any commodity, including emissions, that is to be traded globally.
HDE	ela	Hydroelectric power plants. • Identifies those power plants that are subject to annual or seasonal reservoir management (MR_ARM/MR_SRM). • Used for construction of power plant tables by type in VEDA.
HEATCOOL		Indicator whether the (lth) peaking constraint for heating/cooling will be modeled for the winter (heating) or summer (cooling). Default is 'W'inter.
HLK	hpl	Heat grid inter-connection or link conversion technologies. These technologies consume and produce differently named heat energy carriers. • Heat grids are modeled at the seasonal level (R_THZ), unless defined as externally load managed (xlm). • These technologies contribute to the season based heat equations for the production/consumption portions of the heat balance (MR_BALDH) and the peaking requirement (MR_HPK) equations.
HPL	con tphpl	The subset of conversion plants that produce only low-temperature heat. • For each heating plant not externally load managed (xlm) the activity is tracked by season (z) as a function of the installed capacity (R_CAP) and availability (AF/AF(Z)(Y)) by means of a utilization equation (MR_THZ). • Identifies which technologies contribute to the season based heat equations for the production portion of the heat balance (MR_BALDH) and the peaking requirement (MR_HPK) equations.
LIQ	efs ent	Liquid fossil energy carriers. • Used when building production/consumption standard tables T02-T05, and by VEDA for fuel types.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
LNK	ela	Electric grid inter-connection or link conversion technologies. These technologies consume and produce differently named electric energy carriers, as opposed to storage (stg) technologies that consume/produce the same electricity. • Such grids are modeled at the season/time-of-day level (R_TEZY), unless defined as externally load managed (xlm). • Identifies which technologies contribute to the time-slice based electricity equations for the production/consumption portions of the balance (MR_BALE) equation, the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
LTH	ent h	Low-temperature heat energy carriers, where for regions with multi-grids there may be more that one. • For each low-temperature heat energy carrier, there are a series of seasonally based heat equations generated for the production/consumption balance (MR_BALDH) and the peaking requirements (MR_HPK).
MAT	ent mvo mwt	Those commodities designated as materials. • The production and use of such commodities are modeled using an equality constraint to ensure that the amount produced does not exceed the amount consumed (MR_BAL_E). • Used when building production/consumption Tables in VEDA by fuel types.
MVO	mat	Those materials tracked by volume.
MWT	mat	Those materials tracked by weight.
NLM		Subset of conversion plants that can meet system base load requirements, but do not have to have the same day/night production. • Such technologies are included in the plants meeting the baseload limit (MR_BAS), but operate independently day/night during any season.
NST	dmdnst prcnst	Processes (prc) and demand devices (dmd) that consume electricity only at night while supplying energy and demand services annually. • Such technologies enter only the off-peak (night) electric balance equations (MR_BALE).
NUR		List of reactor types. • Used when building Tables in VEDA by reactor types. • Not available as a User Input Set in ANSWER.
P (c)		The cost step associated with each resource supply option (srcencp). Taken directly as the last character of the name of the resource option. • Note that for the specification of bi-lateral trade, the same cost step (P) must be designated to identify individual trade routes. • In ANSWER, P is not a User Input Set, but rather is the set comprised of the last characters of all the resource options.

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
PRC	pon prcnst tch tpprc zpr	The process technologies. Those non-resource technologies not explicitly characterized as belonging to the power sector (con) or as demand devices (dmd). • For each process not treated as externally load managed (xpr), activity is modeled annually (R_ACT) as a function of the installed capacity (R_CAP) and the associated availability factor (AF). This is controlled by means of a utilization equation (MR_UTLPRC).
REG (r)		Each of the regions in the model. Note that for each region self-contained input datasets are generated containing the model specification for that region.
RNT		Those technologies that are to be excluded from the MARKAL MR_PRICER non-binding total system cost excluding renewable technologies. • Used when building production/consumption Tables in VEDA by fuel types.
SLD	efs ent	Solid fossil energy carriers. • Used when building production/consumption tables T02-T05 and by VEDA for fuel types.
SRCENCP (s)	sep (src,ent,p) tpsep	List of the resource supply options, including imports and exports. The single SRCENCP member names are actually composed of three sub-components (src,ent,p) that are derived by the preprocessor (sep). • For each resource supply option there is an associated variable (R_TSEP). • The resource supply variable appears in all appropriate balance (MR_BAL, MR_BALE), electric peaking (MR_EPK) and base load (MR_BAS), and emission (MR_TENV) constraints, as well as the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). • It is **strongly** recommended that the modeler use the same characters for the commodity part (ENT) of the name (srcENTp, characters 4 through up to 9) as the primary output (OUT(ENT)r)) to avoid a possible mix-up with respect to the actual names employed in the GAMS model run and those provided by the modeler. • For bi-lateral trade, including electricity, there must be a corresponding srcencp entry, where the p-index identifies the particular trade route between the importing/exporting countries.
STG	con ela	Conversion plants that produce electricity only during the day, consuming off-peak night-time electricity to charge the storage (e.g., pumped storage). • In the model structure no night-time activity variable (R_TEZY) is generated, but rather the corresponding daytime variable is used to determine the amount of night electricity needed to refill the reservoir.
TAXSUB (txs)		The name of the tax/subsidy constraints for commodity/technology combinations. • For each tax/subsidy entry an associated constraint (MR_TXSUB) is generated to which each identified commodity/technology combination contributes, and the associated balance variable (R_TXSUB) enters the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).

Set ID/ Index[39,40]	Alias/ Linked[41]	Description[42]
TCH (p)	con dmd prc tptch	The master list of all technologies (process technologies (prc) plus conversion plants (con) plus demand devices (dmd); collectively sometimes referred to as "processes" rather than "technologies"). The set TCH excludes resource supply options (srcencp).
TEG		Those technologies subject to the endogenous technology learning algorithm when ETL is actived (see section 2.8).
TP (t)	ts tc/rtp year	The periods for the current model run. • The series of technology related sets (e.g., tptch, tpcon, tpdmd, tpprc, tpsep) establish the periods for which a technology is available according to the year in which it can first be deployed (START). • An alias (tc) is used for dual period index parameters such as those needed for the capacity transfer constraint.
XLM	elaxlm elaxstg nucxlm hplxlm	Conversion plants (other than coupled heat and power (cpd)) that are externally load managed. These are plants that cannot operate in a flexible manner, and that if built are used according to an associated capacity utilization factor (CF/CF(Z)(Y)) rather than to a maximum availability factor (AF/AF(Z)(Y)). Technologies that are subject explicitly to external factors such as weather (e.g., solar, wind) are usually characterized as externally load managed. • For such conversion plants no activity variable (R_TEZY / R_TCZYH / R_THZ) is generated, but rather the activity is derived directly from the level of installed capacity (R_CAP) as a function of a capacity utilization factor (CF/CF(Z)(Y)); therefore no utilization equation (MR_TEZY/MR_THZ) is needed.
VEDABE VEDABEX VEDAVDD	$SET switch, added manually by the user to the GAMS command files	Series of switches indicating that VEDA is being used for results processing. • VEDABE 'YES' to indicate that VEDA-BE is being used. • VEDABEX 'V4' to indicate that VEDA Version4 is being used (VEDABE switch needed as well). • VEDAVDD 'YES' to indicate that the GDX2VEDA GAMS utility is being used (VEDABE switch needed as well).
YEAR (t)	tp	The periods associated with the input data and thereby available for model runs, though a run may be done for a shorter time horizon if desired.

2.2.4 Internal Sets

This section presents the key internal sets, built from primary sets discussed above and used to control the execution of the model code and the creation of coefficient instances of the matrix.

In Table 2-4 the Set Rules used to define internal sets are presented. In some cases, the Set Rules are not fully elaborated, and only the important aspects are shown.

Often the Set Rules involve either the intersection, or the union, or the negation of primary sets. In Table 2-4 below, the following short-hand notation is adopted:

*	indicates the intersection of two sets (e.g., A*B – in both set A and set B);
+	indicates the union of two sets (e.g., A+B – in either set A or set B);
not	indicates the negation of a set (e.g., not A – not in set A).

Table 2-4. Definition of internal sets[43]

Internal Control Set[44]	Main Input Sets	Set Rule[45]	Description
BASNUC	con	bas*nuc	Nuclear plants that are characterized as base load.
CENBAS CENCPD	con cpd	cen*bas cen*cpd	Centralized base loaded and centralized coupled-production power plants.
CPDBAS	cpd	cpd*bas	Base loaded coupled-production power plants.
DCNCPD	cpd	dcn*cpd	Decentralized coupled-production power plants.
DMD_DM	dmd,dm	dmd* DMD_OUT, where OUT(DM) is largest	Identifies the primary demand sector (dm) serviced by each demand device (dmd) according to which output fraction (DMD_OUT) is the largest, or the first one encountered if more that one demand receives the same fraction. [Set in DMD_DM.ANS]
DMD_DMS	dmd,dm	dmd* DMD_OUT	All the demand sectors (dm) serviced by each demand device (dmd) according to the device output specification (DMD_OUT). [Set in MMFILL.INC]
DMDNST	dmd	dmd*nst	Demand devices that are night storage technologies.
DMD_VINT	dmd	EFF_I/R	List of DMDs for which vintage tracking is requested by providing the investment (EFF_I) and residual (EFF_R) efficiencies, instead of the traditional capital stock efficiency (EFF). This results in the model using the investment (R_INV) and residual capacity (R_RESIDV) variables instead of the capacity variable (R_CAP) for all activity based equations entries. [Set in MMFILL.INC]

[43] Sets without a rule are static and declared in MMINIT.*, most others are established in MMSETS.INC; except where noted in the description. While this list, in tandem with the User Input Sets in Table 2.4, is comprehensive, it is not exhaustive.
[44] The Internal sets are used in the GAMS code as indexes into other sets, the parameters and the matrix row/columns.
[45] In some cases the Set Rule is only given for the first Internal Set, as indicated by …, with the others listed following an identical approach but with different conditional criteria.

Internal Control Set[44]	Main Input Sets	Set Rule[45]	Description
ELA ELABAS ELACEN ELANUC ELASTG ELAXBAS ELAXLM ELAXLSTG	con	ela+stg+cpd ela*bas ela*cen ela*nuc ela*stg ela-bas ela*xlm elaxlm*stg	This set is established by the pre-processor directly from the input data as the combination of the various types of power plants that can generate electricity (conventional (ele), coupled production (cpd) and storage (stg)). • All such power plants are modeled at the season/time-of-day level (R_TEZY). • For each power plant not externally load managed (xlm), activity is modeled by season/time-of-day (R_TEZY) as a function of the installed capacity (R_CAP) and the associated availability factor (AF/AF(Z)(Y)) by means of a utilization equation (MR_TEZY). • Identifies which technologies contribute to the time-slice based electricity equations for the production portion of the balance equations (MR_BALE), the peaking requirements (MR_EPK), and the base load constraint (MR_BAS).
ENV_ENTXI ENV_ENTXO	tch, env	ENV_ENT	An indicator to exclude the inputs/outputs from a particular technology for the ENV_ENT algorithm.
ERNXMAC	ern	not mac	Renewable energy carriers for which no physical resource supply (RNWencp) activity is defined. These energy carriers (e.g., solar, wind, geothermal) have no balance equations constructed, but instead are characterized at the technology level. Their fossil equivalent (FEQ) is tracked by means of non-binding balance constraints (MR_BAL_N). This set also includes the fossil equivalent energy carriers associated with physical renewables that are input in material rather than energy units (e.g., tons of biomass).
FOS	con	not stg	All conversion plants that consume a fossil energy carrier. [Set in ANS2GAMS.ANS/VEDA2GAMS.VFE by examining the inputs (INP(ENT)c) to the technology.]
GWP	env	ENV_GWP	Emission indicators for which a global warming potential is assigned, based upon parameter ENV_GWP(gwp_env,env,t). [Set in MMFILL.INC]
HPLXLM	con	hpl*xlm	Heating plants that are externally load managed.
IE	src srcencp		The exchange resource options for EXPort and IMPort of commodities.
IE_ELC	sep	('IMP'encp+ 'EXP'encp)* elc	An indicator of which resource supply options EXPort and IMPort electricity.

Internal Control Set[44]	Main Input Sets	Set Rule[45]	Description
MAC	ern ernxmac	'RNW'encp* ern	Renewable energy carriers for which a physical resource supply activity is defined and therefore the material must be accounted for. These energy carriers (e.g., MSW, biomass) have energy balance equations (MR_BAL_G) constructed, as opposed to the rest of the renewable energy carriers (ern), that are considered unlimited from the resource availability point of view (and instead characterized at the technology level). These renewables are often modeled in physical units (e.g., tons of biomass), and shadowed by a fossil equivalent (feq) accounting energy carrier.
NOBASIE	elc		An indicator that imports and exports are to be excluded from the base load constraint (MR_BAS).
NUC	con	not stg	All conversion plants that consume a nuclear energy carrier. [Set in ANS2GAMS.ANS/VEDA2GAMS.VFE]
NUCXLM	con	nuc*xlm	Nuclear plants that are externally load managed.
P (c)			List of all 1-digit numbers and upper case letters available to identify the individual supply curves or the cost steps; each member of the list corresponds directly to the last character of the name of a member of set SRCENCP.
PON	tch	prc+con	All process (prc) and conversion (con) technologies.
PRCNST	prc	prc*nst	Processes that are night storage technologies.
QHRZ	z	y='D'+'N'	Fraction of the year for each season, that is the sum of the day and night portions (QHR(Z)(Y)).
REN	con	not stg	All conversion plants that consume a renewable energy carrier. [Set in ANS2GAMS.ANS/VEDA2GAMS.VFE]
RTY			Equation type = 'FIX', 'MAX' (GE), 'MIN' (LE), 'OBJ'.
SEP	src,ent,p	srcencp	Parsed pieces of resource supply names, where first three characters correspond to the source (src), the last character to the supply step (p), and the remaining characters to the commodity (ent). It is strongly recommended that the modeler uses the same characters for the SEPent root as the primary output (OUT(ENT)r)) to avoid missing internal and input/output SRCENCP names, as well as sticking with the first three character naming convention (below). [Established from srcencp in ANS2GAMS.ANS.]
SEP_SRCENC	sep, srcencp		Mapping of the srcencp 10-character name to its sep(src,ent,p) 3-tuple.

Internal Control Set[44]	Main Input Sets	Set Rule[45]	Description
SEPEE SEPIEE	sep,enc, env	ENV_ENT	The resource options involved in commodity-based emission tracking (ENV_ENT(env,ent)). [Set in MMSETEE.INC]
SRC		srcencp	Type of resource supply option[46]: • EXP = Export • IMP = Import • MIN = Mining/Extraction • RNW = Renewable (physical) • STK = Stockpiling (for nuclear material)
TB	tp	first tp	First period of the model run.
TCHEE TCHIEE	tch,enc, env	ENV_ENT	The technologies involved in commodity-based emission tracking (ENV_ENT(env,ent)). [Set in MMSETEE.INC]
TCHAFCK	tch,z,y	AF/AF(Z,Y)	Timeslices that a non-externally load managed technology is permitted to operate. [Set in MMFILL.INC]
TD	z,y		The combination of all season/time-of-day timeslice combinations = Intermediate Day (‘ID’), Intermediate Night (‘IN’), Summer Day (‘SD’), Summer Night (‘SN’), Winter Day (‘WD’), Winter Night (‘WN’). The modeler assigns the fraction of the year that corresponds to each of the six possible divisions via the global parameter QHR(TD). To eliminate a season/time-of-day timeslice do not provide its QHR(TD).
TLAST	tp	last tp	Last period of the model run.
TPTCH TPCON/ TPCPD/ TPDMD/ TPELA/ TPELAXLM/ TPHPL/ TPPRC/ TPTEG/ TPZPR	tp,tch	tp*tch* (VAL(tp) ≥ START) …	List of periods for which each technology is available, beginning with the period in which the technology is first available and running through the modeling horizon.
XPRT	src	srcencp	The export resource option where src = ‘EXP’.
Y (td)			The time-of-day values = Day/Night (‘D’/‘N’), corresponding to the y index in the set of the time-slices (td).
YESRA	adratio	RAT_ACT	To control the handling of RAT_ACT against R_ACT + R_CAP. [Set in MMRATACT.INC]
Z (td)			The season values = Intermediate/Summer/Winter of (‘I’/‘S’/‘W’), corresponding to the z index in the set of timeslices (td).

[46] These specific short acronyms **MUST** be used as the root first 3 characters of the SRCENCP resource name.

Internal Control Set[44]	Main Input Sets	Set Rule[45]	Description
ZPR	prc	xpr + (LIFE<=1 period or no INVCOST)	All externally load managed processes (xpr), and regular processes (prc) with a lifetime (LIFE) <= 1 period or with no investment cost (INVCOST) are included in this extended set, that is used for all subsequent processing. Note that such processes do not have activity variables (R_ACT) generated in the model, rather the capacity variable (R_CAP) is used directly applying the fixed capacity factor (CF).

2.3 Parameters

The parameters are the data input to MARKAL defining the RES. They indicate the inputs and outputs to/from each technology, describe the operations and limits of the individual technologies, and represent the demands for energy services. MARKAL parameters are described below in two sub-sections, 1) User Input Parameters (also called Primary Parameters) and 2) Matrix Coefficients and Internal Model Parameters. The User Input Parameters are those parameters that the modeler sees and defines in the front-end systems ANSWER and VEDA. The Matrix Coefficient and Internal Model Parameters are those parameters either entered by the user or derived by the GAMS pre-processor that appear in the actual matrix generation portion of the code.

In section 2.3.1 the User Input Parameters are briefly described and presented in Table 2-6. The Matrix Coefficients and Internal Model Parameters are then described in section 2.3.2 and presented in Table 2-10. This Table has an entry for every entity appearing in Appendix A that provides a schematic snapshot of the matrix structure. Finally a few other important Key Internal Parameters are presented in Table 2-11. Collectively these Tables highlight almost all the parameters embodied in the model and the associated code, except for some temporary parameters local to a single GAMS routine.

As noted in the introduction to this Chapter, when running multi-region MARKAL the parameters discussed here have "_R" appended to their names. During multi-region runs the matrix generator and report writer buffer into/from a regional data structure to isolate the specific information associated with each region. These parallel data structures have names identical to their single region counterparts, but with the "_R" suffix added to their names, and the first index for all such mapped parameters designating the region (via the index reg or r). In all the descriptions of parameters in this section the regional suffix and index are omitted, though it is understood to be part of the associated data structures, in almost all cases. Where there is an exception to this rule it is noted, for example if trade options identify both from/to regions (and commodities) for parameters that are strictly related to trade.

2.3.1 User Input Parameters (Primary Parameters)

Each User Input Parameter is explained in terms of the:

- Indexes defining the structure;
- Purpose (as noted in Table 2-5);
- Alias or Internal code Name;
- Related Internal Parameters;
- Units, Range & Default;
- Instance indicating when the parameter is expected to be provided; and
- Description.

The Alias/Internal Name is provided only if a name different from the primary parameter (seen by the modeler) appears in the GAMS code. The Related Parameters column serves to map the primary parameters to the key associated internal parameters used in the GAMS code, and vice versa. Only the more significant Related Parameters are listed in the Table. The Units, Range and Default column indicates the units in which the data is to be entered, consistency of which is totally the responsibility of the modeler. The Range and Default indicate the expected range for the value provided, and the default value set in the code if the parameter value is not provided by the user. The Instance column indicates when a parameter is expected to be provided or omitted, as well as highlighting any special conditions that may be imposed on the parameter. The Description column has two purposes. It first provides a brief statement of what the parameter is all about, and then a series of bullet points indicating how the parameter impacts the model structure.

Each parameter imparts a particular kind of information about the RES and its components. To advise the user as to the nature or purpose of each parameter, each has been assigned to one of the following groups.

As mentioned in the introduction to Chapter 2, to assist the reader with recognizing the nature of the various model components, the following conventions are employed in the Tables:

- Sets, and their associated index names, are in lower case, usually within parentheses;
- Literals, values explicitly defined in the code, are in upper case within single quotes;
- Parameters, and scalars (un-indexed parameters) are in upper case;
- Equations are in upper case with a prefix of MR_, and
- Variables are in upper case with a prefix of R_.

As already mentioned, for parameters in this section the regional suffix (_R) and index (reg, r) are not shown in the Tables, except for trade related parameters, but are implied for a multi-region MARKAL model.

Table 2-5. Purpose of user input parameters

Nature or Purpose	ID[47]	Description
Costs	C	All the monetary parameters.
Demand	D	Characterize the demand for energy services.
Environmental Indicators	E	Used to track emissions associated with resource activities and technologies.
Endogenous Technology Learning	L	Those parameters associated with clairvoyant and inter-period Technology Learning algorithms.
Miscellaneous	M	Global and other parameters not covered by one of the other groups.
Technical Operational Characteristics	O	Those parameters describing the operating characteristics of resource options and technologies and the infrastructure (e.g., transmission and distribution system), including limits imposed on them.
Topology & Trade	T	Depict the flow of commodities into and out of technologies and regions, thereby the commodities consumed/produced.
User Constraints	U	Information related to those specialized constraints explicitly built by the modeler to associate variables of the model in ways not otherwise possible.
Model Variant and Switches	V	*Indicators of which model features are to be activated during the current run (e.g., multi-region, time-stepped, market share, endogenous technology learning, and flexible demands). [In a separate* Table 2-13 in section 2.7.]

[47] The 1-character indicator that appears within [] in Table 2.4.7 to indicate the purpose of the attribute.

Table 2-6. Definition of user input parameters[48]

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
AF (p,t) [O]	TCH_AF1	*TEZY_CAP* *THZ_CAP* *TUTL_CON* *UPRC_ACT* *UPRC_CAP*	• Decimal fraction = {hours of production} / {no. of hours in the year}. • [0,1]; default = 1, if no AF(Z)(Y) provided.	• Required for all process (prc) and conversion (con) technologies. • Omit if seasonal availability (AF(Z)(Y)) provided, or technology is externally load managed (xpr, xlm) or is a peaking device (PEAK(CON) _TID) • The capacity factor	The annual availability of a process (prc) or conversion plant (con). That is the fraction of the year that the capacity is available to operate. • The activity of a conventional process (R_ACT) or conversion plant (R_TCZYH, R_TEZY, R_THZ) is related to the total installed capacity (R_CAP) as a function of the availability factor by the utilization equation (MR_UTLPRC, MR_TEZY, MR_THZ). • If a technology is externally load managed (xpr, xlm) and a capacity factor

[48] *Italics* identify parameters that are directly referenced when generating matrix coefficients. In some cases it is the Input Parameter, in which case a superscript of "row/column" indicating the cell in the MARKAL matrix spreadsheet is specified, or referenced Alias/CodeName that appears in the equation generation code, but most often it is a Related Parameter that is conditionally constructed and/or combines several input components that serves as the matrix coefficient. In this Table those parameters directly referenced in the equations are highlighted in this manner, with all such parameters (including those listed in the Parameter and Alias columns) presented in Table 2.4.7 as well. Note that there is at least 1 italic entry for each parameter, as all input data is somehow represented in the matrix.

[49] A Parameter without indexes is considered a Scalar in GAMS, that is it just assumes a single numeric value. The documentation index letters are used here, except when a parameter only applies to a subset of the master index sets.

[50] The purpose indicates the main role played by the parameter, as noted in Table 2.6.

[51] Most Parameters are named differently in the GAMS code than as presented to the modeler. An Alias is a parameter where only the name is different, that is it is not subject to condition or a combination of parameters.

[52] Only the most prominent related parameters are listed here, that encompass most but not necessarily all those parameters associated with a user parameter entry in this Table.

[53] An indication of the circumstances for which the parameter is to be provided or omitted, and any special circumstances.

[54] References to other model components are enclosed in (), after naming them; (set) in lower case, (PARAMETER) in upper case, and (MR_/R_) are references to model equations/variables. This holds for the most part, with the exception that some file names, literals (e.g., 'BPRICE') and MARKAL variants that are also in upper case. The bulleted items indicate how the parameter impacts the matrix.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				(CF) is set to the availability factor (AF) if a plant is externally load managed and CF is not provided. • Peaking devices (PEAK(CON) _TID) apply a peak duration factor (PD(Z)D) to the time-slice (QHR) instead of an availability factor.	(CF/CF(Z)(Y)) is not provided then the availability factor is assigned as the capacity factor. For such technologies no activity variable is created, rather the capacity factor is applied to the capacity variable to determine the technology's fixed operation.
AF(Z)(Y) (p,w,t) [O]	TCH_AF3	*TEZY_CAP* *THZ_CAP*	• Decimal fraction = {hours of production in the time-slice} / {no. of hours in the time-slice}. • [0,1]; no default, but set to AF if not provided.	• When seasonal variability of conversions plants (con) exists. • Omit if annual availability (AF) provided, or technology is externally load managed (xpr, xlm) or is a peaking device (PEAK(CON)_TID). • Once one AF(Z)(Y) is specified the plant will operate only for those time-slices for which AF(Z)(Y) is	The annual seasonal/time-of-day availability of a conversion plant (con). That is the fraction that the capacity is available to operate in each time-slice. • The seasonal/time-of-day availability of a conversion plant (R_TCZYH, R_TEZY, R_THZ) is related to the total installed capacity (R_CAP) as a function of the time-sliced availability factor by the utilization constraint (MR_TEZY, MR_THZ). • The time-sliced availability factors take precedence over the annual AF. When a seasonal AF(Z)(Y) is specified for a power plant, then its availability in each time-slice is conditional upon whether or not that time-slice is explicitly provided.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				provided. • Heating (hpl), base load (bas) and storage (stg) plants should only have a day ('D') slice provided = seasonal availability / no. of hours in the season (z). • Note that since AF(Z)(Y) are the fractions for each time-slice they may very well sum to more than 1. • Peaking devices (PEAK(CON)_TID) apply a peak duration factor (PD(Z)D) to the time-slice (QHR) instead of an availability factor.	• If a technology is externally load managed (xlm) and a time-sliced capacity factor (CF(Z)(Y)) is not provided then the availability factor is assigned as the capacity factor. For such technologies no activity variable is created, rather the capacity factor is applied to the capacity variable to determine the technology's fixed operation. • Note that if a conversion plant is externally load managed (xlm) and a capacity factor (CF/CF(Z)(Y)) is not provided then the availability factor is assigned as the capacity factor.
AF_TID (p) [O]	TCH_AF2	*TEZY_CAP* *THZ_CAP* *TUTL_CON*	• Decimal fraction. • [0,1]; default = 1	• Provided when not all the unavailability of a conversion technology (con) can be scheduled by the model, that is the portion of the unavailability that	The fraction of the unavailability that is forced outage. So, AF/AF_TID imply that o Unavailability = 1 – AF o Forced Outage = AF_TID * (1-AF) o Remaining (scheduled) outage = (1-AF_TID) * (1-AF)

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				corresponds to forced outage. • AF_TID can only be specified with AF (not AF(Z)(Y)).	• Results in the introduction of a maintenance equation (MR_UTLCON) and variable (R_M) that ensures that the unavailability of the technology in any time-slice that the model can schedule is reduced by the forced outage.
ARAF (hde,t) [O]	*TCH_ARAF*		• Fraction. • [0-1]; no default.	• Provided if a hydro plant has an annual limit on the reservoir.	To limit the annual availability of a hydroelectric (hde) reservoir. • Results in an annual utilization equation (MR_ARM) establishing the limit of total hydroelectric generation based upon total capacity (R_CAP) * the annual reservoir factor (ARAF).
BASELOAD (e,t) [O]	T_BASELOAD	*BAS_CAP* *BAS_SEP* *BAS_TCZY* *BAS_TEZY*	• Decimal fraction. • [0,1]; no default.	• To be provided if the base load set (bas) is provided. • Omit otherwise.	The electric base load requirement for each electricity energy carrier. The value indicates the maximum amount (fraction) of the highest night-time demand that can be met by base load (bas) or non-load management (nlm) power plants. • For each electricity energy carrier for which the parameter is provided a seasonal base load constraint (MR_BAS) is generated.
BI_TRDCST$_{25s/28s}$ (r,x,e,c) [C]			• Monetary units (Million US$2000). • [$.001-1000]; no default.	• Omit if not desired.	The additional "transaction" cost for bilateral trading a particular commodity. • Cost multiplier in the regional objective function (MR_OBJ) for a traded commodity (R_TSEP).
BI_ TRDCSTELC	*BI_TRDCSTE*$_{25t,28t}$		• Monetary units (Million US$2000).	• Omit if not desired.	The additional "transaction" cost for bilateral trading of electricity for each time-

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
(r,x,e,c,w) [C]			• [$.001-1000]; no default.		slice. • Cost multiplier in the regional objective function (MR_OBJ) for traded electricity (R_TSEPE).
BOUND(BD) (p,b,t) [O]	*TCH_BND*$_{35f}$		• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Omit if capacity of a technology is not to be explicitly limited.	The limit on total installed capacity (R_CAP = residuals (RESID) + new investments still available (R_INV)) in a period. • Sets a hard bound on the capacity variable (R_CAP) according to the bound type (bd).
BOUND(BD)O (p,b,t) [O]	*TCH_BNDO*$_{35E,13Y}$		• Units of activity (e.g., GWh, PJ). • [open]; no default.	• Omit if activity of a technology is not to be explicitly limited.	The limit on total annual activity in a period. • Sets a hard bound on the activity of a process (R_ACT) according to the bound type (bd). • Sets an annual limit on the activity of a conversion plant by means of a utilization equation (MR_BNDCON) that sums the time-sliced activity variables (ela=R_TEZY+R_TCZYH, hpl=R_THZ).
BOUND(BD)Or (s,b,t) [O]	*SEP_BND*$_{35S}$		• Units of activity (e.g., PJ). • [open]; no default.	• Omit if activity of a resource supply is not to be explicitly limited.	The limit on total activity of a resource supply option in a period. • Sets a hard bound on the activity of a resource option (R_TSEP) according to the bound type (bd).
CAPUNIT (p) [O]	TCH_CAPU	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *BAS_CAP*	• Scalar, units of annual production activity per unit of capacity. • [open]; default =	• Can be omitted if the default of 1 is wanted. • Affects the units of: o technologie	Defines the units of annual production activity. When set to unity the units of capacity and production are identical. Traditionally used to allow conversion technology capacity to be in terms of

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
		DEM_CAP *EPK_CAP* *HPKW_CAP* *TENV_ACT* *TENV_CAP* *TEZY_CAP* *THZ_CAP* *TUTL_CON* *UPRC_CAP*	1.	s FIXOM, INVCOST, BOUND(BD), IBOND(BD), RESID, INP(ENC)_TID o energy carriers DTRANINV, ETRANINV, EDISTINV • For MARKAL, the gigawatts-to-petajoule CAPUNIT = 31.536.	kilowatts/gigawatts while annual production is defined in terms of petajoules or BBTU's. • The CAPUNIT, in tandem with the capacity factor (CF/CF(Z)(Y)), is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • CAPUNIT is applied directly to the activity bound (BOUND(BD)O) when bounding output for externally load managed processes or conversion plants (xpr/xlm).
CEH(Z)(Y) (cpd,w,t) [O]	*T_CPDCEH*	*ANC_TCZY* *BAL_TCZY* *BALE_CZY* *BAS_TCZY* *BND_TCZY* *TENV_CZY* *TEZY_CZY* *HPKW_CPD* *INV_INV*	• Units of electricity lost per unit of heat gained. • [0,1]; no default.	• Required for pass-out turbines.	The pass-out turbine operates by diverting heat from the turbine to the low-temperature heat grid. As heat is produced, electricity production decreases at this rate. The electricity reduction is limited by the ELM parameter. • The heat production variable (R_TCZYH) is generated for each time-slice for which CEH(Z)(Y) is provided. • Total electric production in a time-slice is thus a function of two parameters and variables: $[R\_TEZY_{r,t,p,w} + (CEH_{r,p,w,t} * (1/ELM_{r,p} - 1) * R\_TCZYH_{r,t,p,w})]$

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
CF (p,t) [O]	TCH_CF2	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *DEM_CAP* *EPK_CAP* *HPKW_CAP* *TENV_CAP*	• Decimal fraction. • [0,1]; default = 1.	• To be provided for externally load managed processes (xpr) or conversion plants (xlm) when a seasonal capacity factor is not provided (CF(Z)(Y)), or demand devices (dmd). • The time-sliced capacity factors (CF(Z)(Y)) take precedence over the annual CF if both are specified. • Note that since CF(Z)(Y) are the fractions for each time-slice they may very well sum to more than 1. • Note that if a conversion plant is externally load managed and a capacity factor is not provided then the availability factor	The capacity factor is the fixed fraction of utilization of the installed capacity, as opposed to an availability factor (AF) that allows for flexible operation up to a limit. • The capacity factor, in tandem with the CAPUNIT, is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • The capacity factor is applied directly to the activity bound (BOUND(BD)O) when bounding output for externally load managed processes or conversion plants (xpr/xlm).

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				(AF) is assigned as the capacity factor.	
CF(Z)(Y) (p,w,t) [O]	TCH_CF1	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *BAS_CAP* *EPK_CAP* *HPKW_CAP* *TENV_CAP*	• Decimal fraction. • [0,1]; no default.	• To be provided for externally load managed conversion plants (xlm) when a seasonal capacity factor is not provided (CF(Z)(Y)). • The time-sliced capacity factors take precedence over the annual CF if both are specified. • Once one CF(Z)(Y) is specified the plant will operate only for those time-slices for which CF(Z)(Y) are provided. • As heating plants are operated on a seasonal basis in the model, rather than day/night as for electricity, CF(Z)(Y) should only be provided for y = 'D' as {hours of seasonal availability} / {no. of	The time-sliced capacity factor is the fixed fraction of utilization of the installed capacity in a season/time-of-day, as opposed to an availability factor (AF(Z)(Y)) that allows for flexible operation up to a limit. Note that when CF(Z)(Y) is specified for a power plant, then its availability in each time-slice is conditional upon whether or not that time-slice is explicitly provided for the technology. • The capacity factor, in tandem with the CAPUNIT, is applied to the capacity variable (R_CAP) whenever the level of activity is to be determined directly from the total installed capacity. • The capacity factor is applied directly to the activity bound (BOUND(BD)O) when bounding output for externally load managed processes or conversion plants (xpr/xlm).

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				hours in the season}. • Note that if a conversion plant is externally load managed and a capacity factor is not provided, then if seasonal availability factors (AF(Z)(Y)) are provided they are assigned to the capacity factor.	
COST (s,t) [C]	SEP_COST1	*ANC_TSEP*	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• Provided whenever a cost is incurred to supply a resource. • For 'EXP'orts the user should provide a positive COST value. The matrix generator uses the negative of this value.	Annual cost of supplying a resource. • The resource supply variable (R_TSEP) is entered into the objective function (EQ_PRICE if single-region, MR_OBJ if multi-region). • For electricity (elc) imports ('IMP') transmission and distribution O&M charges are added to the COST.
COST_TID (s) [C]	SEP_STKCOS	*ANC_ZSTK*	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• Provided whenever there is a value associated with a stockpiled resource.	Salvage value of a stockpiled energy carrier at the end of the modeling horizon. • The stockpile terminal level variable (R_ZSTK) is entered into the objective function (EQ_PRICE if single-region, MR_OBJ if multi-region).
DECAY (p,t)	TCH_TDE		• Decimal fraction. • [-2,+3]; no default	• Provided if the rate of decrease in total capacity is to be	The maximum annual decay rate of total installed capacity of a technology in a period.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[O]				limited.	• A technology decay constraint (MR_DETCH) is created for each period for which the parameter is provided. • The constraint limits the rate at which a technology can decrease such that the total installed capacity (R_CAP) in the current period does not drop below (1+DECAY)**-NYRSPER times the level in the previous period. For example, a 10% annual decay rate in a 5-year period model would permit a decrease to (1+.1)**-5 = 0.62092 times the current level of capacity.
DECAYr (s,t) [O]	SEP_TDE		• Decimal fraction. • [-2,+3]; no default	• Provided if the rate of decrease in total resource production is to be limited.	The maximum annual decay rate of total production from a resource option in a period. • A resource decay constraint (MR_DESEP) is created for each period for which the parameter is provided. • The constraint limits the rate at which a resource can decrease such that the total annual production (R_TSEP) in the current period does not drop below (1+DECAYr)**-NYRSPER times the level in the previous period. For example, a 10% annual decay rate in a 5-year period model would permit a decrease to (1+.1)**-5 = 0.62092 times the current level of production.
DELIV(ENT/MA	SEP_DELIV	*ANC_ACT*	• Base year	• If an auxiliary	Annual cost for the delivery of a commodity

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
T) (s,e,t) DELIV(ENT/MAT) (p,e,t) [C]	TCH_DELIV	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	charge is to be levied for the consumption of a commodity by a supply option or technology. • Note that a corresponding commodity input parameter (one of the INP(ENT)s) must be provided.	to a resource activity or a technology. • The cost is multiplied by the activity level (R_TSEP, R_ACT, R_TCZYH, R_TEZY, R_THZ) and enters the objective function as an additional variable cost.
DEMAND (d,t) [D]	DM_DEM		• Units of end-use demand. • [open]; no default.	• Required for each end-use demand.	Annual demand for an energy service. • Demands are exogenous for standard MARKAL, and providing DEMAND controls generation of the demand equation (MR_DEM) or not.
DHDE(Z) (e,z,t) [O]	T_DHDE	*BALH_CAP* *BALH_THZ* *HPWK_CAP* *HPKW_HLK*	• Decimal fraction. [0,1]; no default.	• To be provided if some demand device (dmd) consumes low-temperature heat (lth).	Distribution efficiency for low-temperature heat in each season (z). • Applied to the consumption level for demand devices (dmd) and grids (hlk) using low-temperature heat in the balance (MR_BALDH) and peaking equations (MR_HPKW) to raise the production requirements appropriately to account for the loss.
DISCOUNT [C]		*COST_INV* CRF CRF_RAT PRI_DF PRI_DISC	• Decimal fraction. • [0,0.99]; no default.	• Always required.	Overall long-term annual discount rate for the whole economy. • Used in the calculation of the capital recovery factor (CRF). • Also used to report the discounted costs

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					(e.g., total system cost) to a base year, including taking into consideration any technology-based discount rate (DISCRATE) via a CRF ratio (CRF_RAT).
DISCRATE (p) [C]	TCH_DISC	*COST_INV* CRF CRF_RAT	• Decimal fraction. • [0,0.99]; no default.	• Only provided when a technology-based discount or "hurdle" rate is to be applied.	The discount rate associated with an individual technology, or all technologies in a sector. • Used in the calculation of the capital recovery factor (CRF) to determine the annual payments for investments. • Also used to report the costs (e.g., total system cost) discounted to a base year, including taking into consideration the relationship to the standard discount rate (DISCOUNT) via a CRF ratio (CRF_RAT).
DTRANINV (lth,t) [C]	T_DTRANINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of heat plant capacity (e.g., 2000 M$/PJa). • [open]; no default.	• Provided if additional charges are to be levied for the heat transmission system for centralized (cen) plants.	Investment in low-temperature heat transmission equipment for centralized (cen) heat producing plants (hpl/cpd). It is assumed that grid expansion and/or upgrading is required whenever new capacity is added. • The total cost of investing in the expansion of the heating grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the objective function (EQ_PRICE if single region, MR_OBJ if

Input Parameter (Indexes[49]) [Purpose][50]	**Alias/Internal Name[51]**	**Related Parameters[52]**	**Units/ Range & Defaults**	**Instance[53] (Required/Omit/Special Conditions)**	**Description[54]**
					multi-region).
DTRANOM (lth,t) [C]	T_DTRANOM	*ANC_CAP* *ANC_TCZY* *ANC_THZ*	• Base year monetary units per unit of heat (e.g., 2000M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the heat transmission system for centralized (cen) plants.	Operating and maintenance costs associated with low-temperature heat transmission equipment for centralized plants • The total O&M cost associated with a heating grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, R_THZ, or R_CAP if xlm).
EDISTINV (elc,t) [C]	T_EDISTINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of electricity capacity (e.g., 2000 M$/GW). • [open]; no default.	• Provided if additional charges are to be levied for the electric distribution system.	Investment in electricity distribution equipment. It is assumed that grid expansion and/or upgrading is required whenever new capacity is added. • The total cost of investing in the expansion of the electricity distribution grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
EDISTOM (elc,t) [C]	T_EDISTOM	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	• Base year monetary units per unit of electricity (e.g., 2000 M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the electricity distribution system.	Operating and maintenance costs associated with electricity distribution equipment. • The total O&M cost associated with an electricity distribution grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, R_THZ, or R_CAP if xlm).

Input Parameter (Indexes[49]) [Purpose][50]	**Alias/Internal Name[51]**	**Related Parameters[52]**	**Units/ Range & Defaults**	**Instance[53] (Required/Omit/Special Conditions)**	**Description[54]**
EFF (dmd,t) [O]	DMD_EFF	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *EPK_CAP* *HPKW_CAP*	• Decimal fraction equal to the useful energy service met per unit of commodity consumed. • [0,1] usually; default = 1.	• Provided when a value other than the default of 1 is necessary. • For most demands in energy units the efficiency is usually a decimal fraction. • For transportation the devices (cars, trucks, training) may be providing billion passenger kms or tons of freight and thus the efficiency must also convert from energy units to demand service units. • For devices such as heat pumps, refrigerators, lights, the efficiency may be greater than 1. • Omit if DMD Vintaging is used instead.	The technical efficiency of a demand device. • Applied to the consumption level for demand devices when entering the balance (MR_BAL, MR_BALE, MR_BALDH) and peaking equations (MR_EPK, MR_HPKW) to raise the consumption requirements appropriately, and to the variable costs (DELIV, VAROM) according to activity (R_CAP taking into consideration CAPUNIT and CF) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
EFF_I EFF_R (dmd,t)	DMD_IEFF DMD_REFF	*ANC_CAPR* *ANC_VDI* *BAL_VDI* *BALE_VDI*	• Decimal fraction equal to the useful energy service met per unit of	• When demand device vintaging is desired. • See EFF for the rest	The technical efficiency of a demand device is normally associated with the entire installed base (R_CAP). Providing these parameters instructs the model to associate

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[O]		*BALH_VDI* *DMD_EFF* *EPK_VDI* *HPKW_VDI* *PRI_CAPR* *PRI_VDI*	commodity consumed. • [0,1] usually; default = 1.	of the relevant conditions.	the efficiency of a device with when it was actually built (R_INV for new investments, R_RESIDV for past investments still existing in a period). • Applied to the consumption level for demand devices when entering the balance (MR_BAL, MR_BALE, MR_BALDH) and peaking equations (MR_EPK, MR_HPKW) to raise the consumption requirements appropriately, and to the variable costs (DELIV, VAROM) according to activity (R_INV/R_RESIDV taking into consideration CAPUNIT and CF) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). • NOTE that vintaged demand devices (dmd) do *not* have the ability to "strand" or not use previously installed capacity as is the case with conventional demand devices in MARKAL.
ELCFEQ (c) [D]		T_ELCFEQ	• Decimal fraction. • [1 - any]; no default.	• The fossil equivalent value is set to that the average of the fossil fuel power plants in the region, and used for reporting the primary energy equivalent of renewable energy use.	The fossil equivalent corresponds to the average efficiency (inverse thereof actually) of the fossil fuel power plants in a region. • Used for reporting the primary energy equivalent of renewable energy use.
ELF	DM_ELF	*EPK_CAP*	• Decimal fraction.	• Provided for any	The electricity load factor is a fraction that

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
(d,t) [D]			• [0,1]; no default.	demand (dm) serviced by a device that consumes electricity, when a value other than the default of 1 is necessary.	indicates how much of the electric demand that occurs during the peak time-slice actually coincides with the peak moment. • Controls the amount of electricity consumption by the associated demand devices (dmd) that enters the peak equation (MR_EPK).
ELM (cpd,w,t) [O]	T_CPDELM	*ANC_TCZY* *BAL_TCZY* *BALE_CZY* *BALH_CZY* *BAS_TCZY* *BND_TCZY* *TENV_CZY* *TEZY_CZY* *HPKW_CPD* *INV_INV*	• Decimal fraction. • [0,1]; default = 1.	• The indicator that a coupled production plant is a pass-out turbine. • Usually set to about 0.1.	The electric-loss maximum defines the fractional loss beyond which the total electric production may not be reduced. • Results in the creation of a heat production variable (R_TCZYH). • Total electric production in a time-slice is thus a function of two parameters and variables: $[R\_TEZY_{r,t,p,w} + (CEH_{r,p,w,t} * (1/ELM_{r,p} - 1) * R\_TCZYH_{r,t,p,w})]$
ENV_ACT (p,v,t) [E]	ENV_TACT	*TENV_ACT* *TENV_CAP* *TENV_CZY* *TENV_EZY* *TENV_HZ* env_a env_c	• Quantity of annual emissions (tons or thousand tons) per unit of technology annual activity. • [open]; no default.	• Provided if a technology emits (or removes) an environmental indicator according to its activity. • ENVSCAL may be used to scale all environmental indicators if desired.	Amount of emissions discharged, or reduced (if negative), per unit of output from a process or conversion technology. Note that this coefficient is output based, so if the source data ties the emissions to a commodity it is the modeler's responsibility to convert the factor by reflecting the units of energy consumed per unit output (e.g., the inverse of the commodity efficiency). • Providing ENV_ACT results in the

Input Parameter (Indexes[49]) [Purpose][50]	**Alias/Internal Name[51]**	**Related Parameters[52]**	**Units/ Range & Defaults**	**Instance[53] (Required/Omit/Special Conditions)**	**Description[54]**
					associated activity variable (R_ACT, R_TCZYH, R_TEZY) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied. • If the technology is externally load managed (xpr/xlm) then ENV_ACT is assigned to ENV_CAP by applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)), and time-slice fraction (QHR(Z)(Y)) if a power plant.
ENV_BOUND(BD) (v,b,t) [E]	*EM_BOUND* *EM_FIX*		• Tons or thousand tons. • [open]; no default.	• When there is to be an annual limit imposed.	Limit on the total production of an emission indicator in a period. • Sets a bound on the total annual emission variable (R_TENV).
ENV_CAP (p,v,t) [E]		*TENV_CAP*	• Quantity of annual emissions (tons or thousand tons) per unit of technology capacity. • [open]; no default.	• Provided if a technology emits (or removes) an environmental indicator according to its capacity. • ENVSCAL may be used to scale all environmental indicators if desired.	Amount of emissions discharged, or reduced (if negative), per unit of installed capacity. • Providing ENV_CAP results in the associated capacity variable (R_CAP) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied. • If the technology is externally load managed (xpr/xlm) then ENV_ACT is assigned to ENV_CAP by applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)) , and time-slice fraction (QHR(Z)(Y)) if power plant.
ENV_COST$_{25/28H}$			• Monetary unit	• When there is to be	An emissions "tax" applied to an indicator,

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(v,t) [E]			per unit of emission indicator. • [open]; no default.	a charge applied to an emissions indicator.	and "backed out" of the total system cost (in MARKAL reports). • When provided the annual emissions variable (R_TENV) enters the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
ENV_CUMMAX (v) [E]	*ENV_CUM*		• Tons or thousand tons. • [open]; no default.	• When there is to be a cumulative limit imposed.	Total limit on an emission indicator over the entire modeling horizon. • Sets the right-hand-side of the total emissions constraint (MR_ENV). • Note that the annual emissions variable (R_ENV) is multiplied by the period length (NYRSPER) when entered into MR_ENV.
ENV_ENT/ ENV_ENTr (v,e,t/s) [E]		ENV_ENTXI ENV_ENTXO	• Scalar. • [open]; no default.	• Emission rate per unit of commodity e consumed/ produced.	Permits emission tracking by commodity, rather than explicity for each technology or resource option. MARKAL normally applies the emission rate coefficients to the output of a technology, but often emission rates are commodity dependent. ENV_ENT applies the provided emission rate to the in/out flows of the identified commodities, thus taking into account the efficiency of the technology, which otherwise needed to be embedded in the output oriented emission factor. Consumption flows (other than exports) add emissions to the balance, while production flows result in a reduction. The ENV_ENTXI/O parameters inhibit including in the algorithm for some

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					technologies. • For each emission indicator, each technology and resource producing/consuming the identified commodities, the coefficients used for the balance equations (MR_BAL / MR_BALE / MR_BALDH) are multiplied by the commodity-based emission rate and accumulated in the emission accounting equation (MR_TENV).
ENV_ENTXI/O *(p,v)* [E]		ENV_ENT	• Indicator • [open]; no default.	• When the Input/Output flows associated with this technology are NOT to be included in the ENV_ENT algorithm.	Permits the exclusion of certain input/output commodity flows from being including in the ENV_ENT algorithm for a particular emission indicator. • All the intersections discussed under ENV_ENT are ignored.
ENV_GWP$_{29H}$ *(*gwp,v,t) [E]			• Scalar. • [open]; no default.	• GWPs are included in the environmental indicator set (env), and identified by inclusion (in the first position) in this Table. • Provided for each GHG emissions indicator (env) that is to be accumulated according to its global warming potential. • The ENV_GWP	The global warming potential that relates the relative contribution that various greenhouse gases make to the warming of the atmosphere. • For each emission indicator in the first position an emission accounting equation (MR_TENV) is created where the entries are the total emissions (R_EM) associated with the various contributing greenhouse gases.

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				Table can also be used to accumulate several emission indicators to create a sector or region total if desired.	
ENV_INV (p,v,t) [E]	*TENV_INV*		• Quantity of annual emissions (tons or thousand tons) per unit of technology capacity. • [open]; no default.	• Provided if a technology emits (or removes) an environmental indicator according to its investment. • ENVSCAL may be used to scale all environmental indicators if desired.	Amount of emissions discharged, or reduced (if negative), per unit of investment in new capacity. • Providing ENV_INV results in the associated investment variable (R_INV) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied.
ENV_SCAL$_{29H}$ [E]			• Scalar. • [.000001 - 1]; default = 1.	• Provided if marginals are not reported for constrained emissions.	A global emissions scaling factor that is applied unilaterally to all emission indicators (env). Sometimes used to force reporting of marginal values on emission constraints. • Applied to the period emission total variable (R_EM).
ENV_SEP (s,v,t) [E]	*ENV_TSEP* *TENV_SEP*		• Quantity of annual emissions (tons or thousand tons) per unit of resource production. • [open]; no	• Provided if a resource option emits (or removes) an environmental indicator. • ENVSCAL may be used to scale all	Amount of emissions discharged, or reduced (if negative), per unit of resource activity. • Providing ENV_SEP results in the associated resource supply variable (R_TSEP) entering the equation that tracks the emissions (MR_TENV) with the coefficient applied.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
			default.	environmental indicators if desired.	
ERESERV (elc,t) [O]	T_ERESERV	*EPK_ACT* *EPK_CAP* *EPK_HPL* *EPK_SEP* *EPK_TCZY* *EPK_TEZY*	• Decimal fraction. • [0,1]; no default.	• ERESERV should be provided for each electricity energy carrier (elc) for which a peaking requirement is to be imposed.	The electricity peaking factor looks to ensure that enough capacity is in place to meet the highest average electricity demand during the day of any season. It must encompass both an estimate of the level above the highest average demand that corresponds to the actual peak (moment of highest electric demand) plus a reserve margin of excess capacity. This is to ensure that if some plants are unavailable the demand for electricity can still be met. Note that because ERESERV must encompass both the estimate from average to peak and the actual additional peak reserve it is usually substantially above that of a traditional utility reserve margin. • Providing ERESERV results in the creation of an electricity peaking constraint (MR_EPK) that enforces the above requirement. The equation is generated for each season according to the daytime demand for electricity.
ETRANINV (elc,t) [C]	T_ETRANINV	*COST_INV* INV_INV INVCOST	• Base year monetary units per unit of electric capacity (e.g., 2000 M$/GW). • [open]; no	• Provided if additional charges are to be levied for the electric transmission system for centralized power plants (cen).	Investment in electricity transmission equipment. It is assumed that grid expansion and/or upgrading is required whenever new capacity is added. • The total cost of investing in the expansion of the electricity transmission

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
			default.		grid per unit of new capacity added. The cost is added to the rest of the investment cost (INV_INV) and applied to the investment variable (R_INV) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
ETRANOM (elc,t) [C]	T_ETRANOM	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_TSEP*	• Base year monetary units per unit of electricity (e.g., 2000 M$/PJ). • [open]; no default.	• Provided if additional charges are to be levied for the electricity transmission system.	Operating and maintenance costs associated with electricity transmission equipment. • The total O&M cost associated with an electricity transmission grid per unit of activity. The cost is added to the rest of the variable charges and applied to the activity variable (R_TCZYH, R_TEZY, R_THZ, or R_CAP if xlm).
FIXOM (p,t) [C]	TCH_FIXOM	*ANC_CAP*	• Base year monetary units per unit of installed capacity (e.g., 2000 M$/GW or PJa). • [open]; no default.	• Provided if fixed operating and maintenance charges are to be levied for the total installed capacity.	Annualized fixed operating and maintenance cost for *all* the capacity in place in a period, charged regardless of whether or not the technology operates. • The cost is applied to the capacity variable (R_CAP) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
FR(Z)(Y) (d,w) [D]	DM_FR	*BALH_CAP* *BALE_CAP* *EPK_CAP* *HPKW_CAP*	• Decimal fraction. • [0,1]; default QHR(Z)(Y).	• Potentially provided for demand sectors (dm) for which some device consumes electricity or heat. • When not provided the demand timing	The fraction of the annual demand for an energy service in each time-slice (w) when some device servicing the demand consumes electricity or heat. FR(Z)(Y) thus describes the timing or shape of the load curve. • The capacity of the electricity and heat demand device is chosen such that it is

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				matches that of the supply side, so all time-slices are set to the standard season/time-of-day splits (QHR(Z)(Y)) indicating uniform distribution (UNIFDIST). • If some FR(Z)(Y) are provided, and others are not, in those time-slices where FR(Z)(Y) is omitted it is assumed that there is no associated electric/heat demand.	sufficient to meet the demand according to FR(Z)(Y)/QHR(Z)(Y). • The coefficient contributes to determining the amount of electricity and heat that must be provided (via the balance equations (MR_BALE, MR_BALDH)) and the peaking needs (MR_EPK, MR_HPK) in each time-slice according to the device activity (R_CAP adjusted for capacity factors, efficiency and market allocation).
FR(Z)(Y)(ELC) (s,w) [O]	SEP_FR	*BALE_SEP* *BAS_SEP* *EPK_SEP*	• Decimal fraction. • [0,1]; default QHR(Z)(Y).	• Only provided for electricity imports/exports. • When not provided all time-slices are set to the standard season/time-of-day splits (QHR(Z)(Y)). • If some FR(Z)(Y)(ELC) are provided, and others are not, in those time-	The fraction of annual electricity imports or exports that occur in each time-slice. • The amount of electricity credited to (for imports) or against (for exports) the electricity balance (MR_BALE) and peaking (MR_EPK) in a time-slice is determined by the FR(Z)(Y)(ELC) parameter.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				slices where FR(Z)(Y)(ELC) is omitted it is assumed that there is no associated electricity import/export.	
GEMLIM (v) *GEMLIMT* (v,t) [E]			• Tons or thousand tons. • [any]; no default.	• Provided to limit emissions collectively across multiple regions.	A cap on emissions for a collection of, or all, regions. • An emissions tracking limit equation (MR_GEMLIM/T) is constructed summing the individual regional emission variables (R_EM) and imposing an overall or period-based limit on the emissions.
GROWTH (p,t) [O]	*TCH_TGR*		• Decimal fraction. A 15% per annum growth rate is specified as 1.15. • [-2,+3]; no default.	• Provided if the rate of increase in total capacity of a technology is to be limited. • The expansion limit constraint may need to be seeded by a maximum initial build (see GROWTH_TID).	The maximum annual growth rate of total installed capacity in a period. • A technology growth constraint (MR_GRTCH) is created for each period for which the parameter is provided. • The constraint limits the rate at which capacity can expand such that the total capacity (R_CAP) in the current period does not exceed (1+GROWTH)**NYRSPER times the level in the previous period. For example, a 10% annual growth rate in a 5-year period model would permit an increase of (1+.1)**5 = 1.61051 times the current level of installed capacity.
GROWTH_TID (p)	*TCH_GRTI*		• .Units of capacity (e.g., GW, PJa).	• A growth constraint cannot be applied if	The growth constraint seed indicates the initial capacity level permitted over and

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[O]			• [open]; no default.	the technology has not been deployed (that is the growth from 0 is 0 no matter what the rate).	above the growth constraint for the purposes of getting the growth constraint going. • GROWTH_TID is the right-hand-side of the technology growth constraint (MR_GRTCH). • Thus for new technologies not yet introduced to the energy system the constraint takes the form of initial investment < GROWTH_TID until that first investment is made.
GROWTH_TIDr (s) [O]	*SEP_GRTI*		• .Units of a commodity (e.g., PJ). • [open]; no default.	• A growth constraint cannot be applied if a resource has not been tapped (that is the growth from 0 is 0 no matter what the rate).	The growth constraint seed indicates the initial production level permitted over and above the growth constraint for the purposes of getting the growth constraint going. • GROWTH_TIDr is the right-hand-side of the resource growth constraint (MR_GRSEP). • Thus for new resources not yet introduced to the energy system the constraint takes the form of initial resource activity < GROWTH_TIDr until that first use of the resource option takes place.
GROWTHr (s,t) [O]	*SEP_TGR*		• Decimal fraction. A 15% per annum growth rate is specified as 1.15. • [-2,+3]; no default	• Provided if the rate of increase in total resource production is to be limited. • The expansion limit constraint may need to be seeded by a	The maximum annual growth rate of total production from a resource option in a period. • A resource growth constraint (MR_GRSEP) is created for each period for which the parameter is provided. • The constraint limits the rate at which a

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				maximum initial build (see GROWTH_TIDr).	resource can expand such that the total annual production (R_TSEP) in the current period does not exceed (1+GROWTHr)**NYRSPER times the level in the previous period. For example, a 10% annual growth rate in a 5-year period model would permit an increase of (1+.1)**5 = 1.61051 times the current level of production.
GobjZscal [M]			• Scalar, • [.000001 – 1], default 1.	• Provided to scale the objective function value. •	The internal scaling of the model to improve stability and speed the solution of large problems. • Appears only in the individual objective functions (EQ_PRICE if single region, MR_OBJ if multi-region).
HRESERV (lth) [O]	T_HRESERV	*HPKW_ACT* *HPKW_CAP* *HPKW_CPD*	• Decimal fraction. • [0,1]; no default.	• HRESERV should be provided for each low-temperature heat energy carrier (lth) for which a peaking requirement is to be imposed.	The low-temperature heat peaking factor looks to ensure that enough capacity is in place to meet the highest average heating demand in the winter (heatcool) season. It must encompass both an estimate of the level above the highest average demand that corresponds to the actual peak (moment of highest heat demand) plus a reserve margin of excess capacity. This is to ensure that if some plants are unavailable the demand for heat can still be met. Note that because HRESERV must encompass both the estimate from average to peak and the actual additional peak reserve it is usual substantially above that of a tradition utility

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					reserve margin. • Providing HRESERV results in the creation of a low-temperature heat peaking constraint (MR_HPKW) that enforces the above requirement.
IBOND(BD) (p,b,t) [O]	*TCH_IBND*		• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Omit if investment in a technology is not to be explicitly limited.	The limit on new investments (R_INV) in a period. • Sets a hard bound on the investment variable (R_INV) according to the bound type (bd).
INP(ENC)_TID (con,enc) INP(MAT)_TIDp (prc,mat) [T]	CON_INP1 PRC_INP1 prc_enc snk_enc	*BAL_INV* T08	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is consumed as part of putting a new investment in place.	Energy/material required at the time an investment in new capacity is made. Typically used to represent the initial load of fuel into the core of a nuclear reactor. • This amount enters the balance equation (MR_BAL) in the period in which the investment takes place, except for nuclear energy carriers (enu) for which it is charged to the previous period. • If more than one commodity is consumed at investment time then each must have a corresponding INP(ENT/MAT)_TID parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)c (con,ent,t) INP(MAT)c (con,mat,t)	CON_INP2 con_enc con_elc con_lth	*ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_THZ* *BAL_CAP*	• Units of commodity consumed per unit of production. • [open]; no	• If a commodity is consumed by a power plant. • At least one input must be specified.	Energy/material consumed to generate one unit of electricity and/or low-temperature heat leaving the facility, that is, after any on-site consumption. Expressed as the inverse of the efficiency of the power plant.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[T]		*BALE_CAP* *BALE_EZY* *BALE_HPL* *BALH_CAP* *BALH_THZ* *EPK_CAP* *EPK_HPL* *EPK_TEZY* *HPKW_CAP* *HPKW_HLK* TCH_DELIV	default.		• This amount enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK, MR_HPKW) equations, and contributes to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the facility (R_TCZYH, R_TEZY, R_THZ). • If more than one commodity is consumed by a power plant then each must have a corresponding INP(ENT/MAT)c parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)p (prc,ent,t) {But not ent=LTH} INP(MAT)p (prc,mat,t) [T]	PRC_INP2 prc_enc prc_elc	*ANC_ACT* *ANC_CAP* *BAL_ACT* *BAL_CAP* *BALE_ACT* *BALE_CAP* *EPK_TEZY* TCH_DELIV	• Units of commodity consumed per unit of production. • [open]; no default.	• If a commodity is consumed by a process. • At least one input must be specified.	Energy/material consumed to generate one unit output from a process. • This amount enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contributes to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the process (R_ACT). • If more than one commodity is consumed by a process then each must have a corresponding INP(ENT/MAT)p parameter. • Contributes to defining the flow of

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					commodities through the RES.
INP(ENT)r (s,ent,t) {But not ent=LTH} INP(MAT)r (s,mat,t) [T]	SEP_INP sep_ent	*ANC_TSEP* *BAL_SENT* *EPK_SEP* TCH_DELIV	• Units of commodity consumed per unit of production. • [open]; no default.	• If an auxiliary commodity is consumed as part of a resource supply option.	Energy/material consumed to produce one unit of output from a resource activity. • This amount enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contributes to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a delivery cost (DELIV) for the commodity, according to the amount of resource produced (R_TSEP). • If more than one commodity is consumed by a resource activity then each must have a corresponding INP(ENT/MAT)r parameter. • Contributes to defining the flow of commodities through the RES.
INP(ENT)x (s,ent,t) {But not ent=LTH} INP(MAT)x (s,mat,t) [T]	srcencp sep	*ANC_TSEP* *BAL_SENT* *EPK_TEZY*	• Indicator. • [1]; no default.	• If a commodity is exported. • Identifies the name of the commodity, and should always be specified with a value of 1. • Only one INP(ENT/MAT)x should be specified per export option.	Energy/material "consumed" as part of an exporting activity, that is the actual exported commodity. • This amount enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contributes to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a delivery cost (DELIV) for the commodity, according to the amount of resource produced (R_TSEP). • Contributes to defining the flow of commodities through the RES.

Input Parameter (Indexes[49]) [Purpose][50]	**Alias/Internal Name[51]**	**Related Parameters[52]**	**Units/ Range & Defaults**	**Instance[53] (Required/Omit/Special Conditions)**	**Description[54]**
INVCOST (p,t) [C]	TCH_INVCOS	*COST_INV* CRF FL/FRLIFE FRACLIFE INV_INV *PRI_INV*	• Base year monetary units per unit of capacity (e.g., 2000 M$/GW or PJa). • [open]; no default.	• Provided for all technologies for which an investment cost applies. • If no investment cost or lifetime (LIFE) is provided for processes (prc) then they are considered to be externally load managed (xpr).	The total cost of investments associated with the installation of a unit of new capacity in a given period. • The technology investment cost is spread (COST_INV) over the lifetime (LIFE) of the technology by applying a capital recover factor (CRF), and adjusting for any fraction of the last period an investment is available (FRACLIFE/FRLIFE), if necessary. CRF in turn depends upon the discount rate (DISCOUNT or DISCRATE) as well as the technical lifetime (and period length NYRSPER). • If the technology is a conversion technology (con), then additional costs for the transmission (if centralized, ETRANINV) and distribution (EDISTINV) infrastructure are added to the facility investment cost to determine the total investment cost (INV_INV). • The "spread" annualized cost (COST_INV) is applied to the investment variable (R_INV). Note that the investment cost is taken from the input data in the period in which the investment takes place, and from the first period of all the remaining residual capacity.
LAG(ENC) (prc,enc,t)	*PRC_LAG*	*BAL_ACT* *BAL_CAP*	• Decimal fraction. • [0,1]; no default.	• Should be provided if the process has an	The fraction of output energy carrier (enc) or material (mat) that becomes available in

Input Parameter (Indexes[49]) [Purpose][50]	**Alias/Internal Name[51]**	**Related Parameters[52]**	**Units/ Range & Defaults**	**Instance[53] (Required/Omit/Special Conditions)**	**Description[54]**
LAG(MAT) (prc,mat,t) [T]				output energy or material production, part of which is delayed until period t+1.	period t+1. • This option is mostly used for nuclear processes to allow for cooling and reprocessing of fuel.
LED(ENT) (prc,ent,t) LED(MAT) (prc,mat,t) [T]	*PRC_LED*	*ANC_ACT* *ANC_CAP* *BAL_ACT* *BAL_CAP* *BALE_ACT* *BALE_CAP* *EPK_ACT* *EPK_CAP*	• Decimal fraction. • [0,1]; no default.	• Should be provided if the process has an input energy or material requirement, part of which is needed in the previous period t-1.	The fraction of energy carrier (ent) or material (mat) that is input to the process and that is required in period t-1. • This one period lead between input fuel consumption and process activity is generally required for nuclear processes.
LIFE (p) [O]	*TCH_LIFE*	*BAL_INV* *COST_INV* *CPT_CAP* *CPT_INV* CRF CRF_RAT FL/FRLIFE FRACLIFE INV_INV *PRI_INV*	• Number of years. • [open]; no default.	• Should be provided for all technologies, except dummy (dum) processes. • If no investment cost (INVCOST) or lifetime is provided for processes (prc) then they are considered to be externally load managed (xpr).	The number of years that a technology is available from the year of initial installation. • The technology investment cost (INVCOST) is spread (COST_INV) over the lifetime of the technology by applying a capital recover factor (CRF), and adjusting for any fraction of the last period an investment is available (FRACLIFE/FRLIFE), if necessary. • The "spread" annualized cost (COST_INV) is applied to the investment variable (R_INV). • The availability of new investments (R_INV) is tracked by means of the capacity transfer equation (MR_CPT), where the investment remains available until the lifetime is reached, including a

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					fraction thereof if LIFE is not a multiple of the number of year per period (NYRSPER).
LIMIT (prc,t) [O]	PRC_LIM	*BAL_ACT* *BAL_CAP*	• Sum of all outputs per unit of production (e.g., PJ/PJ). • [0,5]; no default.	• Provided for processes that permit flexibility among the possible outputs.	Indicates that a process is to be modeled permitting flexible output of the various commodities produced by the process. When LIMIT is provided the commodity output fractions (OUT(ENC)p) are interpreted as *maximum* outputs (per unit of process activity), rather than as rigid fixed ratios. The value of LIMIT defines the overall efficiency of the process. • The LIMIT parameter results in a new process/commodity variable (R_LOUT) being created for each output. • The individual output flows are controlled by a limit equation (MR_LIM) for each commodity. • The overall efficiency of the process is ensured by a process balance equation (MR_PBL) that sums the individual output flows (R_LOUT) and ensures that they equal LIMIT * process activity (R_ACT).
MA(ENC)_TID (dmd,enc) MA(MAT)_TID (dmd,mat) [T]	DMD_MATI snk_enc	*BAL_INV* T08	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is consumed as part of putting a new investment in place.	Energy/material required at the time an investment in new capacity is made. • This amount enters the balance equation (MR_BAL) in the period in which the investment takes place. • If more than one commodity is consumed at investment time then each

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					must have a corresponding MA(ENT/MAT)_TID parameter. • Contributes to defining the flow of commodities through the RES.
MA(ENT) (dmd,ent,p) MA(MAT) (dmd,mat,p) [T]	DMD_MA dd_ma	*ANC_CAP* *BAL_CAP* *BALE_CAP* *BALH_CAP* *EPK_CAP* *HPKW_CAP* TCH_DELIV	• Units of commodity per unit of device output (e.g., PJ/PJ). • [open]; no default.	• If a commodity is consumed by a demand device.	The share of energy/material consumed to service one unit of demand from a demand device. • This amount enters the balance (MR_BAL, MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPKW) equations, and contributes to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a delivery cost (DELIV) for the commodity, according to the level of operation of the facility (R_TCZYH, R_TEZY, R_THZ). • If more than one commodity is consumed by a power plant then each must have a corresponding MA(ENT/MAT) parameter. • Contributes to defining the flow of commodities through the RES.
MED-BASEANNC [C]	MEDBANNC		• Base year monetary units (e.g., 2000 M$). • [open]; no default.	• Obtained from $SET MARKALED 'BPRICE' run as written to <reg>.EDD file.]	Annualized costs from the MED reference run used to calculate the change in annual producer/consumer surplus in the reports.
MED-BASEOBJ [C]	BASEOBJ		• Base year monetary units (e.g., 2000 M$).	• Obtained from $SET MARKALED 'BPRICE' run as	Total system cost for the MED reference run used to report change in total producer/consumer surplus.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
			• [open]; no default.	written to <reg>.EDD file.	
MED-DMBPRICE (d,t) [C]	*DMBPRICE*	*DM_ELAST* *DM_STEP* *DM_VAR*	• Base year monetary units (e.g., 2000 M$). • [open]; no default.	• Obtained from $SET MARKALED 'BPRICE' run as written to <reg>.EDD file.	Marginal cost of the individual flexible demands for the MED reference run. • Used in the calculation of the cost to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) as a result of up/down movement of the elastic demands (R_ELAST) according to MED-ELAST, MED-STEP, MED-VAR.
MED-ELAST(BD) (d,b,t) [D]	*DM_ELAST*	*DMBPRICE* *DM_STEP* *DM_VAR*	• Scalar exponent. • [open]; no default.	• An elasticity is required for each step/direction the demand is permitted to move. • A different value may be provided for each direction, thus curves may be asymmetric.	Elasticity of demand indicating how much the demand rises/falls in response to a unit change in the marginal cost of meeting the elastic demands. Contributes to determining (according to MED-ELAST, MED-STEP, MED-VAR) the amount the demand can move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (EQ_PRICE if single

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$$MED\_STEP(BD)_{r,d,b,t} \sum_{b,u=1} \left\{ \left\langle -\left\{ \begin{matrix} [1+(MED\_VAR_{r,d,b,t} / MED\_STEP(BD)_{r,d,b,t} *(u-.5))]** \\ (1/MED\_ELAST_{r,d,b,t}) \end{matrix} \right\} \$b='UP' + \left\{ \begin{matrix} [1-(MED\_VAR_{r,d,b,t} / MED\_STEP(BD)_{r,d,b,t} *(u-.5))]** \\ (1/MED\_ELAST_{r,d,b,t}) \end{matrix} \right\} \$b='LO' \right\rangle * R\_ELAST_{r,t,d,b} \right\}$$

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					region, MR_OBJ if multi-region). The core calculation is shown in the footnote[55], and is multiplied by MED-BPRICE when entering the objective function.
MED-STEP(BD) (d,b,t) [D]	*DM_STEP*	*DMBPRICE* *DM_ELAST* *DM_VAR*	• Count. • [open]; no default.	• The number of steps is required for each step/direction the demand is permitted to move. • A different value may be provided for each direction, thus curves may be asymmetric.	The number of steps to use for the approximation of the change in producer/consumer surplus when running MED. • Contributes to determining (according to MED-ELAST/MED-STEP/MED-VAR) the amount the demand can move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). [See MED-ELAST(BD) for calculation.]
MED-VAR(BD) (d,b,t) [D]	*DM_VAR*	*DM_BPRICE* *DM_ELAST* *DM_STEP*	• Scalar • [0,1]; no default.	• A variation is expected for each step/direction the demand is permitted to move. • A different value may be provided for each direction, thus curves may be asymmetric.	Variation of demand indicating how much the demand rises/falls in response to a unit change in the marginal cost of meeting the elastic demands. • Contributes to determining (according to MED-ELAST, MED-STEP, MED-VAR) the amount the demand can move (R_ELAST) in the demand equation (MR_DEM), and the associated cost to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). [See MED-ELAST(BD) for calculation.]
MM-SCALE			• Scalar. • [.000001 to 1];		The internal scaling of the model when running MARKAL-MACRO to scale the

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[M]			default = 1.		energy component (EC) relative to the overall economy. • Appears in every equation of the model.
MO(ENC)_TID (dmd,enc) MO(MAT)_TID (dmd,mat) [T]	DMD_MOTIREL_ENC dd_mo	*BAL_INV*	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is released as part of decommissioning an investment made during the modeling horizon.	Energy/material released per unit of investment at the end of the useful lifetime (LIFE) of a demand device. • This amount enters the balance (MR_BAL) for the commodity according to the level of investment (R_INV). • If more than one commodity is produced at the end of the lifetime (LIFE) of a demand device then each commodity must have a corresponding MO(ENC/MAT)_TID parameter. • Contributes to defining the flow of commodities through the RES.
MO(ENC) (dmd,enc,t) MO(MAT) (dmd,mat,t) [T]	DMD_MO dd_mo	*BAL_CAP*	• Units of commodity released per unit of energy service provided (e.g., PJ/PJ). • [0,1]; no default.	• If a commodity is produced as part of servicing a demand.	Energy/material released per unit of activity of a demand device. • This amount enters the balance (MR_BAL) for the commodity according to the level of operation of the device (R_CAP with consideration for capacity factor and units). • If more than one commodity is produced by a device then each commodity must have a corresponding MO(ENC/MAT) parameter. • Contributes to defining the flow of commodities through the RES.
OUT(DM)	DMD_OUT	*BALE_CAP*	• End-use demand	• Provided for each	The fraction of the output from a demand

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
(dmd,d,t) [T/D/O]	DMD_OUTX dmd_dm dmd_dms	*BALH_CAP* *DEM_CAP* *EPK_CAP* *HPKW_CAP*	service met per unit of device activity. • [0,1]; no default.	demand sector serviced by a demand device. • Each device is expected to service at least one sector. • For a given demand device the OUT(DM) should sum to 1.	device that services a particular demand. • The amount that output (R_CAP, taking into consideration the capacity factor and units) contributes to the meeting of the demand (MR_DEM). • When electricity/heat is involved the load is adjusted according to the weighted FR(Z)(Y) for each demand serviced, affecting the balance (MR_BALE, MR_BALDH) and peak (MR_EPK, MR_HPKW) equations. • If more than one demand is serviced by a device then each demand must have a corresponding OUT(DM) parameter. • Contributes to defining the flow of commodities through the RES.
OUT(ELC)_TID (con,elc) OUT(LTH)_TID (con.,lth) [T]	*CON_GRID* CON_HGRD tch_enc	*ANC_TCZY* *ANC_TEZY* *ANC_THZ* *BALE_CAP* *BALE_EZY* *BALE_HPL* *BALH_CAP* *BALH_CYZ* *BALH_THZ* *EPK_CAP* *EPK_HPL* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP*	• Indicator. • [1]; no default.	• Identifies the grid to which a conversion technology is connected. • A conversion plant may only feed a single electricity/heat grid.	An indicator (always 1) as to which electric/low-temperature heat grid a power plant is connected. • The grid controls to which electric/heat balance MR_BALE/MR_BALDH) and peaking (MR_EPK/MR_HPK) equations a conversion plant contributes, and which transmission and distribution costs (ETRANOM, EDISTOM, DTRANOM) are to be charged to the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). • Contributes to defining the flow of commodities through the RES.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
		HPKW_CPD *HPKW_HLK* INV_INV TCH_DELIV T04			
OUT(ENC)c (con,enc,t) OUT(MAT)c (con,mat,t) OUT(ENC)p (prc,enc,t) OUT(MAT)p (prc,mat,t) [T]	CON_OUT con_enc PRC_OUT prc_enc	*BAL_ACT* *BAL_CAP* *BAL_TCZY* *BAL_TEZY* *BAL_THZ*	• Units of commodity released per unit of activity (e.g., PJ/PJ). • [0,1 usually]; no default.	• If a commodity is produced by a process or conversion technology.	The amount of a commodity released per unit of activity. In the case of conversion plants this is a by-product of the electric/heat generation activity. For processes this is the fixed or maximum (in the case of flexible LIMIT processes) primary output from the process. • This amount enters the balance (MR_BAL) equation according to the level of operation of the facility (R_TCZYH, R_TEZY, R_THZ). • If more than one commodity is produced by a process or as a by-product from a conversion plant then each commodity must have a corresponding OUT(ENC/MAT)_TIDc/p parameter. • Contributes to defining the flow of commodities through the RES.
OUT(ENT)r (s,ent,t) {not ent = lth} OUT(MAT)r (s,mat,t)	sep tpsep	*ANC_TSEP* *BAL_TSEP* *BALE_SEP* *BAS_SEP* *CUM_TSEP* *EPK_SEP*	• Indicator. • [1]; no default.	• Identifies the commodity produced, as well as the grid for electricity.	For resource activities, other than exports, an indicator (should be 1) as to the commodity produced from a resource supply option. • The commodity enters the balance (MR_BAL, MR_BALE) and peaking (MR_EPK) equations, and contributes to

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[T]					the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) if there is a resource cost (COST) for the commodity, according to the amount of resource produced (R_TSEP). • Contributes to defining the flow of commodities through the RES.
OUT(MAT)_TIDc (con,mat) OUT(MAT)_TIDp (prc,mat) [T]	CON_OUT1 PRC_OUT1 prc_enc rel_enc	*BAL_INV*	• Units of commodity per unit of capacity (e.g., PJ/PJa). • [open]; no default.	• If a commodity is produced as part of retiring a new investment at the end of its lifetime (LIFE).	Material released per unit of investment at the end of the useful lifetime (LIFE) of a process or conversion technology. • If more than one commodity is produced at the end of the lifetime then each commodity must have a corresponding OUT(MAT)_TIDc/p parameter. • Contributes to defining the flow of commodities through the RES.
PD(Z)D (con,z,t) {except con=cpd} [O]	TCH_PD	EPK_TEZY HPKW_ACT PEAK_TID TEZY_CAP THZ_CAP	• Fraction. • [0,1]; default = 1, when PEAK_TID (CON) provided.	• Most often left to the default of 1. • Not available for coupled heat and power plants (cpd).	For a power generating plant considered to be a peaking device (PEAK_TID(CON)) whose contribution to the peaking constraint is only to be credited according to its operational level, PD(Z)D is the fraction of the daytime in a season for electricity (or season for heat) that the plant is available. • The activity of such plants is controlled by the utilization equation (MR_TEZY, MR_THZ) by limiting the activity (R_TEZY, R_THZ) to the peak duration * fraction of the year (QHR(Z)(Y), QHRZ). • In the peaking constraint (MR_EPK,

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					MR_HPKW) the activity variable enters the equation taking into consideration both the fractional capacity that can be credited to the peak (PEAK(CON)) and the duration of operation within the time-slice.
PEAK(CON) (con,z,t) [O]	PEAK1	*EPK_CAP* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP* *HPKW_CPD*	• Fraction. • [0,1]; default = 1.	• For most plants it is assumed that all the capacity is available to meet the peak, however for intermittent technologies (e.g., solar, wind) a value less 1than is normally provided.	For a power generating plant, the fraction of the plant's capacity that should be credited towards the peaking requirement. • The multiplier applied to the capacity variable (R_CAP), or activity variable (R_TEZY/R_THZ) if a peaking device (PEAK_TID(CON)) taking into consideration the duration factor (PD(Z)D), in the peaking constraint (MR_EPK, MR_HPKW).
PEAK(CON)_TID (con) [O]	PEAK_TID	*BAS_TEZY* *EPK_CAP* *EPK_TEZY* *HPKW_ACT* *HPKW_CAP* *HPKW_CPD* *PD(Z)D* *TEZY_CAP* *THZ_CAP*	• Indicator. • [1]; no default.	• Used for peaking-only devices that have low capital costs but high operating costs, which sometimes results in the model building but not using such technologies. If such decisions were made by the model only to meet the peaking requirements, then activating this tends to result in the running of such	An indicator that a power generating plant is only to be credited towards the peaking requirement based upon its operation, not its capacity. • The indicator serves as a switch that inhibits the generation of an activity variable (R_TEZY, R_THZ) at off-peak times, thus allowing such plants to only operate at potential peak times. • The activity of such plants is controlled by the utilization equation (MR_TEZY, MR_THZ) by limiting the activity to the peak duration (PD(Z)D) * fraction of the year (QHR(Z)(Y), QHRZ). • In the peaking constraint (MR_EPK,

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				plants, or the selection of some other technology. • Not permitted for coupled production (cpd) or base load (bas) plants.	MR_HPKW) the activity variable enters the equation taking into consideration both the fractional capacity that can be credited to the peak (PEAK(CON)) and the duration of operation within the time-slice.
PEAKDA(PRC) (prc,t) [O]	PEAKDA_PRC	*EPK_ACT* *EPK_CAP*	• Fraction. • [0,1]; default = 1.	• Only provided if some part of the demand for electricity is know not to occur during the peak time.	For processes that use electricity the fraction of the consumption that is coincident with the peak. • A multiplier adjusting the amount of electricity consumed (R_ACT/R_CAP (xpr) consumption) that contributes to peaking requirements (MR_EPK).
PEAKDA(SEP) (s,t) [O]	*PKDA_SEP*	*EPK_SEP*	• Fraction. • [0,1]; default = 1.	• Only provided if some part of the import/export/ consumption of electricity does not occur during the peak time. • For resources consuming electricity during the extraction process, it indicates the fraction of the consumption that is coincident with the peak.	For the import and export of electricity, and for ancillary use of electricity, the fraction of the supply/export/consumption that is coincident with the peak. • A multiplier adjusting the amount of electricity delivered/exported/consumed (R_TSEP, latter adjusted for consumption) that contributes to meeting (for imports) or adds to the requirements for peak capacity (MR_EPK).
QHR(Z)(Y)	*QHR_{18D}*	*$QHRZ_{8D}$*	• Fraction.	• Must be provided	The fraction of the year represented by each

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
(w) [M]			• [0,1]; no default.	for each time-slice to be represented. • Omit for any time-slice *not* to be tracked.	time-slice. The year may be divided up into six time-slices according to season (Z=Winter/Intermediate/Summer) and time-of-day (Y=Day/Night). The time-slices are used to shape electricity and heat load to discriminate between times of high and low demand. • The electricity and heat balance (MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPK) equations are only generated for those time-slices specified. • All power plants (con) operate based upon these fractions of the year, with variables (R_TCZYH, R_TEZY, R_THZ) only created for those time-slices specified. • The power sector generation in each time-slice is matched against the load accumulated according to the various demands shapes (FR(Z)(Y)), that default to QHR(Z)(Y) when not provided.
RAT_RHS (a,b,t) [U]	*RAT_RTY1* *RAT_RTY*		• Fraction. • [open]; no default.	• MR_ADRATn user constraints are only generated when the user explicitly provides a value (including 0) for the RHS in a period.	RAT_RHS has three purposes. It is a designator as to the sense ('LO', 'FX', 'UP', 'NON', corresponding to MR_ADRAT1,2,3,4 respectively) of a user-defined ad hoc constraint, indicating in which periods the constraint applies (whenever a value is explicitly provided for period), and the value of the RHS. • A user-defined constraint

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					(MR_ADRAT1-4, according to the sense of the equation) generated by applying the associated multipliers (the RAT_* parameters below) directly to the appropriate associated variables.
RAT_ACT$_{6E,6P,6Q,6R}$ (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided. • Note that the same coefficient is used for all time-slices when RAT_ACT is used for conversion plants. If the coefficient is to vary according to time-slice, then the individual RAT_TCZY or RAT_TEZY or RAT_HPL parameter need to be used.	The coefficient in a user-defined constraint (MR_ADRAT1-4) to be applied to the annual activity of a technology. • The multiplier to be applied to the activity variable (R_ACT (for prc), R_CAP (for dmd), R_TCZYH (pass-out cpd), R_TEZY (ela), R_THZ (hpl)) for each time-slice. • For externally load managed (xlm, xpr) and demand (dmd) technologies, the coefficient is applied to the capacity variable (R_CAP) according to the operation of the technology.
RAT_CAP$_{6F}$ (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint to be applied to the capacity of a technology. • An intersection is created in the user-defined equation (MR_ADRAT1-4) for a technology according to its total installed capacity (R_CAP).

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
RAT_HPL₆R (a,hpl,z,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period and season for which a coefficient is provided. • The multiplier is applied to the seasonal activity variable (R_THZ) only for those seasons for which the parameter is provided.	The coefficient in a user-defined constraint to be applied to the seasonal (z) activity of a heating plant (hpl). • An intersection is created in the user-defined equation (MR_ADRAT1-4) for a technology according to the seasonal activity of the heating plant (R_THZ).
RAT_INV₆J (a,p,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint applied to new investment in a technology. • An intersection is created in the user-defined equation (MR_ADRAT1-4) for the technology according to the level of new investment (R_INV).
RAT_SEP (a,s,t) [U]	*RAT_TSEP₆R*		• Constant. • [open]; no default.	• An intersection is created for each period in which a coefficient is provided.	The coefficient in a user-defined constraint applied to the level of production from a resource supply option (srcencp). • An intersection is created in the user-defined equation (MR_ADRAT1-4) for the resource supply option according to the resource activity level (R_TSEP).
RAT_TCZY₆P (a,cpd,w,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period and time-slice for which a coefficient is	The coefficient in a user-defined constraint applied to the time-slice (w) activity of a coupled heat and power (cpd) pass-out turbine (ELM/CEH). • An intersection is created in the user-

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				provided. • The multiplier is applied to the activity variable (R_TCYZH) only for those time-slices for which the parameter is provided.	defined equation (MR_ADRAT1-4) for a technology according to the time-slice activity of the heat output from a pass-out turbine (R_TCZYH).
RAT_TEZY[6Q] (a,ela,w,t) [U]			• Constant. • [open]; no default.	• An intersection is created for each period and time-slice for which a coefficient is provided. • The multiplier is applied to the activity variable (R_TEZY) only for those time-slices for which the parameter is provided.	The coefficient in a user-defined constraint according to the time-slice (w) activity of an electric plant (ela). • An intersection is created in the user-defined equation (MR_ADRAT1-4) for the technology according to the time-slice activity of the electricity output (R_TEZY).
REG_XCVT(ent)[11S, 11T,21I] (r,ent) *REG_XCVT(env)*[21I] (r,v) *REG_XCVT(mat)*[11S, 11T,21I] (r,mat)			• Constant. • [open]; default = 1.	• Provided if the units of any commodity are different between regions.	Commodity conversion factor by region for bilateral and global trade. • Applied to the import/export (R_TSEP, R_TSEPE) and global (R_GTRD) trade variables in the bi-lateral (MR_BITRD, MR_BITRDE) and global (MR_GTRD) constraints.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[T]					
REG_XMONY$_{24M,24N}$ (r) [T/C]			• Constant. • [open]; • Default = 1.	• Provided if the monetary units in any region are different.	Monetary conversion factor by region for the total costs of the energy system in each region. • Applied to the regional total system cost variable (R_objZ) for each region as it enters the final objective function (MR_OBJ).
REH (cpd,t) [O]	*T_CPDREH*	*ANC_TEZY* *BALH_CYZ* *HPKW_CPD* *HPKW_HLK*	• Unit of electricity produced per unit of heat produced. • [0,1]; no default.	• Provided if a coupled-heat and power plant is a back-pressure turbine.	For a back-pressure turbine coupled production plant (cpd), that is one having a fixed electricity-to-heat ratio, the ratio of electricity per unit of heat produced. • Since there is a fixed relationship between the amount of electricity and heat, the electric production variable (R_TEZY) is used to represent both. REH is applied to R_TEZY to reflect the heat contribution from such a plant to the heat balance (MR_BALH) and peaking (MR_HPKW) constraint. • If there are O&M charges for the heat distribution system (DTRANOM) then the variable also appears in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
RESID (p,t) [O]	*TCH_RES*		• Unit of capacity (e.g., GW, PJa). • [open]; no default.	• Provided for each period for which capacity installed prior to the beginning	For technologies available at the beginning of the modeling horizon, that is those that were in place before the first year of the model, the "retirement" trajectory as those

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				of the model horizon is still in place.	technologies reach their useful lifetime. Thus for each period RESID is the exogenous specification of the residual capacity left around for those technologies inherited by the model when it started. • The RESID appears as the right-hand-side of the capacity transfer constraint (MR_CPT1/2/3).
SAL_REL (c,t) [C]	SALV_INV INV_INV *PRI_INV*		• Monetary units. (M$2000) • [any], none.	• When material released during the modeling horizon is to be credited to the objective function.	Upon the release of sunk materials their value is credited back to the objective function. • The value of salvaged material is applied to the investment variable (R_INV) upon reaching its technical lifetime (LIFE).
SAL_SNK (c,t) [C]	SALV_INV INV_INV *PRI_INV*		• Monetary units. (M$2000) • [any], none.	• When material released at the end of the modeling horizon is to be credited to the objective function.	At the end of the modeling horizon all sunk materials embodied in investments still in place have their value credited back to the objective function. • The value of sunk material is applied to the investment variable (R_INV) upon reaching its technical lifetime (LIFE).
SECURITY (c,t) [O]	*TSEC_SEP* *SEC_SEP*		• Any. • [any], none.	• When the security accounting equation is to be built for a commodity.	A multiplier applied to each resource supply option, if desired. • Entry made in the security accounting equation (MR_TSECURI) for each resource option to be tracked.
SLKCOST (c,t)			• Monetary units. (Million US$2000) • [any], none.	• When the user wishes to make an energy carrier (enc)	An indicator that results in the reformulation of the energy and low-temperature balance equations as slack constraints. This is

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[C]				balance constraint an =E= with a slack variable (R_SLKENC / R_SLKLTH), rather than the traditional =G=. • The value of SLKCOST should be very high.	sometime useful when infeasibilities are occurring as a result of fixed output technologies. • A slack variable (R_SLKENC/R_SLKLTH) is added to the standard balance constraints (MR_BAL/MR_BALDH) and the equations changed to an equality =E=. • The slack variable also enters the objective function (EQ_PRICE if single region, MR_OBJ if multi-region) multiplied by the SLKCOST.
SRAF(Z) (hde,z,t) [O]	*TCH_SRAF*		• Fraction. • [0-1]; no default.	• Provided if a hydro plant has a seasonal limit on the reservoir availability.	To limit the seasonal availability of a hydroelectric (hde) reservoir. • Results in a seasonal utilization equation (MR_SRM) establishing the limit on total hydroelectric generation in a season based upon total capacity (R_CAP) * the seasonal reservoir factor (SRAF).
SRCENCP[56] [T]	sep tpsep	*ANC_TSEP* *ANC_ZSTK* *BAL_SENT* *BALE_SEP* *BAS_SEP* *CUM_TSEP* *EPK_SEP*	• List. • [alpha-numeric according to naming convention]; no default.	• Provided for each resource supply options, including bi-lateral imports/exports. • A mapping set (sep) splits the single SRCENCP name into	List of resource supply options. • For each element of the set SRCENCP (sep) a resource variable is created (R_TSEP), beginning from the year the variable is first available. • The resource variable enters the balance constraint (MR_BAL, MR_BALE) and if electricity (elc) the peaking constraint

[56] SRCENCP is actually a list created from the user input, and is managed as a Set in the GAMS code. However, the entries themselves, constructed with the named outputs from the various resource options, control the entry in the various cost, balance and peaking equations. Therefore it is listed here as well.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				its 3 components (src, e, c). This short form is often used in the code. • [Note, it is recommended that the modeler use the same characters for the SEPent root as the primary output (OUT(ENT)r/x)) to avoid mixing internal and input/output SRCENCP names.]	(MR_EPK) and base load (MR_BAS) constraint, as well as the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). • If the total amount available from a source over the entire time horizon is limited by cumulative proven reserves (CUM), then the resource variable also enters a special cumulative constraint (MR_CUM). • For the stockpile activities there is a special variable created for the recovered material in the stockpile at the end of the model horizon (R_ZSTK).
START (p/s) [O]	TCH_STRT SEP_STRT tptch tpsep et al		• Year. • [1990-2200]; default = first year of the model horizon.	• Used when establishing the master control set for technologies (tptch), and technology subsets (e.g., tpela, tpcpd, tpsep) that is used throughout the code to control from which period a technology is available.	The year in which a technology or resource option is initially available. • Controls from which period technology-based equations (e.g., MR_CPT, MR_TEZY, MR_THZ) and technology and resource based variables (all with p, and R_TSEP respectively) are to be generated.
STARTYRS [M]		*PRI_DF* PRI_DISC	• Number of years. • [open]; default = 2.5.	• For MARKAL part of calculating the cost coefficients for the objective function.	Shift from the first year of the model to the year relative to which all discounted costs are to be calculated. • A period runs from Jan 1 to Dec 31 over

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					some number of years. Thus a 5-year period is represented by the middle of the middle year, 2.5 years from the beginning or June 30th of the second year. As discounting begins from the first day of the first period, to move to the mid-point the discounting calculation needs to be shifted accordingly.
TE(ENT) (ent,t) TE(MAT) (mat,t) [O]	T_TE T_TEENC TE_CON	*BAL_TSEP* *ANC_ZSTK* *BAL_ACT* *BAL_CAP* *BAL_TCZY* *BAL_TEZY* *BAL_THZ* *BALE_CAP* *BALE_SEP* *BALE_CZY* *BALE_EZY* *BAS_CAP* *BAS_SEP* *BAS_TCZY* *BAS_TEZY* *EPK_CAP* *EPK_SEP* *EPK_TEZY*	• Fraction. • [0,1]; default = 1.	• Usually a non-unity value is only provided for electricity and low-temperature heat transmission systems (from central station plants).	The overall efficiency of the infrastructure associated with each energy carrier/material. The purpose of the commodity based efficiency factor is to reflect losses encountered due to transmission or other movement of a commodity. When less than the default of 1, the effect being to increase the amount of the commodity that may be produced. • The transmission efficiency appears in almost every equation tracking energy/material (MR_BAL, MR_BALDH, MR_BALE, MR_EPK, MR_HPK, MR_BAS), where it is applied to the technology and resource activity variables (all with p, and R_TSEP respectively).
TRD_BND[361] (e/v) [T]			• Units of the commodity. • No default.	• Provided if a bound is to be applied.	Limit on the import or export of a globally traded commodity into/from a region. • Applied to the global trade (R_GTRD) variable for the region, period and

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					import/export operation.
TRD_COST$_{25S,28S,251}$ (e/v) [T/C]			• Base year monetary units (e.g., 2000 M$). • [open]; default = 1.	• Provided if a transaction cost is to be added to the cost of bi-lateral trade for a globally traded commodity.	Additional (transaction) cost associated with global trade in a commodity. • Applied to the import/export (R_TSEP) and global (R_GTRD) trade variables in the objective function (EQ_PRICE if single region, and MR_OBJ if multi-region).
TRD_FROM$_{21D}$ (e/v) [T]			• Year. • [open]; default = first period.	• Provided if trade in a global commodity is not to start in the first period.	Year from which global trade in a commodity may commence. • The global trade constraint (MR_GTRD) is only generated beginning from the TRD_FROM period. • Entries for the global trade variable (R_GTRD) are only made in the associated balance (MR_BAL_G/E) and emissions (MR_TENV) equations beginning from the TRD_FROM period.
TRNEFF(Z)(Y) (cpd,w,t) [O]	T_CPDEFF	*BALH_CZY* *HPKW_CPD*	• Decimal fraction. • [0,1]; default = 1.	• Provided if default of 1 is not desired, the time-slices have different efficiencies, or only some time-slices are available. • Note that this efficiency is used in place of the commodity-based transmission efficiency (TE(ENT))	Transmission efficiency of low-temperature heat from coupled heat and power plants (cpd) according to season and time-of-day. • Affects the low-temperature heat balance (MR_BALDH) and peaking (MR_HPK) constraints by adjusting the amount of heat delivered (R_TCZYH, R_TEZY) to reflect the loss.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
				for coupled heat and power plants.	
TSUB_BND(BD) (txs,b,t) [C]			• Monetary units. • No default.		Bound applied to a tax/subsidy as the total tax/subsidy permitted in a period.
TSUB_COST$_{25/28T,32D}$ (txs,t) [C]			• Monetary units. • No default.	• Provided if a tax (+) or subsidy (-) is to be associated with a commodity /technology.	Cost of a tax/subsidy applied to selected energy carriers and the technologies producing and/or consuming it. • The cost is applied to the accumulated total in the objective function (EQ_PRICE if single region, and MR_OBJ if multi-region).
TSUB_ENT (txs,e,t) *TSUB_MAT* (tx,mat,t) [C]			• Indicator. • (0,1), no default.	• Provided for the commodity into/from the technology to which the tax (+) or subsidy (-) is to be associated.	Commodity to which a tax/subsidy applies. • All commodities listed for a particular tax/subsidy constraint are accumulated in each tax/subsidy equation (MR_TXSUB) based upon the amount flowing in/out of the identified resources and technologies.
TSUB_SEP$_{32S}$ (txs,p,t) [C]			• Indicator. • (0,1), no default.	• Provided for the technology to which the tax (+) or subsidy (-) is to be associated.	Contribution of a resource activity to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed. • All resources listed for a particular tax/subsidy constraint are accumulated in each tax/subsidy equation (MR_TXSUB).
TSUB_TECH$_{32E/F/J/O/P/Q/R}$ (txs,p,t)			• Indicator. • (0,1), no default.	• Provided for the technology to which the tax (+) or subsidy (-) is to be associated.	Contribution of a technology to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed.

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
[C]					• All technologies listed for a particular tax/subsidy constraint are accumulated in each tax/subsidy equation (MR_TXSUB).
UNIFDIST (d) [D]	DM_UDIS	*BALE_CAP* *BALH_CAP* DM_FR *EPK_CAP* *HPKW_CAP*	• Indicator. • [0,1]; default = 1, if no demand fraction (FR(Z)(Y)) is provided.	• Results in the demand fractions (FR(Z)(Y)) being set = QHR(Z)(Y).	An indicator that the electric and heat load shapes are a unified distribution matching that of the standard time-slices (QHR(Z)(Y)). When non-standard the demand shape is specified by time-slice by means of the demand fractions (FR(Z)(Y)). • FR(Z)(Y) controls the fraction of the annual service demand met by electricity/heat that occurs in each time-slice. As such it controls the amount required for the balance (MR_BALE, MR_BALDH) and peaking (MR_EPK, MR_HPKW) constraints.
VAROM (p,t) [C]	TCH_VAROM	*ANC_ACT* *ANC_CAP* *ANC_TCZY* *ANC_TEZY* *ANC_THZ*	• Base year monetary units per unit of activity (e.g., 2000 M$/PJ). • [open]; no default.	• Provided if there are variable costs associated with the operation of a technology.	The variable operating and maintenance costs associated with the activity of a technology. • The cost to be applied to the activity variable (R_ACT (for prc), R_CAP (for dmd), R_TCZYH (pass-out cpd), R_TEZY (ela), R_THZ (hpl)) to charge the variable costs to the objective function (EQ_PRICE if single region, or MR_OBJ if multi-region). • For externally load managed (xlm, xpr) and demand (dmd) technologies, the cost is applied to the capacity variable (R_CAP) according to the operation of the

Input Parameter (Indexes[49]) [Purpose][50]	Alias/Internal Name[51]	Related Parameters[52]	Units/ Range & Defaults	Instance[53] (Required/Omit/Special Conditions)	Description[54]
					technology.
XRATRHS (a,b,t) [U]	XARATRTY		• Numeric value • [open]; no default.	• MR_XARAT cross-region user constraints are only generated when the user explicitly provides a value (including 0) for the RHS in a period.	XRATRHS has three purposes. It is a designator as to the sense ('LO', 'FX', 'UP', 'NON', corresponding to MR_XARAT1,2,3,4 respectively) of a user-defined, cross-region ad hoc constraints, indicates in which periods the constraint applies (whenever a value is explicitly provided for period), and the value of the RHS. • A user-defined constraint (MR_XARAT1-4, according to the sense of the equation) generated by applying the associated multipliers (the RAT_* parameters above) directly to the appropriate associated variables.

2.3.2 Matrix Coefficients and Internal Model Parameters

The parameters described in this section relate to the coefficients and internal parameters used in the MARKAL GAMS code to control the translation of the model equations and data input to a matrix ready for solution. They constitute those parameters actually referenced in the equations themselves, and the key internal parameters used in the MARKAL GAMS code when building the matrix coefficient parameters. As such they constitute the interim and final LP matrix coefficients calculated as part of the model generation process.

Section 2.3.2.1 first lists and describes all of the sets and parameters appearing in the MARKAL matrix. This includes input data parameters that appear directly in the equation generation code as well. In Section 2.3.2.2 the internal parameters that are crucial to building the matrix coefficients (e.g., the capital recovery factor (CRF) for annualizing the investment costs) are described.

2.3.2.1 Matrix Control Sets and Parameters

There are quite a few input parameters and attributes associated with technologies that affect which variables and matrix intersections are populated in the matrix. Under normal circumstances both activity and capacity variables are generated, annually or at the time-slice level according to the nature of the technology. Some of the key aspects controlling how a technology is modeled are presented in Table 2-7 below.

Table 2-7. Matrix Component Control

Input	Type[57]	Implications
BAS	S	Conversion technologies characterized as base load are modeled using only daytime production vectors (R_TCZYH, R_TEZY). The level of these vectors reflects both the day and night-time production, that by definition are equal.
CPD	S	Coupled heat and power technologies are characterized as either back-pressure (REH) or pass-out (ELM, CEH(Z)(Y)). In the case of the former, the heat output is directly derived from the electric production (R_TEZY), in the case of the latter the heat output is modeled (R_TCZYH) and the electricity then derived. These variables are modeled at the time-slice level, and the associated parameters must take into consideration both the type of the plant as well as for which time-slices the technology is available.

[57] P = parameter describing the technical nature of a technology, S = set characterizing the nature of a technology.

Input	Type[57]	Implications
DMD	S	Demand devices are modeled by means of a capacity variable (R_CAP) only, with the activity derived by applying the appropriate capacity factors (CF) and unit conversion (CAPUNIT), unless the demand device is "vintaged," in which case the activity is derived from the investment variable (R_INV) and a separate variable is used to represent the (fixed) residual capacity (R_RESIDV).
ELE	S	Electric generating plants are modeled at the time-slice level, and the associated parameters must take into consideration the type of the plant (e.g., conventional, externally load managed, base load, storage, coupled heat and power) as well as for which time-slices the technology is available.
HPL	S	Heat generating plants are modeled at the seasonal level, and the associated parameters must take into consideration the type of the plant (e.g., conventional, externally load managed) as well as for which seasons the technology is available.
NST	S	Night storage technologies are special processes (prc) and demand devices (dmd) that consume off-peak electricity at night and provide commodities or demand services during the day.
PEAK_TID(CON)	P	Peaking only technologies have their activity controlled to ensure they operate at peak times only (e.g., daytime).
START	P	The period from which a technology or resource option is first available is used to inhibit generation of all related equations and variables before that time.
XPR/XLM	S	Externally load managed process and conversion plants are forced to operate according to the explicit capacity factors (CF/CF(Z)(Y)) provided by the user. As such no activity variable is generated, but rather the activity level is determined from the installed capacity (R_CAP).

To manage this complexity a series of internal parameters are built by the preprocessor of the matrix generator to ensure that each technology is properly modeled. These parameters indicate which technologies belong in the various constraints and under what circumstances, thus enabling them to control the permitted intersections in the matrix and make the equation code rather straightforward.

To the extent possible parameter prefixes "match" the root of the equation to which the coefficient applies. These are summarized in Table 2-8 below.

Table 2-8. Description of matrix parameters prefixes[58]

Parameter Prefix	Description
ANC_	Annualized costs appearing in the regional objective function (MR_OBJ).
BAL_	Contribution (+ for production, - for consumption) to the energy and material balance constraints (MR_BAL).
BALE_	Contribution (+ for production, - for consumption) to the electricity balance constraints (MR_BALE), including an indication of in which time slices.
BALH_	Contribution (+ for production, - for consumption) to the low temperature heat balance constraints (MR_BALDH), including an indication of in which season.
BAS_	Contribution (+ for base load, zero for non base load) to the electricity base load constraint (MR_BAS).
BND_	Annual bound to be applied to time sliced variables (R_TCZYH, R_TEZY, R_THZ) by means of associated constraints (MR_BNDCON).
CPT_	Capacity transfer constraint (MR_CPT).
DM_	Demand for energy services (MR_DEM).
ENV_	Total emission over the entire modeling horizon (MR_ENV).
EPK_	Electricity peaking constraint (MR_EPK).
HPKW_	Heat peaking constraint (MR_HPK).
PRC_	Limit processes (MR_LIM, MR_PBL).
PRI_	Discounted costs (MR_PRICE).
RAT_	User-defined constraints (MR_ADRAT).
REG_	Multi region linkage (MR_OBJ).
SEP_	Limits applied to resources (MR_CUM and R_TSEP annual bound).
TCH_	Technology characterization, thus applying to various constraints.
TENV_	Annual emission constraint (MR_TENV).
TEZY_	Electricity plant utilization (MR_TEZY).
THZ_	Heating plant utilization (MR_THZ).
TRD_	Bi-lateral trade (MR_BITRD, MR_BITRDE).
UPRC_	Process utilization constraint (MR_UTLPRC).

[58] Only those prefixes that apply to more that 1 parameter are listed here.

Similarly, the suffix on many of the parameter names also follows a convention. Whenever possible it reflects the variables to which the parameter is applied, as noted in Table 2-9 below.

Table 2-9. Description of matrix parameters suffixes

Parameter Suffix	Description & Variables
ACT	Related to the annual activity of a technology. Most often relates to the annual activity of a process (R_ACT). But also an annual value to be used in all time-slices for conversion technologies (R_TCZYH, R_TEZY, R_THZ), or the activity rather that the capacity of demand devices (R_CAP).
CAP	Related to the total installed capacity of a technology (R_CAP).
INV	Related to the newly installed capacity of a technology (R_INV).
TCZY	Related to the production of heat (and electricity) from coupled heat and power pass-out turbines in each time-slice (R_TCZYH).
TEZY	Related to the production of electricity from power plants in each time-slice (R_TEZY).
THZ	Related to the production of low-temperature heat from heating plants in a season (R_THZ).
TSEP	Related to the annual production of a resource supply option (R_TSEP).
ZTSK	Related to the final release of a stockpiled commodity (R_ZSTK).

Table 2-10 below describes each of the parameters and sets involved in the matrix. A heavy emphasis is put on identifying the key input parameters from which these important control parameters are derived. The matrix intersections to which the parameter applies are referenced as the row and column intersection in the MARKAL matrix, and can be often be derived from the prefix/suffix comprising many of the internal parameter names. The input parameters are themselves defined in Table 2-6.

Table 2-10. Definition of matrix coefficient parameters and controls[59]

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
ANC_ACT$_{7E}$ (t,prc)	I	Annualized cost of operating a process (R_ACT) comprising the variable operating costs (VAROM), and any delivery costs (DELIV) according to how much of the commodity is consumed in the current period (taking into consideration both the input amount (INP(ent/mat)p) and any lead or pre-capacity based commodity requirements (LED(enc/mat)). Note that for technologies where MARKAL does not construct an activity variable, the capacity variable is used instead; see ANC_CAP.
ANC_CAP$_{7F}$ *ANC_CAPR* *ANC_VDI* (t,p)	I	Annualized cost associated with the total capacity in place (R_CAP) of a technology. Note that in the case of technologies for which MARKAL does not build an activity variable, that is for demand devices (dmd) and externally load managed processes or conversion plants (xpr, xlm), the coefficient associated with the activity of the technology must be associated with the capacity variable (instead of with the activity variable as would normally be the case).
(t,con)		For a conversion plant (con), the annual fixed operation and maintenance cost (FIXOM) as input by the modeler. For externally load managed plants (xlm), any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ENT/MAT)c) and capacity-to-activity and unit conversion factors (CAPUNIT, CF/CF(Z)(Y)). Note that for electricity and district heat, the transmission (ETRANOM, DTRANOM) and distribution (EDISTOM, only when electricity) costs are also accounted for. Furthermore, note that the season/day night nature of the operation of these plants is also taken into consideration (via CF(Z)(Y) and QHR(Z)(Y)).

[59] Those parameters and mathematics related to special MARKAL instances such as MARKAL-MACRO, endogenous technology learning (ETL), environmental damage (EV_Damage), etc, are not included here as they do not explicitly appear in the actual matrix of the model passed to the solver(s).

[60] For certain parameters there may be substantively different conditions/code to warrant splitting out the details of an internal parameter according to the main user sets. The documentation index letters are used here, except when a parameter only applies to a subset of the master index sets.

[61] Type indicates the nature of the parameter/control as follows:

- A – Alias of a user input parameter;
- I – Internal parameters, most often derived from a combination of user input parameters and other conditions;
- S – Set (either internal or by user), and
- U – User input parameter.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
(t,dmd)		For a demand device (dmd), the annual fixed operation and maintenance cost (FIXOM) plus any associated variable operating (VAROM) and delivery (DELIV) costs, taking into consideration the appropriate consumption/performance (MA(ENT/MAT), EFF) information and capacity-to-activity and unit conversion factors (CAPUNIT, CF). The ANC_CAPR and ANC_VDI instances apply only to demand devices (dmd) that are vintaged, where the cost is applied to the residual capacity (R_RESIDV) and vintaged investment (R_INV) using the associated efficiency (EFF_R, EFF_I).
(t,prc)		For a process (prc), the annual fixed operation and maintenance cost (FIXOM) as input by the modeler. For externally load managed processes (xpr), and those processes with no investment cost (INVCOST) and life (LIFE) specified, any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ENT/MAT)p/LED(ENT/MAT)) and capacity-to-activity and unit conversion factors (CAPUNIT, CF).
ANC_TCZY$_{7p}$ (t,cpd)	I	For a coupled heat and power plant that is a pass-out turbine (ELM, CEH(Z)(Y)) according to the amount of heat produced (R_TCYZH), the annual variable operating (VAROM) and delivery (DELIV) costs are accounted for, taking into consideration consumption/performance (INP(ENT/MAT)c, ELM, CEH(Z)(Y)) and transmission (ETRANOM, DTRANOM for centralized plants (cen)) and distribution (EDISTOM, only when electricity) costs. The code must then perform some exception handling for base load (bas) technologies (these are modeled using only a 'D'ay variable).
ANC_TEZY$_{7Q}$ (t,ela)	I	For a standard electric power plant (ela), according to the amount of electricity produced (R_TEZY), the annual fixed operation and maintenance cost (FIXOM) along with any associated variable operating (VAROM) and delivery (DELIV) costs are also accounted for. For the latter pair taking into consideration consumption/performance (INP(ENT/MAT)c) and capacity-to-activity and for electricity and district heat the transmission (ETRANOM, DTRANOM for centralized plants (cen)) and distribution (EDISTOM) costs. The code must then perform some exception handling for peaking-by-activity (PEAK_TID) (with variables only for peaking times) and base load (bas) and storage (STG) technologies (these are modeled using only a 'D'ay variable).
ANC_THZ$_{7R}$ (t,hpl)	I	For a standard heating plant (hpl) according to the amount of heat produced (R_THZ), the annual variable operating (VAROM) and delivery (DELIV) costs are accounted for, taking into consideration consumption/performance (INP(ENT/MAT)c) and transmission (DTRANOM for centralized plants (cen)) costs. The code must then perform some exception handling for base load (bas) technologies (these are modeled using only a 'D'ay variable).

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
ANC_TSEP$_{7T}$ (t,e,c)	I	The cost (COST) of obtaining a resource according to its activity (R_TSEP). This includes the operation and maintenance cost of power lines (ETRANOM, EDISTOM) if importing electricity. Assumes a negative value for exports. If there are delivery costs (DELIV) associated with the consumption of auxiliary commodities (INP(ENT)r) they are reflected as well.
ANC_ZSTK$_{7W}$ (t,enc,c)	I	Value of stockpiled material at the end of the modeling horizon (R_ZSTK) according to the unit cost (COST_TID).
BAL_ACT$_{9E,38E}$ (t,t,prc,enc)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)p) or produced (OUT(ENT/MAT)p) in a time period by processes as a function of the overall activity (R_ACT) of the process. Any transmission losses (TE(ENT/MAT)) are taken into consideration. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes are needed.
BAL_CAP$_{9F,38F}$ *BAL_VDI* (t,t,p,enc)	I	The amount of each commodity (other than electricity and heat) consumed (MA(ENT/MAT), INP(ENT/MAT)p) or auxiliary commodity produced (MO(ENT/MAT), OUT(ENT/MAT)p) in a time period by demand devices (dmd) or externally load managed technologies (xlm, xpr) based upon total installed capacity (R_CAP). The capacity factor (CF/CF(Z)(Y)) and capacity-to-activity factor (CAPUNIT), along with efficiency (EFF) for demand devices is applied as well. Any transmission losses (TE(ENT/MAT)) are taken into consideration. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes are needed. The BAL_VDI instances apply only to demand devices (dmd) that are vintaged, where the balance coefficient is applied to the vintaged investment (R_INV) using the associated efficiency (EFF_I). The BAL_CAP is similarly applied to the residual capacity variable (R_RESIDV) using EFF_R.
BAL_INV$_{9J,38J}$ (t,t,p,e)	I	The amount of each commodity (other than electricity and heat) required (MA(ENT/MAT)_TID, INP(ENT/MAT)_TID) at the time an investment is made in new capacity (R_INV). Any transmission losses (TE(ENT/MAT)) are taken into consideration. For nuclear fuel (enu) the commodity flow occurs in the previous time period (INP(ENC)_TID), for all others in the period the investment takes place. Thus double period indexes are needed.
BAL_SENT$_{9T,11T,22T}$ (t,s,e)	I	The amount of an auxiliary commodity (other than electricity and heat) consumed (INP(ENT/MAT)r) in a time period by a resource supply activity (R_TSEP). Any transmission losses (TE(ENT/MAT)) are taken into consideration.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
BAL_TCZY$_{9P,38P}$ (t,cpd,enc,w)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by a coupled production pass-out turbine according to the heat generation in each time-slice (R_TCZYH). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_TEZY$_{9Q,38Q}$ (t,p,enc,w)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by an electric generation plant according to the level of activity in each time-slice (R_TEZY). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_THZ$_{9R,38R}$ (t,hpl,enc,z)	I	The amount of each commodity (other than electricity and heat) consumed (INP(ENT/MAT)c) or auxiliary commodity produced (OUT(ENT/MAT)c) in a time period by a heating plant according to the heat generated in a season (R_THZ). Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_TSEP$_{9Y,38T}$ (t,t,s)	I	The amount of a commodity (other than electricity and heat) delivered as part of a resource activity (OUT(ENT/MAT)r) in a time period by a resource supply activity (R_TSEP). For stockpiled resources ('STK') there is a carry-over of the current amount to the next period, thus double period indexes are needed. Any transmission losses (TE(ENT/MAT)) are taken into consideration.
BAL_ZSTK$_{9w}$ (t,enc,c)	I	The amount of a commodity (other than electricity and heat) salvaged from a stockpile (R_ZSTK) in the final period, taking into consideration any transmission losses (TE(ENT/MAT)).
BALE_ACT$_{11E,38E}$ (t,t,prc,elc,w)	I	The amount of electricity consumed (INP(ENT)p) in a time period by processes as a function of the overall activity (R_ACT) of the process. The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes are needed.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
BALE_CAP$_{11F,38F}$ *BALE_VDI* (t,t,p,elc,w)	I	The amount of electricity consumed (MA(ENT), INP(ENT)p) in a time period by a demand device (dmd) or externally load managed process (xlm) based upon total installed capacity (R_CAP). The capacity factor (CF) and capacity-to-activity factor (CAPUNIT), along with efficiency (EFF) for demand devices is applied as well. In addition, generation of electricity (OUT(ELC)_TID) from externally load managed power plants (xlm) is accounted for according to the appropriate capacity factor (CF(Z)(Y)) and capacity-to-activity factor (CAPUNIT), and time-slice duration (QHR(Z)(Y)). The parameter must take into consideration any pre/post commodity flows (LED/LAG), as well as flows related to investments (INP(ENC/MAT)_TID), thus double period indexes are needed. The BALE_VDI instances apply only to demand devices (dmd) that are vintaged, where the balance coefficient is applied to the vintaged investment (R_INV) using the associated efficiency (EFF_I). The BALE_CAP is similarly applied to the residual capacity variable (R_RESIDV) using EFF_R.
BALE_CZY$_{11P}$ (t,cpd,elc,w)	I	The amount of electricity (OUT(ELC)_TID) produced (R_TCZYH) in a time period by a coupled production pass-out turbine according to the heat generation in each time-slice (adjusted for base load (bas) plants appropriately) by applying the electricity/heat relationship parameters (ELM, CEH(Z)(Y)). Any transmission losses (TE(ENT)) are taken into consideration.
BALE_EZY$_{11Q,38Q}$ (t,ela,elc,w)	I	The amount of electricity (OUT(ELC)_TID) produced (R_TEZY) in a time period by an electric plant in each time-slice (QHR(Z)(Y), adjusted for base load (bas) and peaking (PEAK(CON)_TID) plants appropriately). Any transmission losses (TE(ENT)) are taken into consideration for centralized (cen) plants. The parameter also takes account of consumption of electricity (INP(ENT)c) by storage (stg) and grid link (lnk) technologies.
BALE_HPL$_{11R,38R}$ (t,hpl,elc,w)	I	The amount of electricity consumed (INP(ENT)p) in a time period and season (QHR(Z)) by heating plants as a function of the heat produced (R_HPL).
BALE_SEP$_{11T,38T}$ (t,s,w)	I	The import/export of electricity, taking into consideration the time-slice fractions (FR(Z)(Y)(ELC)), as well as losses (TE(ENT)) during importing.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
BALH_CAP$_{10F,38F,38P}$ *BALH_VDI* (t,t,p,lth,w)	I	The amount of low-temperature heat consumed (MA(ENT)) in a time period by a demand device (dmd) based upon the activity from total installed capacity (R_CAP). The capacity factor (CF), efficiency (EFF) and capacity-to-activity factor (CAPUNIT), as well as the split to the demand (OUT(DM)) and associated season load shape (FR(Z)(Y)), are applied. In addition, the seasonal distribution efficiency (DHDE(Z)) is applied. Generation of heat (OUT(LTH)_TID) from externally load managed power plants (xlm) is accounted for according to the appropriate capacity factor (CF(Z)(Y)) and capacity-to-activity factor (CAPUNIT) and time-slice duration (QHR(Z)(Y)). The BALH_VDI instances apply only to demand devices (dmd) that are vintaged, where the balance coefficient is applied to the vintaged investment (R_INV) using the associated efficiency (EFF_I). The BALH_CAP is similarly applied to the residual capacity variable (R_RESIDV) using EFF_R.
BALH_TCZY$_{10P,38P}$ (t,cpd,lth,w)	I	The amount of heat (OUT(ELC)_TID) produced (R_TCZYH for pass-out turbine, R_TEZY for back-pressure turbines) in a time period by a coupled production plant in each time-slice (adjusted for base load (bas) appropriately) by applying the electricity/heat relationship parameters (ELM, CEH(Z)(Y). Any seasonal transmission losses (TRNEFF(Z)(Y)) are taken into consideration for the centralized (cen) plants.
BALH_THZ$_{10R,38R}$ (t,hpl,lth,z)	I	The amount of heat (OUT(LTH)_TID) produced (R_THZ) in a time period by a heating plant in a season (QHR(Z), adjusted for base load (bas) and peaking (PEAK(CON)_TID) plants appropriately). Any transmission losses (TE(ENT)) are taken into consideration for centralized (cen) plants.
BAS_CAP$_{12F}$ (t,tch,elc,w)	I	Reflects the base load (bas) and non-base load power plant contribution (BASELOAD) to the base load constraint (MR_BAS) for externally load managed (xlm) plants based upon the total installed capacity (R_CAP), the capacity factor (CF(Z)(Y)) and capacity-to-activity factor (CAPUNIT), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).
BAS_TCZY$_{12P}$ (t,cpd,elc,w)	I	Reflects the base load (bas) and non-base load contribution (BASELOAD) to the base load constraint (MR_BAS) from coupled heat and power (cpd) plants based upon the generation (R_TEZY for back-pressure and R_TCYZH for pass-out), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).
BAS_TEZY$_{12Q}$ (t,ela,elc,w)	I	Reflects the base load (bas) and non-base load contribution (BASELOAD) to the base load constraint (MR_BAS) from electricity (ela) plants based upon the generation (R_TEZY), as well as the time-slice duration (QHR(Z)(Y)) and any transmission losses (TE(ENT)).

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
BAS_SEP$_{12T}$ (t,s,z)	I	Reflects the base load ('IMP') and non-base load ('EXP') contribution (BASELOAD) to the base load constraint (MR_BAS) from electricity exchanges based upon the level of resource activity (R_TSEP), as well as the time-slice duration (FR(Z)(Y)(ELC)) and any transmission losses (TE(ENT)).
BI_TRDCST$_{7T,32T}$ (r,s)	U	The addition cost, or transaction cost, to be applied to bi-laterally traded commodities (BI_TRDENT) according to the level of resource activity (R_TSEP).
BI_TRDCSTE$_{7U,32U}$ (r,s,w)	A	The addition cost, or transaction cost, to be applied to bi-laterally traded electricity (BI_TRDELC) according to the level of exchange in a time-slice (R_TSEPE).
BND_TCZY$_{15P}$ (cpd,w)	I	The electric portion of the annual bound (BOUND(BD)O) to be applied to the output from a coupled heat and power pass-out turbine (R_TCZYH), taking into consideration ELM, CEH(Z)(Y) and base load (bas) characteristics if appropriate. Both it and the heat portion are subject to the total annual production bound (MR_BNDCON).
BND_TEZY$_{15Q}$ (ela,w)	I	An indicator of when a time-slice electricity production variable (R_TEZY) is to be generated taking into consideration base load (bas), peaking (PEAK(CON)_TID), storage (stg), and externally load managed (xlm) characteristics of a conversion technology. It thereby controls the actual time-sliced variables summed in the annual production constraint (MR_BNDCON) according to the limit (BOUND(BD)O).
BND_THZ$_{15R}$ (hpl,z)	I	An indicator of when a seasonal heat production variable (R_THZ) is to be generated taking into consideration peaking (PEAK(CON)_TID), and externally load managed (xlm) characteristics of a heating plant. It thereby controls the actual seasonal variables summed in the annual production constraint (MR_BNDCON) according to the limit (BOUND(BD)O).
COST_INV$_{7J,7N}$ (t,t,p)	I	Technology annualized investment costs, where all the conditions associated with calculating the full lump-sum investment costs have already been taken into consideration. Thus COST_INV distributes these costs annually according to the capital recovery factor (CRF) and the lifetime (LIFE) of the technology, including any fraction of a period (FRLIFE). Note that COST_INV has dual period indexes where the first represents the initial period in which the investment took place, and the second is the current period for which the payment is being calculated.
CPT_CAP$_{16F}$ (t,p)	I	An indicator of from when a technology is available (START) and a capacity transfer constraint (MR_CPT) is needed to accumulate the total installed capacity (R_CAP). The latter is the case whenever a lifetime (LIFE) is provided.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
CPT_INV$_{16J}$ (t,t,p)	I	An indicator of from when a technology is available (START) for new investment and a capacity transfer constraint (MR_CPT) is needed (LIFE provided), as well as the amount of the investment (R_INV) available in the period. The latter is 1 in all periods from the installation period until the last period, where it may reflect that only a fraction (FRLIFE) is available owing to the lifetime not being a multiple of the period length (NYRSPER).
CUM_TSEP$_{17T}$ (t,s)	I	Indicator that a resource option is subject to a cumulative resource limit (CUM). Set to the number of years per period (NYRSPER), except stockpiles ('STK'), to drive the cumulative resource constraint (MR_CUM).
DD_MA$_{28D}$ (dmd,e)	S	A mapping set that indicates which commodities are consumed (MA(ENT/MAT), MA(ENT/MAT)_TID) by a demand device.
DEM_CAP$_{20F}$ (t,dmd,d)	I	The amount of useful energy demand services (dm) delivered from a demand device, taking into consideration the capacity factor (CF) and capacity-to-activity factor (CAPUNIT), as well as the split for devices serving more than one demand sector (OUT(DM)).
DM_DEM$_{20D,20G,42G}$ (d,t)	A	The projected demand for useful energy services that must be met (MR_DEM right-hand-side) in the reference run, and from which the elastic demands (R_ELAST) move in response to price.
DM_ELAST$_{7G}$ (d,b,t)	A	The elasticity (MED-ELAST) associated with a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and objective function (EQ_PRICE if single region, and MR_OBJ if multi-region).
DM_STEP$_{7G,42G}$ (d,b)	A	The number of divisions of the demand curve (MED-STEP) to be used for a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and objective function (EQ_PRICE if single region, and MR_OBJ if multi-region).
DM_VAR$_{7G,42G}$ (d,b,t)	A	The variation (MED-VAR) associated with a flexible demand (R_ELAST) in the demand constraint (MR_DEM) and objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
DMBPRICE$_{7G}$ (d,t)	A	The marginal price of meeting the demand in the reference run. Used for determining the change in cost (DM_ELAST,DM_STEP,DM_VAR) associated with moving along the demand curve (R_ELAST) in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
EM_BOUND$_{42H}$ (v,t)	U	The bound on annual emissions (R_EM).
ENV_COST$_{7H,32H}$ (v,t)	U	The tax/subsidy to be applied to an emission (R_EM) in the regional objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
ENV_CUM$_{21Z}$ (v)	A	The cumulative limit (ENV_CUMMAX) on the production of an emission indicator over the entire modeling horizon (MR_CUM).

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
ENV_GWP$_{35H}$ (v,v,t)	U	The coefficient that is used to convert one emission variable (R_EM) into another one. An example is the global warming potential of say methane emissions, which converts one unit of methane into one unit of CO2 equivalent.emission.
ENV_SCAL$_{21H,21Z,23H,24H,35H}$ (v)	U	A multiplier that is applied to scale all emissions variables (R_EM), turned to if the marginals are only being reported as 'EPS'.
EPK_ACT$_{22E}$ (t,t,prc,elc,z)	I	The amount of electricity required (INP(ENT)p) by a process (prc) activity (R_ACT) that occurs during the peak time (PEAKDA(PRC)), taking into consideration any lead requirements (LED) and night storage (nst) operation in the peaking constraint (MR_EPK).
EPK_CAP$_{22F}$ *EPK_VDI* (t,t,p,elc,z)	I	The amount of electricity (MA(ENT)) required (CF, EFF, CAPUNIT) by a demand device (dmd) activity (R_CAP) taking into consideration the shape of the load being served (FR(Z)(Y), OUT(DM)), as well as the peaking coincidence of the demand (ELF) and the length of the season (QHRZ), ignoring night storage (nst) devices. The electricity consumed by processes (INP(ENT)p) that are externally load managed (xpr) taking into consideration the capacity factor (CF) and capacity-to-activity factor (CAPUNIT), as well as LED requirements and peak coincidence (PEAKDA(PRC)). In addition, the parameter handles the contribution (OUT(ELC)_TID) to the peaking requirement (ERESERV) made by the individual power plant's capacity (R_CAP) taking into consideration the peak contribution (PEAK(CON)) capacity-to-activity factor (CAPUNIT), and transmission losses (TE(ENT)), as well as inhibiting for peaking only plants (PEAK(CON)_TID, forcing activity) in the peaking constraint (MR_EPK). The EPK_VDI instance applies only to demand devices (dmd) that are vintaged, where the peak coefficient is applied to the vintaged investment (R_INV) using the associated efficiency (EFF_I). The EPK_CAP is similarly applied to the residual capacity variable (R_RESIDV) using EFF_R.
EPK_HPL$_{22R}$ (t,hpl,elc,z)	I	The amount of electricity required (INP(ENT)c) by a heating plant (hpl) activity (R_THZ) that occurs during the peak time (PEAKDA(PRC)), taking into consideration the length of the season (QHRZ) in the peaking constraint (MR_EPK).
EPK_SEP$_{22T}$ (t,s,elc,z)	I	The amount of electricity (INP(ENT)x) exported (R_TSEP) by a resource option (srcencp) that occurs during the peak time (PEAKDA(SEP)), taking into consideration any restrictions on the amount of the annual total available in a season (FR(Z)(Y)(ELC)). The parameter also covers the amount of electricity imported (OUT(ENT)r) by a resource option (srcencp) that occurs during the peak time (PEAKDA(SEP)), taking into consideration any restrictions on the amount of the annual total available in a season (FR(Z)(Y)(ELC)) in the peaking constraint (MR_EPK).

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
EPK_TEZY$_{22Q}$ (t,t,ela,elc,w)	I	The contribution (OUT(ELC)_TID) to the peaking requirement (ERESERV) from peaking only plants (PEAK(CON)_TID) according to activity (R_TEZY) that occurs at peaking time (PD(Z)D) and the amount of the output to credit (PEAK(CON)), taking into consideration any efficiency loss (TE(ENT)) in the peaking constraint (MR_EPK).
GobjZscal$_{32M}$	U	A scalar applied to the regional total system cost to scale the entire model for the regional objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
HPKW_ACT$_{28R}$ (t,con,lth)	I	The contribution (OUT(LTH)_TID) to the peaking requirement (HRESERV) from peaking only plants (PEAK(CON)_TID) according to activity (R_THZ) that occurs at peaking time (PD(Z)D) and the amount of the output to credit (PEAK(CON)) according to the season length (QHRZ) in the peaking constraint (MR_HPKW).
HPKW_CAP$_{29F}$ *HPKW_VDI* (t,con,lth)	I	The amount of heat (MA(ENT)) required (CF, EFF, CAPUNIT) by a demand device (dmd) activity (R_CAP) taking into consideration the shape of the load being served (FR(Z)(Y), OUT(DM)), as well as the distribution losses (DHDE(Z)) and the length of the season (QHRZ), ignoring night storage (nst) devices. In addition, the parameter handles the contribution (OUT(LTH)_TID) to the peaking requirement (HRESERV) made by the individual power plants capacity (R_CAP) taking into consideration the peak contribution (PEAK(CON)), capacity-to-activity factor (CAPUNIT), and transmission losses (TE(ENT)), as well as inhibiting for peaking only plants (PEAK(CON)_TID, forcing activity). These demands and capacity contributions are “balanced” in the peaking constraint (MR_HPKW). The HPKW_VDI instance applies only to demand devices (dmd) that are vintaged, where the peak coefficient is applied to the vintaged investment (R_INV) using the associated efficiency (EFF_I). The HPKW_CAP is similarly applied to the residual capacity variable (R_RESIDV) using EFF_R.
HPKW_CPD$_{29F}$ (t,cpd,lth)	I	The amount of heat (OUT(LTH)_TID) related capacity available from coupled heat and power plants (R_TEZY for back-pressure, R_TCZYH for pass-out), taking into consideration operational characteristics (CEH(Z)(Y), ELM, REH), reserve margin (HRESERV) and transmission efficiency (TRNEFF(Z)(Y)) that contributes to the peaking requirement (MR_HPKW).
HPKW_HLK$_{28R}$ (t,hlk,lth)	I	The consumption (R_THZ) of heat (INP(ENT)c) by grid links (hlk) taking into consideration distribution efficiency (DHDE(Z)) and seasonal fraction (QHRZ) in the peaking constraint (MR_HPKW).
NYRSPER$_{21H,23H,35J}$	I	The number of year per period, as determined by the difference between the first and second years of the model.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
PRC_LIM$_{23D,23E,23F}$ (prc,t)	U	The overall efficiency of a flexible output process, applied to the activity variable (R_ACT) to ensure that the sum of all the individual commodity outputs (R_LOUT) is properly limited in the limit equation (MR_LIM).
PRC_OUT$_{29D,29F}$ (prc,enc,t)	U	The maximum share (OUT(ENC)p) that is applied to the overall process activity (R_ACT) to limit the individual commodity flows leaving such processes (R_LOUT) in the process operation equation (MR_PBL).
PRI_ACT$_{32E}$ (t,prc)	I	Same as ANC_ACT, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_CAP$_{32F}$ *PRI_CAPR* *PRI_VDI* (t,p)	I	Same as ANC_CAP, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_DF$_{32H, 32I,32O,32T, 32U,32V}$ (t)	I	Period discount factor (see MR_OBJ in the Section 2.5 for details).
PRI_INV$_{32J}$ (t,tch)	I	Lump-sum discounted (by applying PRI_DISC, that takes into consideration any technology specific "hurdle" rate (DISCRATE)) investment for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region), and taking into consideration any salvage associated with installed new capacity still available at the end of the modeling horizon (see MR_OBJ in the Section 2.5 for details).
PRI_TCZY$_{32P}$ (t,cpd)	I	Same as ANC_TCZY, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_TEZY$_{32Q}$ (t,ela)	I	Same as ANC_TEZY, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_THZ$_{32R}$ (t,hpl)	I	Same as ANC_THZ, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_TSEP$_{32T}$ (t,e,c)	I	Same as ANC_TSEP, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
PRI_ZSTK$_{32W}$ (t,enc,c)	I	Same as ANC_ZSTK, above, except discounted (by applying PRI_DF) for the MARKAL objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
QHR$_{,22D,22U,}$ (w)	U	The duration of each time-slice as a fraction of the year. Where a QHR instance is omitted for a time-slice, no equations or variables are generated for this time-slice.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
QHRZ$_{10D,11D,11T,12D,\ 36D,\ ,37D}$ (z)	I	The duration of each season as a fraction of the year, based upon the individual time-slice splits (QHR). Where both the day and night QHR instances are omitted for a season, no equations or variables are generated for this season.
RAT_ACT$_{6E}$ (a,p,t)	U	The multiplier to be applied to the activity variable (R_ACT, R_TCZYH, R_TEZY, R_THZ) in a user-defined constraint (MR_ADRAT).
RAT_CAP$_{6F}$ (a,p,t)	U	The multiplier to be applied to the capacity variable (R_CAP) in a user-defined constraint (MR_ADRAT).
RAT_HPL$_{6R}$ (a,hpl,z,t)	U	The multiplier to be applied to the activity of a heating plant (R_THZ) in a season for a user-defined constraint (MR_ADRAT).
RAT_INV$_{6J}$ (a,p,t)	U	The multiplier to be applied to the investment variable (R_INV) in a user-defined constraint (MR_ADRAT).
RAT_RTY2$_{6Z}$ (a,b)	A	The indicator specifying the sense (bd) of a user-defined constraint (MR_ADRAT).
RAT_RTY1$_{6D}$ (a,b,t)	A	The right-hand-side value associated with a user-defined constraint (MR_ADRAT), where the equation is only generated for those periods in which a value is provided.
RAT_TCZY$_{6P}$ (a,cpd,w,t)	U	The multiplier to be applied to the activity of a coupled heat and power plant (R_TCZYH, R_TEZY) in a time-slice and for a user-defined constraint (MR_ADRAT).
RAT_TEZY$_{6Q}$ (a,ela,w,t)	U	The multiplier to be applied to the activity of an electric generating plant (R_TEZY) in a time-slice and for a user-defined constraint (MR_ADRAT).
RAT_TSEP$_{6T}$ (a,s,t)	A	The multiplier to be applied to the resource supply variable (R_TSEP) in a user-defined constraint (MR_ADRAT).
REG_XCVT$_{13T14U,27I}$ (r,e)	U	A multiplier applied to a commodity in a region in the trade constraints (MR_BITRD, MR_BITRDE, MR_GTRD) to convert the commodity to a standard common unit, if needed.
REG_XMONY$_{30M}$ (r)	U	A multiplier applied to the costs in a region to convert the regional objective function (MR_OBJ) terms to a standard common unit, if needed.
SEP_BND$_{42T}$ (s,b,t)	A	The resource bound (BOUND(BD)Or) applied directly to limit the annual production from a supply option (R_TSEP) in each period for which a value is provided.
SEP_CUM$_{17D,17Z}$ (s)	A	The cumulative resource limit (CUM) applied as the right-hand-side of the cumulative resource constraint (MR_CUM) to limit the total supply of an option (R_TSEP) over the entire modeling horizon.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
SEP_GRTI$_{25D,25W}$ (s)	A	The "seed" used as the right-hand-side of the resource growth constraint (MR_GRSEP) to set the maximum penetration of a resource (R_TSEP) if it was not used in the previous period.
SEP_STRT$_{18D,25D}$ (s)	A	The initial period from which a resource supply option (R_TSEP) is available.
SEP_TDE$_{18T}$ (s,t)	A	The annual decay of a resource option (R_TSEP) permitted, as controlled by the resource decay constraint (MR_DESEP) in a period when a value is provided.
SEP_TGR$_{25T}$ (s,t)	A	The annual growth of a resource option (R_TSEP) permitted, as controlled by the resource growth constraint (MR_GRSEP) in a period when a value is provided.
SLKCOST$_{7O,32O}$ (e,t)	I	The slack value cost, that also serves as the trigger to change the nature of the commodity balance equation to an equality constraint (MR_BAL_S or MR_BALDH_S) incorporating a slack variable (R_SLKENC, R_SLKLTH). The value is applied to the slack variable in the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
TCH_BND$_{42F}$ (p,b,t)	A	The limit (BOUND(BD)) directly imposed on the total installed capacity (R_CAP) in each period for which a value is provided.
TCH_BNDO$_{15D,15Z,42E,42F}$ (p,b,t)	A	The limit (BOUND(BD)O) imposed on annual activity in each period for which a value is provided. For regular processes (prc) the bound is applied directly to the activity variable (R_ACT). Conventional conversion plants are limited annually by means of a constraint (MR_BNDCON) that sums the individual time-slice activity variables (R_TCZYH, R_TEZY, R_THZ). For externally load managed technologies (xlm/xpr) as applied to the capacity (R_CAP) taking into consideration the capacity factor (CF) and capacity-to-activity factor (CAPUNIT).
TCH_CAPU$_{31F,42F}$ (p,t)	A	The capacity-to-activity conversion factor to ensure that the units of annual production activity match that of the overall model. When set to unity the units of capacity and production are identical. Traditionally used to allow conversion technology (con) capacity to be in terms of kilowatts/gigawatts while annual production is defined in terms of petajoules.
TCH_CF2$_{29F,31F,42F}$ (p,w,t)	A	The fixed capacity utilization factor (CF/CF(Z)(Y)) controlling the activity of demand devices (dmd) and externally load managed technologies (xlm, xpr) according to the total installed capacity (R_CAP). The seasonal CF(Z)(Y) takes precedence over the annual CF, with the latter assigned to the former when the time-sliced values are not provided for conversion plants. Also, the maximum availability factor (AF/AF(Z)(Y)) is assigned to the capacity factor for externally load managed technology when it is provided and the CFs are not.

Matrix Controls & Coefficients (indexes)[60]	Type[61]	Description & Calculations
TCH_GRTI$_{26D,26Z}$ (p)	A	The "seed" used as the right-hand-side of the technology growth constraint (MR_GRTCH) to set the maximum penetration of capacity (R_CAP) if it was not used in the previous period.
TCH_IBND$_{42J}$ (p,b,t)	A	The limit (IBOND(BD)) directly imposed on investments in new capacity (R_INV) in each period for which a value is provided.
TCH_LIFE$_{16D}$ (p)	A	The number of periods (can be fractional, e.g., 3.5) that a technology is available (LIFE) from the year of initial installation (R_INV), where the number of years per period (NYRSPER) is determined in the code from the first two periods of the model run. Used to control the need for and longevity of new investment in the capacity transfer equation (MR_CPT). The LIFE parameter also impacts the annualized cost of new investments as embodied by the capital recovery factor (CRF).
TCH_RES$_{16D,16Z,42N}$ (p)	A	The amount of pre-existing or residual capacity (RESID) originally available prior to the modeling horizon and still available in the current period. The RESID parameter serves as the right-hand-side of the capacity transfer equation (MR_CPT).
TCH_STRT$_{19D,26D}$ (p)	A	The period from which a technology is available (START), controlling when equations and variables are permitted. It is imbedded into almost all the other technology related internal parameters so that it does not need to be tested repeatedly throughout the code.
TCH_TDE$_{19F,19Z}$ (p,b,t)	A	The annual decay of installed capacity (R_CAP) permitted, as controlled by the technology decay constraint (MR_DETCH) in a period when a value is provided.
TCH_TGR$_{26F,26Z}$ (p,b,t)	A	The annual growth of installed capacity (R_CAP) permitted, as controlled by the technology growth constraint (MR_GRTCH) in a period when a value is provided.
TENV_ACT$_{35E}$ (t,p,v)	I	Amount of emissions (ENV_ACT) discharged, or reduced (if negative), per unit of output from a process (prc) and credited into the annual emissions constraint (MR_TENV). If the coefficient ENV_ACT was originally provided for a conversion plant (con) or demand device (dmd) it is assigned to an associated internal parameter (TENV_CAP / TENV_TCZY / TENV_TEZY / TENV_THZ).
TENV_CAP$_{35F}$ (t,p,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of installed capacity (R_CAP) as tracked in the annual emissions constraint (MR_TENV). If the technology is a demand device or is externally load managed (xpr/xlm) and ENV_ACT is specified then it is assigned to ENV_CAP applying the appropriate capacity conversion factors (CAPUNIT, CF/CF(Z)(Y)) , and time-slice fraction (QHR(Z)(Y)) if a power plant.
TENV_INV$_{35J}$ (t,p,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of new investment in capacity (R_INV) as tracked in the annual emissions constraint (MR_TENV).

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
TENV_TCZY$_{359P}$ (t,cpd,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of pass-out turbine production of heat (R_TCZYH) as tracked in the annual emissions constraint (MR_TENV) by applying the heat ratio factor CEH(Z)(Y)/ELM. It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.
TENV_TEZY$_{35Q}$ (t,ela,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of electricity production (R_TEZY) as tracked in the annual emissions constraint (MR_TENV). It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.
TENV_HZ$_{35R}$ (t,hpl,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of heat production (R_THZ) as tracked in the annual emissions constraint (MR_TENV). It is set from ENV_ACT ensuring that matrix intersections only occur for permitted technologies, periods and time-slices.
TENV_SEP$_{35T}$ (t,s,v)	I	Amount of emissions discharged, or reduced (if negative), per unit of resource activity. Specifying ENV_SEP results in the associated resource supply variable (R_TSEP) entering the equation tracking annual emissions (MR_TENV) with the coefficient applied.
TEZY_CAP$_{36F}$ (t,ela,w)	I	The amount of total installed capacity (R_CAP) available in a time-slice (AF/AF(Z)(Y)), taking into consideration any forced outage (AF_TID); as well as base load (bas), storage (stg) and peaking only (PEAK(CON)_TID) attributes. With regard to the latter the peak duration (PD(Z)(D)) is used in place of the availability.
TEZY_EMT$_{36Q}$ (t,cpd,w)	I	Maintenance indicator (AF_TID) forcing the inclusion of a forced maintenance variable (R_M) in the utilization constraint (MR_TEZY).
TEZY_TCZY$_{36P}$ (t,cpd,w)	I	An indicator for pass-out turbine production of heat (R_TCZYH) electric output (CEH(Z)(Y)/ELM) in a particular period and time-slice, taking into consideration base load (bas), as it contributes to the utilization constraint (MR_TEZY).
TEZY_TEZY$_{36Q}$ (t,cpd,w)	I	An indicator that a conventional conversion plant (non-xlm) is permitted to run (R_TEZY) in a particular period and time-slice in the conversion plant utilization equation (MR_TEZY), taking into consideration base load (bas), storage (stg), and peaking only (PEAK(CON)_TID) attributes.
THZ_CAP$_{37F}$ (t,hpl,z)	I	The amount of the total installed capacity (R_CAP) available in a season (AF/AF(Z)(Y)) and period in the conversion plant utilization equation (MR_TEZY), taking into consideration any forced outage (AF_TID); as well as peaking only (PEAK(CON)_TID) attributes. With regard to the later the peak duration (PD(Z)(D)) is used in place of the availability.

Matrix Controls & Coefficients (indexes) [60]	Type[61]	Description & Calculations
THZ_THZ$_{39R}$ (t,hpl,z)	I	An indicator that a conventional heating plant (non-xlm) is permitted to run (R_THZ) in a particular period and season in the heating plant utilization equation (MR_THZ), taking into consideration base load (bas), storage (stg), and peaking only (PEAK(CON)_TID) attributes.
TRD_BND$_{42I}$ (r,g,ie,b,t)	A	The bound on the import/export of a globally traded commodity (R_GTRD) in a region/period.
TRD_COST$_{7I,7T,32I,32T,28I,28S}$ (r,e/v)	A	An additional (transaction) cost applied to traded commodities (R_GTRD, R_TSEP, R_TSEPE) in the regional objective function (MR_OBJ).
TRD_FROM$_{27D}$ (r,e/v)	A	The year from which trade in a global commodity is permitted to take place.
TSUB_BND$_{42V,28U,32D}$ (txs,t)	U	Bound applied to a tax/subsidy as the total tax/subsidy permitted in a period.
TSUB_COST$_{7V,32V,28U,32D}$ (txs,t)	U	Cost of a tax/subsidy applied to selected energy carriers and the technologies producing and/or consuming it.
TSUB_SEP$_{38T}$ (txs,s,t)	A	Contributions of a resource supply option to a tax/subsidy, where the value is applied to the level of resource activity.
TSUB_TCH$_{38E,38F,38J,38P,38Q,38R}$ (txs,p,t)	A	Contribution of a technology to a tax/subsidy, where the value is applied to the amount of the associated commodity produced/consumed.
UPRC_ACT$_{40E}$ (t,prc)	I	An indicator that a conventional process (non-xpr) is permitted to run (R_ACT) in a particular period, and thus appear in the process utilization equation (MR_UTLPRC).
UPRC_CAP$_{40F}$ (t,prc)	I	The amount of the total installed capacity (R_CAP) available in a period (AF) in the process utilization equation (MR_UTLPRC).
ZPR$_{d0D}$ (prc)	S	The expanded list of externally load managed processes (xpr) plus those process with a short lifetime (LIFE <= 1 period).

2.3.2.2 Other Key Internal Parameters

Besides the parameters presented in the previous two Tables, there are several other internal parameters that are central to the calculation of key other parameters. These are listed in Table 2-11 below.

Table 2-11. Definition of other key internal parameters

Internal Parameter (indexes)	Related Parameters	Description
CRF (p)	DISCOUNT DISCRATE LIFE	The capital recovery factor for each technology; based upon the applicable discount rate, the technology lifetime, and the number of years per period. The CRF is calculated as *x=1/(1+DISCOUNTorDISCRATE), and then* $CRF=\{1-x\}/\{1-x^{LIFE}\}$.
CRF_RAT (p)	CRF DISCOUNT LIFE	The ratio of the capital recovery factor for technologies with "hurdle rates," calculated as {the technology CRF} / {the CRF using the standard system-wide discount rate}.
FRACLIFE (p)	COST_INV PRI_DISC	The fraction of a period that an investment is available. This is 1 for all periods up until the period from installation until the lifetime (LIFE), and then (number of years in the last period of existence / number of years per period (NYRSPER)) if LIFE is not a multiple of NYRSPER.
FLLIFE (p)	COST_INV	Last period during which a technology is available based upon its LIFE.
FRLIFE (p)	COST_INV LIFE	Adjustment to the capital recovery factor (CRF) if the lifetime (LIFE) is not a multiple of the number of years per period (NYRSPER).
INV_INV (t,p)	PRI_INV	The lump-sum investment cost (INVCOST), augmented for any additional costs associated with the electricity or heat transmission (ETRANINV/DTRANINV) or distribution (EDISTINV) for the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
NYRSPER	TP	The number of years per period. As MARKAL permits only periods of equal length the value of NYRSPER is determined by the number of years between the first two periods provided for the set YEAR.
PRI_DISC (t,p)	CRF_RAT FRACLIFE	The spread discounting of the lump-sum investment (R_INV) for the objective function (EQ_PRICE if single region, MR_OBJ if multi-region). Calculated by taking into consideration any technology specific discount rate (DISCRATE, but reflected in CRF_RAT) and any fraction of the period for lifetimes not a multiple of the period length (FRACLIFE) as $PRI\_DISC=(1/((1+DISCOUNT)^{(-STARTYRS+NYRSPER*(ORD(TP)-1))))} * CRF\_RAT * FRACLIFE$.
SALV_INV (t,p)	PRI_INV	The amount of remaining investment salvaged, that is credited against the lump-sum cost investment charge for the objective function (EQ_PRICE if single region, MR_OBJ if multi-region).
SIGN (src)		For EXPorts = -1, for the other four resources options (IMPort, MINing, ReNeWable, StocKpile) = +1.
T04/E/H/O (t,p,*)	BAL/BALE/ BALH_*	Electricity and heat production by plant, along with production of auxiliary commodities, for the VEDA reports.

Internal Parameter (indexes)	Related Parameters	Description
T08/ T08ENT (e,p,t)/(e,p)	BAL_*	Energy/material consumption by technology for the VEDA reports.
T27V (t,p,v)	TENV_*	Emissions activity by technology and resource for the VEDA reports.

2.4 Variables

This section details the role of each primal decision variable of the MARKAL Linear Program. These variables, along with the sets and parameters described in previous sections, enter the mathematical expressions that constitute the MARKAL constraints that are listed and explained in section 2.5. The primal decision variables of a Linear Program are often referred to as *Columns*, whereas constraints or Equations are referred to as *Rows*. This terminology stems from the position of these two types of entity in the matrix of the Linear Program. For the same reason, the phrase *Row/Column Intersection* is often used to designate the coefficient of a single MARKAL variable in a particular constraint. A fairly complete rendition of the MARKAL LP matrix is provided in Appendix A, and is available as an Excel spreadsheet.

The same syntax conventions used in the earlier parts of Chapter 2 are used here as well. Namely:

- Sets, and their associated index names, are in lower case, usually within parentheses;
- Literals, values explicitly defined in the code, are in upper case within single quotes;[62]
- Parameters, and scalars (un-indexed parameters) are in upper case;
- Equations are in upper case with a prefix of MR_; and
- Variables are in upper case with a prefix of R_.

Table 2-12 lists the indexes referenced in this section. As noted previously in section 2.2 the process of constructing the parameters, variables and equations that control the intersections in the MARKAL Matrix is controlled by subsets of the referenced indexes, particularly with regard to the technology references. For the most part these indexes correspond directly to user input sets discussed in Section 2.2. The reader is referred to that section for a full description of each index, and for an indication of how entries in the sets influence the resulting model.

[62] Examples of literals in the code are:

- 'EXP' and 'IMP' are reserved for exports and imports;
- 'LO', 'FX', and 'UP' identify lower, fixed and upper bound types, and
- 'S', 'I', and 'W' for Summer, Intermediate, and Winter seasons, and 'D' and 'N' for day and night components of the sub-annual timeslices.

Table 2-12. Indexes and associated user sets

Index	Input Set[63]	Description
a	adratio	The names of the user-defined *Ad hoc* constraints (referred to in the code by the set (adratio)) that are used to guide market splits, group limits for technologies, etc.
b	bd	The **B**ounds or right-hand-side constraint types ('LO', 'FX', 'UP', and variations thereof (e.g., 'MIN', 'FIX', 'MAX')).
c	p	The **C**osts, or step indicators, for the resource supply options. Correspond to the last character of each member of the resource supply set (srcencp).
d	dm	The names of useful energy **D**emand sub-sectors.
e	ent/mat	The names of **E**nergy carriers (ent) and materials (mat).
g	trd	The names of **G**lobally traded (as opposed to bi-lateral) commodities ((ent) or (env).
m	mkt_id	The names of **M**arket share group indicators.
p	tch	The names of the technologies or **P**rocesses.
r	reg	The names of **R**egions.
s	srcencp/sep	The names of resource **S**upply options, including those serving as the trade variables into/out of the regions.
t	year/tp	The years for which data is provided, and the (possible) subset thereof of **T**ime periods requested for the current model run.
txs	taxsub	The names of **T**a**X**es/**S**ubsidies imposed on the system.
u	med-step(bd)	The steps employed for the discretization of the price elastic demand curve, and for expressing the loss of **U**tility. Provided for each demand sector for which flexible demands are employed.
v	env	The names of emissions or en**V**ironmental indicators.
w	td/z+y	The (pre-defined) season/time-of-day compound indexes (season (z) plus time-of-day (y)) identifying the time-slice **W**hen an activity occurs as part of producing/consuming electricity.
x	ie	The export/import e**X**change indicator. Note: the definition of the set (ie) is internal to the model and fixed at 'EXP' for exports and 'IMP' for imports. But for any trade variables defined over (ie) there are only variables for the instances for which the user has explicitly provided data.
z	z	The (pre-defined) seasons for heat generation and consumption.

The variables of the MARKAL model correspond to the columns of the LP matrix. Each variable is indexed by region, for a multi-region model, and by the appropriate set of other indexes necessary to fully and uniquely identify the model component. The variables in the model are generally tied to a region (for a multi-region model), a technology or commodity, and a time period[64]. The main variables are used to:

[63] Input Set identifies the user input set/sets corresponding to each 1-character index used in the definition of the matrix components in this chapter.
[64] The only exceptions are the regional total system cost variable with only a region index, the objective function variable without any index, and the tax/subsidy variable with the named tax/subsidy as an index.

- o track the timing of new investments in each technology in each time period;
- o accumulate the total installed capacity of each technology in each time period;
- o determine the activity (output) of each technology in each time period, and time-slice for electricity and heat producing technologies;
- o accumulate the total emission level of each emission in each time period, and
- o account for the change in demand levels of each end-use demand in each time period.

In all models built upon the MARKAL framework technologies (except resources) are portrayed based upon their level of activity, total installed capacity and amount of new investment. Dividing the activity level by the technology's efficiency derives consumption requirements associated with each technology. In MARKAL there are no "consumption" (input flow) variables, except for exports. Depending upon the nature of a technology there are either individual "production" (output) flow variables for the primary output commodities (e.g., imports, electricity/heat from conversion plants, commodities from flexible (LIMIT) processes), or the various outputs are derived from technological activity or capacity levels (e.g., commodities from regular process, ancillary commodities, useful energy services from demand devices). Thus the energy and material balance equations, as well as peaking and base load requirements and the variable operation costs, are constructed by either employing the flow variables where they exist, or more commonly by applying the appropriate coefficients to determine the amount of each commodity into or out of the technologies based upon the activity or on total installed capacity of the technologies. In addition, individual variables track the blending operations carried out within the flexible refinery sub-model (see chapter 1, section 1.4.5).

The variables of the model are listed alphabetically, by the name assigned to them in the MARKAL GAMS code, in
Table *2-13*, with a sequence number in the first column. Column 2 of the Table indicates the variable name and the associated indexes needed to uniquely define the instances of the variable. The last column describes the variable, with particular emphasis on the conditions that trigger the creation of each variable. The modeler is referred to Appendix A[65] for a global view of the variables involved in the model, which constraints they are involved in, and what parameters (and sets) determine the actual coefficient of each row/column intersection. By looking down a variable column of the MARKAL Matrix one can quickly see all the non-empty intersections, thereby identifying the constraints and associated parameters determining the coefficients of that variable in the various constraints. Also, recall that every set and parameter appearing in the MARKAL Matrix is explained in the Sets & Parameters section 2.2, Table 2-10.

Note that in the source code regionalization is handled by using a MR_ prefix for equation names (MR_<root>)[66] and adding a R_ prefix to variable names (R_<root>),

[65] Appendix A's MARKAL matrix is provided as an Excel file.
[66] In a single region model the energy balance equation is named EQ_BAL_G, while for a multi-region run the regionalized equation has the EQ_ prefix replaced by MR_; for a variable the R_ prefix is added to the root variable name, e.g., the single region INV variable is named R_INV for multi-region MARKAL.

and a _R suffix to set and parameter names (<root>_R). During multi-region runs the matrix generator and report writer generate the parameters and process the model results region-by-region. For matrix generation this is accomplished by moving the single region data into parallel multi-region data structures. The multi-region model variables and constraints, and parameters, if needed, are moved into their corresponding single region instances for the reports. These parallel data structures have the identical names as their single region counterparts, but with the appropriate prefix/suffix added to their names and the first index designating the region (via the index reg/r). In all the descriptions of sets and parameters in this chapter the regional suffix is omitted, though it is understood to be part of the associated data structures for multi-region models. Where there is an exception to this rule it is noted, for example if trade options identify both from/to regions.

Table 2-13. Model variables[67,68,69]

VAR Ref	Variable (Indexes)	Variable Description
1	R_ACT (r,t,p)	The activity of a process technology (prc). Note that only process technologies have R_ACT variables. For power sector technologies (con), variables defined for each time-slice are employed instead, and for demand devices (dmd) an activity variable is not generated as the activity of a demand device is derived directly from the associated capacity variable.
2	R_CAP (r,t,p)	The total installed capacity of a technology. This variable represents the total residual capacity plus investments in new capacity made up to and including the current period and that are still available to the model (i.e. whose life has not expired yet).
3	R_ELAST (r,t,d,b,u)	The movement (up or down) of an energy service demand in response to energy service price when running the model with elastic demands.
4	R_EM (r,t,v)	The level of an emission indicator. This variable is free to take positive or negative values.
5	R_GTRD (r,t,e/v, x)	The total amount of a globally traded commodity (energy carrier or material (e)/emission (v)) imported/exported by a region, where x = ‘IMP’/‘EXP’.
6	R_INV (r,t,p)	The addition of new technology capacity (new investment) in a period. The investment is assumed to take place at the beginning of the period, and so is fully available to the model in the period in which the investment takes place.
7	R_LOUT (r,t,p,e)	The amount of an energy carrier or material produced by a flexible (LIMIT) “refinery” or other mixing process (prc) producing multiple commodities that are not necessarily produced in fixed proportions.
8	R_M (r,t,p,z)	The scheduled maintenance in season z for a power plant for which the user has specified that some fraction of annual unavailability is scheduled [AF_TID] and thus not allowed to be optimally scheduled by the model.

[67] The VAR Ref is the sequence number according to the variable’s alphabetic ordering in this chapter. Each variable is also listed in the corresponding order in the MARKAL model matrix spreadsheet presented in Appendix A: Table A

[68] In the Description column references within parentheses refer to (sets), (PARAMETERS), (MR_/R_ model rows/columns), and (‘LITERAL’).

[69] All variables are positive, except for the objective function variable, and the emissions monitoring variables that are free to take positive or negative values.

VAR Ref	Variable (Indexes)	Variable Description
9	R_objZ (r)	The total cost of a MARKAL region, expressed as the discounted present value to some chosen year. For single region models, this expression is minimized by the model, and this is equivalent to maximizing the total surplus (producer plus consumer surpluses). For multi-region models the regional costs are added and their sum constitutes the objective function to minimize (see R_TOTobjZ below).
10	R_RESIDV (r,t,p)	The exogenously specified residual (RESID) level of investment in a technology made prior to the modeling horizon that is still available in a period. The R_RESIDV variable is used to charge the annualized operating, maintenance and fuel costs associated with demand devices (dmd) employing vintage tracking to the objective function. [Note that this variable is not subject to optimization, as it is completely determined by the RESID input parameter.]
11	R_SLKENC (r,t,e) R_SLKLTH (r,t,e,z)	The slack variable added to a commodity balance equation to change the equation to an equality (MR_BAL_S). This change is triggered by the user providing a cost (SLKCOST) to be applied to the variable in the objective function (see the Equations section).
12	R_TCZYH (r,t,p,w)	The amount of low-temperature heat (lth) produced in each time-slice (w) by a coupled heat and power plant (cpd) that is modeled as a pass-out turbine with a flexible electricity/heat relationship (CEH).
13	R_TEZY (r,t,p,w)	The amount of electricity (elc) produced in each time-slice (w) by a power plant (ela).
14	R_THZ (r,t,p,z)	The amount of low-temperature heat (lth) produced in season (z) by a heating plant (hpl).
15	R_TOTobjZ	The MARKAL objective function to be minimized for multi-region models, consisting of the sum of the regional discounted costs.
16	R_TSEP (r,t,s)	The level of resource production, including imports and exports, arising from each step (price level) of the supply curve (srcencp).
17	R_TSEPE (r,t,s,w)	For bi-lateral electricity trade (bi_trdelc, srcencp), the amount of electricity (elc) exchanged in each time-slice (w).
18	R_TXSUB (r,t,txs)	The tax/subsidy level accumulated from all technology/commodity pairs involved in a tax or subsidy (cost per unit of activity) imposed on the energy system.
19	R_ZSTK (r,e,c)	The carry-over from the final time period of stockpiled material (src = 'STK') that needs to be credited to the objective function.

In the rest of this section each of the variables is described in more detail. On the variable sub-header record the variable name and index is given, along with a reference to the associated column in the Appendix A's MARKAL Matrix. It is helpful to either print the MARKAL Matrix or have it open as you use the description of the structure of MARKAL. For each variable the following information is provided.

Description: A short description of the nature of the variable.

Purpose and A brief explanation of why the variable is needed, along with a general description

Occurrence: of the data circumstances and model rules that control the instances of the variable in the various constraints.

Units: The units associated with the variable.

Bounds: An indication of how limits (lower, upper, fixed) can be directly and indirectly applied to the variable.

2.4.1 R_ACT(r,t,p) MARKAL Matrix, Column e

Description: The level of **ACT**ivity of each process technology (prc)[70]. The variable represents the total annual output of the process technology.

Purpose and Occurrence: The activity variable enables tracking of the level of activity associated with each process technology (prc). It determines the production of each of the energy carriers and materials produced by a process technology, as well as the energy and material required by the process technology and any emissions based upon activity. The variable is used to distinguish the running of a process technology from the level of installed capacity, and enables the variable costs to be separated from the fixed.

Under normal circumstances this variable is generated for each process technology for all time periods beginning from the period that the technology is first available (START). However, generation is conditional upon LIFE being provided, and greater than one period in length, and the process not being described as externally load managed (xpr). [In either of these excluded cases the activity is determined directly by substituting the currently installed capacity (R_CAP) times the capacity factor parameter (CF) for the activity variable (R_ACT)]. The activity variable also serves to ensure that the level of each commodity produced by a flexible process (LIMIT, see MR_LIM, MR_PBL), does not exceed its maximum permitted share of the total output. Since the amount of each commodity consumed or produced by a process is a function of the process activity, the R_ACT variable appears in the commodity balance equations (MR_BAL_E/G/S, MR_BALE1/2), as well as contributing to the electricity peaking requirements (MR_EPK). Since R_ACT represents the overall activity of the process, it appears in the objective function (MR_PRICE) to account for the variable operating and maintenance (VAROM) and any delivery (DELIV) costs associated with the process. R_ACT may also appear in user-defined constraints (MR_ADRATn if RAT_ACT), emission (MR_TENV if ENV_ACT) and tax/subsidy (MR_TXSUB if TSUB_TCH) constraints if the associated data is provided.

Units: Usually energy units, most often PJ, but may be any other unit defined by the user for activity of processes (e.g., material). If activity units differ from capacity units, the conversion is specified by the capacity unit parameters (CAPUNIT = capacity unit/activity unit). The same units must be used for bound (BOUND(BD)O) parameters related to the technology activity.

[70] Reminder: demand device (dmd) activity is derived directly from the capacity variable (R_CAP), and conversion plants (con) have time-sliced activity variables (R_TCZYH, R_TEZY, R_THZ).

Bounds: The process activity variable may be directly bounded by specifying an annual activity bound (BOUND(BD)O, where BD = 'LO'/'UP'/'FX'). It may be indirectly limited by bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)), owing to the fact that the activity variable cannot exceed the capacity variable with the associated availability factor applied.

2.4.2 R_CAP(r,t,p) MARKAL Matrix, Column f

Description: The level of total installed **CAP**acity available in each period, defined for each technology (i.e. each process (prc), conversion (con) and demand (dmd) technology).

Purpose and Occurrence: The capacity variable accumulates the total level of capacity in each period as part of monitoring the turnover of capital stock.

The capacity variable is generated for each technology in all time periods, beginning from the period that the technology is first available (START). Its level is a function of the residual capacity (RESID) from technologies deployed prior to the modeling horizon that remain in the current period, plus those investments in new technologies (R_INV) made earlier than or in the current period that have not yet exhausted their useful lifetime (LIFE). The variable appears in the capacity transfer constraint (MR_CPT) that oversees the tracking of vintages and lifetimes of technologies, and all constraints (MR_TEZY, MR_THZ, MR_UTLPRC) that relate the activity of technologies (R_ACT, R_TCZYH, R_TEZY, R_THZ) to installed capacity. R_CAP directly contributes to the electricity and heat peaking constraint (MR_EPK and MR_HPKW). Note also that R_CAP may be used as a surrogate for *activity* level for externally load managed, or rigid, processes (xpr) and power plants (xlm), substituting for activity variables with the appropriate capacity factors (CF/CF(Z)(Y)) applied. Furthermore, as it is assumed that all demand devices (dmd) operate in proportion to the level of the installed capacity according to a fixed utilization factor (derived from CAPUNIT, CF, EFF and MA(ENT)) no activity variable (R_ACT) is created for demand devices, and commodity flows are directly determined from R_CAP. R_CAP thus also enters in the determination of the level of demand services (according to OUT(DM)) provided by a technology to help satisfy the demand constraint (MR_DEM). It appears in a growth constraint (MR_GRTCH) if requested (GROWTH). Since R_CAP represents the total installed capacity of each technology, it also appears in the objective function (MR_PRICE) to account for the fixed operating and maintenance costs (FIXOM) associated with the technology, as well as the variable costs (VAROM + DELIV) for externally load managed and demand device technologies. R_CAP may (as any other MARKAL variable) also appear in a user-defined constraint (MR_ADRATn) and in emission constraints (MR_TENV) if the associated data is provided by the user.

> Note that for vintaged demand devices (having EFF_I, EFF_R for efficiency in place of an overall EFF), residual (R_RESIDV) and investment variables substitute for the capacity (R_CAP) in all commodity related matrix entries, as the efficiency of such devices is tied to its vintage (year of installation) rather than the current period. A consequence of vintaging a demand device is that the model cannot choose to abandon (not operate) a device once it is invested in, c.f. the case for traditional devices represented by R_CAP owing to the =G= nature of the capacity transfer constraint (MR CPT).

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. The same units are used for the investment (R_INV) variable, and for the residual capacity (RESID), and capacity bound (BOUND(BD)) parameters related to the technology.

Bounds: The capacity variable may be directly bounded by specifying a capacity bound (BOUND(BD)), with BD = ‘LO’/‘UP’/‘FX’. It may be indirectly limited by capacity growth rates (GROWTH), or by bounds on investment variables (IBOND(BD)).

2.4.3 R_ELAST(r,t,d,b,u) MARKAL Matrix, Column g

Description: The u^{th} component of the change (increase or decrease) of an **ELAST**ic demand for a useful energy service in response to the associated own price elasticity.

Purpose and Occurrence: The elastic demand variable allows for the step-wise constant representation of the consumer demand curve for each energy service, if that option is selected by the user. The demand in a particular run is equal to the demand in the base case plus the sum of the components of the demand variation, as explained in sub-section 1.6.2.3.

The elastic demand variable appears in the demand equation (MR_DEM) whenever a non-zero number of steps (MED-STEP(BD)) is provided for a direction (‘LO’/ ‘UP’), and in each period for which an elasticity (MED-ELAST(BD)) is provided for that direction. In such a case, the variable also appears in the objective function (MR_PRICE).

Units: PJ, bkmt, or any other unit in which a demand for an energy service is specified.

Bounds: For each direction (b) and step (u) an upper bound is set such that R_ELAST does not exceed an amplitude that is a function of the reference case demand level (DEMAND), a percent variation parameter (MED-VAR), and the desired number of steps (MED-STEP) for the discrete approximation for that direction, according to the formula step limit = (DEMAND)*(MED-VAR)/(MED-STEP).

2.4.4 R_EM(r,t,v) MARKAL Matrix, Column h

Description: The total level of each **EM**ission indicator in each period.

Purpose and Occurrence: The emission tracking variable accumulates the total (net) level of an emission emanating from all sources, adjusted for those leaving the energy system (due to exports or "removal" (e.g., scrubbing, sequestration)).

Emissions indicators (env) are tracked for each indicator in each period for any period in which an emission is produced or absorbed by any technology or resource. Emissions are tied to technologies according to their activity (ENV_ACT), capacity (ENV_CAP), or investment (ENV_INV), and resources according to their activities (ENV_SEP), where corresponding coefficients are applied to the associated variables for each technology/resource emitting, or consuming, the indicator. Emissions may also be tracked by commodity (ENV_ENT), in which case the preprocessor will determine the associated emissions taking into consideration the technical characteristics of each technology producing/consuming such commodities. Since R_EM represents the total level of emissions, it may also appear in the objective function (MR_PRICE) if an emission tax (ENV_COST) is specified.

> Note that the R_EM variable is free to take positive or negative values, unlike most other model variables, which are non-negative by default.

Units: Tons, Thousand tons, or any other unit defined by the analyst as a measure of the emission per unit of activity, installed capacity, or investment. The same units are used if a bound (ENV_BOUND(BD), ENV_CUM, ENV_MAXEM) is imposed. Consistent units must also be used if an emission "tax" is to be applied (ENV_COST).

Bounds: The total level of an emission may be directly bounded by specifying a cap (ENV_MAXEM) or bound (ENV_BOUND(BD)) on annual emissions, or a cumulative limit (ENV_CUM) on an indicator over the entire modeling horizon. For multi-region models, cross-region cumulative or single period constraints (MR_ GEMLIM or MR_GEMLIMT) may also be imposed.

2.4.5 R_GTRD(r,t,e/v,x) MARKAL Matrix, Column i

Description: The total amount of a **G**lobally **TR**a**D**ed commodity (energy carrier/material (e), or emission (v)) imported/exported (x) to/from a region in each time period.

Purpose and Occurrence: The global trade variable permits trade of a commodity (energy, material or emission) between exporting and importing regions without setting up specific trade routes.

R_GTRD only comes into play in multi-region models. The global trade variable enters the commodity balance equations (MR_BAL, MR_BALE, or MR_TENV), the objective function (MR_PRICE) where any transaction cost (TRD_COST) is applied, and the global trade balance equation (MR_GTRD) where the total exported must equal the total imported, for each region. Note that trade starts beginning at the TRD_FROM period.

Units: Units of the commodity traded, e.g., PJ for energy, Tons for material, Tons for emission or emission permit.

Bounds: The amount of a global traded commodity exported/imported from/to a region, in a period, may be limited by the TRD_BND parameter.

2.4.6 R_INV(r,t,p) MARKAL Matrix, Column j

Description: The level of additional capacity (**INV**estment) in each period. The level of the variable represents the total addition of new capacity occurring during the entire period, but is credited and assumed available from the beginning of the period.

Purpose and Occurrence: The R_INV variable tracks investments in new capacity. Having a separate variable for new construction also enables separating the contribution to the total discounted system cost of investment costs from the contribution of other costs. In the case of vintaged demand devices (dmd with EFF_I) the variable is also used to determine the activity of the device.

This variable is generated for each process (prc), conversion (con) and demand (dmd) technology (but not for resource options) in all time periods beginning from the period that the technology is first available (START), whenever the lifetime (LIFE) parameter has been specified or set by the preprocessor. If any cost parameter is provided, but LIFE is not, then LIFE is set equal to 1 period. If the LIFE parameter is not specified, and no cost parameter is provided then the investment variable is not generated. Whenever LIFE is greater than 1 period the investment variable appears in the capacity transfer constraint (MR_CPT) by which the investments are accumulated over time to define the current installed capacity (R_CAP) in each period. Since R_INV represents the investment in new capacity of each technology, it also appears in the objective function (MR_PRICE) to account for the investment costs (INVCOST) associated with the technology. R_INV appears in a user-defined constraint (MR_ADRATn) if RAT_INV is specified), energy or material balance constraints (MR_BAL, MR_BALE if INP(ENT)_TID or MA(ENT)_TID are specified, and emission constraint (MR_TENV) if ENV_INV is specified; as long as LIFE has been provided.

> Note that for a vintaged demand device (having EFF_I and EFF_R for efficiency instead of an overall EFF) residual (R_RESIDV) and investment variables substitute for the capacity (R_CAP) in all commodity related matrix entries. This permits the efficiency of such a device to be tied to its vintage (year of installation) rather than the current period. A consequence of vintaging a demand device is that the model cannot choose to abandon (not operate) a device once it is invested in, c.f. the case for traditional devices represented by R_CAP owing to the =G= nature of the capacity transfer constraint (MR_CPT).

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. Consistent units must be used for the investment (R_INV) variable, and for the residual capacity (RESID), cost (INVCOST, FIXOM), and investment bound (IBOND(BD)) parameters related to the technology.

Bounds: The investment variable may be directly bounded by specifying an investment bound (IBOND(BD)). It may be indirectly limited by bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or activity (BOUND(BD)O).

2.4.7 R_LOUT(r,t,p,e) MARKAL Matrix, Column k

Description: The output flow of an energy carrier (enc) or material (mat) from a flexible (**L**IMIT) multiple **OUT**put process technology (prc).

Purpose and Occurrence: This flow variable tracks output from "limit" processes, permitting the model to determine the level of each output energy carrier or material flow independently of one another.

There is a collection of R_LOUT variables for each flexible process. The output parameters (OUT(ENC)p/OUT(MAT)p) serve to both identify which commodities are produced by the process, as well as indicate the maximum share each commodity can have of the overall output. Note that while the sum of these maximum shares may be more than one, the total of the actual shares (flows) represented by R_LOUT must not exceed the overall efficiency (LIMIT) of the flexible process. Each R_LOUT variable is tied to the activity variable (R_ACT) of such processes according to its maximum share (MR_LIM), with the overall efficiency controlled by the flexible process output balance constraint (MR_PBL), and contributing to the balance equation for each output commodity (MR_BAL).

Units: Most often PJ, but may be any other unit defined by the analyst for activity of processes. Consistent units must be used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The individual flow of each commodity is bounded by the OUT(ENC)p/OUT(MAT)p parameter. The flow may also be indirectly limited by an annual activity bound (BOUND(BD)O) on the process, bounds specified on capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

2.4.8 R_M(r,t,p,z) MARKAL Matrix, Column l

Description: The level of scheduled **M**aintenance in season z for a conversion plant (con) for which the user has specified that scheduled maintenance is an option.

Purpose and Occurrence: The maintenance variable allocates the total annual scheduled maintenance to the different seasons. The non-scheduled maintenance is considered to be forced outage and occurs uniformly throughout the year. The AF_TID parameter specifies how much of the total maintenance may be scheduled by the model, the rest being forced outage.

The variable enters the conversion plant activity (MR_TEZY) and utilization (MR_UTLCON) constraints, and is triggered by the user providing a scheduled outage (AF_TID) less than 1.

Units: Energy units (PJ/a, etc.).

Bounds: None, other than that imposed by the utilization constraint (MR_UTLCON).

2.4.9 R_objZ(r) MARKAL Matrix, Column m

Description: A variable equal to the **Obj**ective function expression for each region (r), embodying the total discounted system cost for that region.

Purpose and Occurrence: The variable appears only in the constraint expressing the objective function (MR_PRICE). For single region models the variable (without the R_ prefix and the region index) is minimized directly, while for multi-region models the regional variables are added to form the final global objective function (MR_OBJ). The regional cost variables may be weighted if different monetary units (REG_XMONY) are employed in some regions.

Units: Million 2000 US$, or any other unit in which costs are tracked. Note that a regional monetary conversion factor (REG_XMONY) may be applied if different monetary units are used for the various regions.

Bounds: None.

2.4.10 R_RESIDV(r,t,p) MARKAL Matrix, Column n

Description: The **RESID**ual capacity of demand devices (dmd) still available during the modeling horizon when **V**intaging is used.

Purpose and Occurrence: The residual variable represents the level of residual capacity still in place in a period for each vintaged (by the user providing EFF_I and/or EFF_R) demand device.

Normally the capacity variable (R_CAP) incorporates the residual capacity level. However, the R_RESIDV variable substitutes for the capacity variable (R_CAP) in the various flow related equations (MR_BAL, MR_BALE, MR_BALDH, MR_EPK, MR_HPK, MR_BAS) so that the associated efficiency (EFF_R) can be reflected, rather than an overall efficiency (EFF) for all installed capacity in a period. It also enables the annual variable charges (VAROM and DELIV) associated with past investments to be accounted for in the objective function, as a function of the efficiency of the residual capacity of vintaged demand devices. The variable is a pure accounting variable fixed explicitly at the level provided in the input data (RESID), and corresponds directly to the constant value set as the right-hand-side of the capacity transfer constraint (MR_CPT).

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent capacity. Consistent units must be used for the investment (R_INV) variable, and for the residual capacity (RESID), cost (INVCOST, FIXOM), and capacity bound (BOUND(BD)) parameters related to the technology.

Bounds: The variable is fixed explicitly at the level provided in the input data (RESID).

2.4.11 R_SLKENC(r,t,e)/R_SLKLTH(r,t,e,z) MARKAL Matrix, Column o

Description: The **SL**ac**K** variable added to a commodity balance equation associated with an energy carrier (either enc or lth) to change the equation to an equality.

Purpose and Occurrence: A slack variable added to the energy balance equation when the constraint type is changed from an inequality (=G=) to an equality (=E=).

The variable also appears in the objective function (MR_PRICE) for each period and each energy carrier for which a slack penalty (cost) is provided (SLKCOST), and the associated equality slack energy balance constraint (MR_BAL_S), rather than the conventional energy carrier constraint (MR_BAL_G). The SLKCOST serves as the trigger to change the sense of the balance equation.

> While the R_SLKENC/LTH variable is handy during the debugging stages of constructing a model, under normal circumstances the variable should have zero level in the solution. If non-zero it is an indication that some fixed output process(es) are forcing overproduction of the commodity. Therefore, the level of these variable(s) in the solution should be checked to ensure that they are zero. As a corollary, the value of SLKCOST should be rather large.

Units: Energy units.

Bounds: None.

2.4.12 R_TCZYH(r,t,p,w) MARKAL Matrix, Column p

Description: The amount of low-temperature **H**eat (lth) produced in a **T**ime period by a **C**oupled heat and power (cpd) pass-out turbine in each time-slice w. Such power plants have the flexibility to trade-off the amount of electricity versus heat produced (CEH(Z)(Y)), by controlling how much steam is sent to the turbines as opposed to the heat grid.

Purpose and Occurrence: The flow variable tracks the amount of low-temperature heat produced, and determines the amount of electricity, from a flexible pass-out turbine in each time-slice, thereby establishing the overall activity of such technologies.

For a coupled heat and power pass-out turbine there is a R_TCZYH heat variable for each time-slice (w=td(z,y)). The variable represents the amount of heat produced, and thus appears in all low-temperature heat related constraints (MR_BALDH, MR_HPK, for the connected grid (OUT(LTH)_TID)), and to determine the amount of energy consumed per unit production (MR_BAL). The activity is derived directly from the current installed capacity (R_CAP) by applying an availability factor (AF) that limits the plant's maximum output (MR_TEZY) in each time-slice. Once the amount of heat that is to be produced from the plant is determined, the R_TCZYH variable is used as a surrogate for the electricity production from the power plant in all the electricity-related constraints (MR_BALE, MR_BAS for the connected grid (OUT(ELC)_TID, and MR_EPK if peak contribution is a function of the plant activity rather than capacity via PEAK(CON)_TID) since it is a direct function of the amount of heat production. As R_TCZYH defines the overall activity of the technology it also appears in the objective function (MR_PRICE) to represent the variable operating and maintenance costs (VAROM) and any commodity related delivery charges (DELIV). R_TCZYH may also appear in a user-defined constraint (MR_ADRATn if RAT_TCZY or RAT_ACT), emission (MR_TENV if ENV_ACT) and tax/subsidy (MR_TXSUB if TSUB_TCH) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for the activity of conversion technologies. Consistent units must be used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in constraint (MR_BNDCON) to sum the time-sliced activity variables. Indirect limits may be imposed on the activity of pass-out turbines if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

2.4.13 R_TEZY(r,t,p,w) MARKAL Matrix, Column q

Description: The amount of electricity (elc) produced in a **T**ime period by an **E**lectricity generating plant (ela) in each time-slice w (where w = td(**z**,**y**)).

Purpose and Occurrence: R_TEZY(r,t,p,w) is the equivalent of the R_ACT(r,t,p) activity variable defined for process technologies, but the latter is only defined annually whereas R_TEZY(r,t,p,w) is defined for each time-slice. It is defined for all electricity generating facilities (electric only (ele), coupled heat and power (cpd) back-pressure turbines and storage plants (stg)), and thus depicts the activity of such technologies.

The variable represents the amount of electricity produced, and thus appears in each electricity related constraint (MR_BALE and MR_BAS) for the connected grid (OUT(ELC)_TID) and MR_EPK if peak contribution is a function of the plant activity rather than capacity via PEAK(CON)_TID). This variable also determines the amount of energy required (in the commodity balance equation MR_BAL) per unit of production. For a standard power plant the activity is upper bounded by the current installed capacity (R_CAP), by applying an availability factor (AF/AF(Z)(Y)) that limits the plant's maximum output in each time-slice. If the power plant is defined as base load (bas) there is only a single daytime variable (y='D') created, and this same variable is used for both the day and night (y='N') to ensure that the plant operates at the same level day and night. For an externally load managed (or rigid) plant the R_TEZY is not generated, but instead the user-provided capacity factor (CF/CF(Z)(Y)) is applied to the capacity variable (R_CAP) to determine the activity in each time-slice. As the level of R_TEZY defines the overall activity of the technology in each time-slice it also appears in the objective function (MR_PRICE) to represent the variable operating and maintenance costs (VAROM) and any commodity related delivery charges (DELIV). R_TEZY may also appear in a user-defined constraint (MR_ADRATn if RAT_TEZY or RAT_ACT), emission (MR_TENV if ENV_ACT), and tax/subsidy (MR_TXSUB if TSUB_TCH) constraints if the associated data is provided.

Units: Energy unit. Most often PJ, but may be any other unit defined by the analyst for activity of conversion technologies. Consistent units must be used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in a constraint (MR_BNDCON) that sums the time-sliced activity variables. Indirect limits may be imposed on the activity of power plants if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

2.4.14 R_THZ(r,t,p,z) *MARKAL Matrix, Column r*

Description: The amount of low-temperature heat (lth) produced in a **T**ime period by a **H**eating plant (hpl) in each season (**z**).

Purpose and Occurrence: The flow variable tracks the amount of low-temperature heat produced in each season by a heat generating facility, and thus depicts the activity of such technologies.

For a power plant generating only low-temperature heat there is a R_THZ heat variable for each season. The variable represents the amount of low-temperature heat produced, and thus appears in each heat related constraint (MR_BALDH, for the grid connected (OUT(LTH)_TID)); and in MR_HPKW if peak contribution is a function of the plant activity rather than capacity via PEAK(CON)_TID)). This variable also determines the amount of energy required (in the commodity and electricity balance equations MR_BAL, and MR_BALE respectively) per unit production. The activity is upper-bounded for the current installed capacity (R_CAP) by applying an availability factor (AF/AF(Z)'D') that limits the plant's maximum output (MR_THZ) in each season. For externally load managed, or rigid, plants the R_THZ is not generated, but instead a capacity factor (CF/CF(Z)'D') is applied to the capacity variable (R_CAP) to determine the activity in each season. As the level of R_THZ defines the overall activity of the technology in each season it also appears in the objective function (MR_PRICE) to report the variable operating and maintenance costs (VAROM) and any commodity related delivery charges (DELIV). R_THZ may also appear in a user-defined constraint (MR_ADRATn if RAT_HPL or RAT_ACT), emission (MR_TENV if ENV_ACT), and tax/subsidy (MR_TXSUB if TSUB_TCH) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of conversion technologies. Consistent units must be used for the variable cost (VAROM) and bound (BOUND(BD)O) parameters related to the technology.

Bounds: The annual output of a conversion plant may be bounded (BOUND(BD)O), resulting in constraint (MR_BNDCON) to sum the seasonal activity variables. Indirect limits may be imposed on the activity of power plants if bounds are specified for capacity (BOUND(BD)), capacity growth rates (GROWTH), or investment (IBOND(BD)).

2.4.15 R_TOTobjZ MARKAL Matrix, Column s

Description: For multi-region models the MARKAL global **TOT**al **OBJ**ective function variable corresponding to the sum of all regional discounted costs over all the time periods.

Purpose and Occurrence: The variable represents the global objective function to be minimized (MR_OBJ). For multi-region models only.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Bounds: None

2.4.16 R_TSEP(r,t,s) MARKAL Matrix, Column t

Description: The level of activity in a **T**ime period of each resource supply or export option (srcencp = src,ent,p = **sep**). The variable represents the total annual amount of the main energy carrier or material produced, imported, or for exports consumed, by the resource option.

Purpose and Occurrence: The flow variable tracks the level of activity associated with each resource supply and export option.

This variable is generated for each resource supply/export option in all time periods beginning from the period that the option is first available (START). The variable appears in the commodity balance equations (MR_BAL, MR_BALE), as well as contributing to the base load and peaking requirements (MR_BAS, MR_EPK) if involving electricity (elc). For multi-region models, if the source is an import or export (src = ‘IMP’/‘EXP’) and the commodity is involved in bi-lateral trade (BI_TRD(ENT/MAT)) then it also appears in the exchange balance constraints (MR_BITRD). If electricity is traded according to season/time-of-day, then the variable balances against the total over all such time-slices (R_TSEPE) by means of an annual total constraint (MR_REGELC). Since R_TSEP represents the overall activity of the supply option, it appears in the objective function (MR_PRICE) to account for the cost of the resource (COST). R_TSEP may also appear in a user-defined constraint (MR_ADRATn if RAT_SEP), emission (MR_TENV if ENV_SEP) and tax/subsidy (MR_TXSUB if TSUB_SEP) constraints if the associated data is provided.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options. Consistent units must be used for the cost (COST) and bound (BOUND(BD)r) parameters related to the resource option.

Bounds: The resource supply option may be directly bounded by specifying an annual activity bound (BOUND(BD)r). It may also be limited by bounds specified on growth rates (GROWTHr) between periods, as well as cumulative limits (CUM) imposed on the resource over the entire time horizon.

2.4.17 R_TSEPE(r,t,s,w) MARKAL Matrix, Column u

Description: The amount of electricity exchanged (BI_TRDELC, src = 'IMP'/'EXP', elc, p = **sep**) in a **T**ime period between two regions in each time-slice w (where w = td(z,y)).

Purpose and Occurrence: The flow variable tracks the level of activity associated with each bi-lateral exchange of electricity in a time-slice.

R_TSEPE is available only in multi-region models. This variable is generated for each bi-laterally traded electricity exchange in all time periods beginning from the period that the option is first available (START) for each time-slice for which such exchanges are permitted (BI_TRDELC). Since the traded electricity is tracked according to season/time-of-day, it must be balanced against the total annual production (R_TSEP), which is then used to link up with the rest of the model, by means of an annual constraint (MR_REGELC). But as the variable represents the amount of electricity in each time-slice, it appears directly in the appropriate electric balance (MR_BALE), peak (MR_EPK) and base load (MR_BAS) constraints rather than the annual resource variable.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options.

Bounds: Only the annual exchange variables (T_SEP) may be limited by means of a direct bound on the level (BOUND(BD)r), as well as by bounds specified on growth rates (GROWTHr) between periods, or cumulative limits (CUM) imposed on the exchange over the entire time horizon.

2.4.18 R_TXSUB(r,t,txs) MARKAL Matrix, Column v

Description: The total level of a tax/subsidy imposed on the energy system.

Purpose and Occurrence: The tax/subsidy variable accumulates the revenue (a tax is positive) or expenditure (a subsidy is negative) associated with a user-named tax/subsidy.

This variable is generated for each tax/subsidy (txs) for which a cost (TSUB_COST) is provided by the user. The commodities (TSUB_ENT) and resources (TSUB_SEP) and technologies (TSUB_TCH) that are involved with this tax/subsidy must be elaborated. Then according to the TSUB_SEP/TSUB_TCH value and the activity of the individual resources and technologies the total tax/subsidy is determined.

Units: Million 2000 US$ or whatever monetary unit is used in the model.

Bounds: The total tax/subsidy level may be limited by the TSUB_BND parameter if desired.

2.4.19 R_ZSTK(r,e,c) MARKAL Matrix, Column w

Description: The total amount of stockpiled material, if any, at the end of the modeling horizon.

Purpose and Occurrence: This variable accumulates the net remaining energy/material in stockpiles and credits their value back to the objective function.

The variable appears in the commodity balance equations (MR_BAL) and the objective function (MR_PRICE) when a price (COST) is associated with the stockpiled (src = 'STK') material.

Units: Most often PJ, but may be any other unit defined by the analyst for activity of resource options.

Bounds: A resource limit (BOUND(BD)Or) may be applied to the stockpiled material in any period.

2.5 Equations (constraints and objective function)

The equations (constraints, accounting equations, and objective function) of a MARKAL model correspond to the rows of the LP matrix. Each equation is indexed by region, for multi-region models, and the appropriate set of other indexes necessary to fully and uniquely identify the model component. In this document, the word *equation* is not used in the strict mathematical sense of equality between two mathematical expressions. Rather, it is used (rather loosely) to encompass equations, inequalities, or even free expressions, as described below.

MARKAL equations fall into two distinct categories:

- *Binding equations* are the constraints of the Linear Program. Each such constraint expresses a logical condition that any solution of the MARKAL Linear Program must satisfy. The condition is expressed as a mathematical relation of the form:

$$\boldsymbol{LHS} \;\; \Re \;\; \boldsymbol{RHS}$$

Where:
LHS is a linear expression involving the MARKAL variables and attributes
$\Re$ is one of the three relational signs: =, ≥, or ≤
RHS is an expression involving only constants and attributes (may be 0 in several cases).

- *Non Binding equation*s are *free* expressions that are used for reporting a useful quantity (accounting equations). These "equations" have no relational sign and no right-hand-side.

As discussed in section 1.5, the objective of MARKAL is to minimize the discounted total system cost for all regions together, obtained by adding the discounted periods' total annual cost (comprising: annual costs, annualized investment costs, and a cost representing the welfare loss incurred when demands for energy services are reduced due to their price elasticity). This objective is equivalent to maximizing the total surplus (consumers' plus producers' surpluses).

The equations of the model[71] are listed alphabetically in Table 2-14, with a sequence number in the first column. Column 2 of the Table indicates the equation name and the associated indexes, elaborated in Table 2-12, needed to define the unique instances of the equation. The third column describes the equation, with particular emphasis on conditions that control the creation or not of that equation. The modeler is referred to Appendix A's MARKAL Matrix[72], that presents a global view of all the equations involved in the model, which variables are involved in each equation, and what parameters (and sets) determine the actual coefficient of each potential equation/variable intersection. The rows (equations) are ordered alphabetically in this spreadsheet as well as in the Table presented here and the detail specification sheets for each equation that follow. Thus by simply looking across the associated row of the MARKAL Matrix one can quickly see all the variables and parameters that might influence an intersection in the matrix for each equation. However, the parameters listed in the MARKAL Matrix are those that are actually used in the code when creating the coefficient, and thus may not correspond directly to user input data. Every set and parameter appearing on the MARKAL Matrix is explained in the Sets and Parameters section 2.2, Table 2-10 of this chapter. From that Table the user can dig deeper into the details to gain a full understanding of the matrix coefficients, including the input parameter(s) they are dependent upon, if desired.

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[71] The non-binding accounting equations, that are for the most part used for some report table entries, are not discussed in this document.
[72] Appendix A's MARKAL Matrix is provided both as a text Table for those who have a magnifying glass handy and as an inserted Excel file for the rest of us.

Table 2-14. Model constraints[73,74]

EQRef	Constraints[75] (Indexes)	Constraint Description	GAMS Ref
1	MR_ADRAT 1/2/3/4 (r,t,a)	The specialized, user-defined constraints that link variables of the model together for some special purpose (e.g., set market shares for a group of technologies). [Note: the cross-region user-defined constraint MR_XARATn (n=1/2/3/4) is presented with this constraint in the math section.]	MMEQARAT
2	MR_ANNCOST (r,t)	Annual total system cost for each region.	MMEQANC
3	MR_ARM (r,t,p)	Annual reservoir management for hydro plants. [p = hde]	MMEQARM
4	MR_BAL_ G/E/S (r,t,e)	The balance equation(s) that ensures that the production of each commodity equals or exceeds the total consumption of that commodity. When the commodity is an energy carrier (enc), then the MR_BAL_G (≥) constraint permitting production to exceed consumption is applied, or MR_BAL_S which introduces a slack variable for a soft equality constraint (=), where the slack variable enters the OBJ with a user-specified penalty. When the commodity is a material (mat), then the MR_BAL_E hard equality constraint (=) applies that forces production to match consumption. [e = enc/mat]	MMEQBAL
5	MR_BALDH (r,t,e,z)	The balance equation that ensures that the total amount of low-temperature heat produced in each season meets or exceeds that demanded in that season. [e = lth]	MMEQBALH
6	MR_BALE1/2 (r,t,e,w)	The balance equation that ensures that the total amount of electricity produced in each time-slice (season/time-of-day) meets or exceeds that demanded in that time-slice. Equation 1 is for daytime constraints (w = z, 'D') and equation 2 is for night-time constraints (w = z, 'N'). [e = elc]	MMEQBALE
7	MR_BAS (r,t,e,z)	Ensures that a power plant designated as base load (bas), thereby operating at the same level in the day and night, does not exceed a percentage of the highest night-time electricity demand (according to BASELOAD). [e = elc]	MMEQBAS
8	MR_BITRD (r,e,r,t,e,c)	Matches up the bi-lateral trade of a commodity between two regions. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (c) must be used.	MMEQUA.REG

[73] The EQ Ref is the sequence number according to the equation's alphabetic ordering in this chapter. Each equation is also listed in the corresponding order in the MARKAL Matrix spreadsheet presented in Appendix A.
[74] The GAMS Ref column identifies the main routines in which the equations are constructed. A series of core pre-processor routines (MMSETS (prepare control sets), MMFILL (perform interpolation), FRACLIFE (calculate fractional period lifetimes), MMCOEF (derive the matrix coefficients)), along with some minor associated subroutines, prepare the input data for direct use in the actual equation specification routines listed here. [Note that all core code has an extension of .INC unless noted otherwise (e.g., multi-region specific code has .REG extension, SAGE time-stepped specific code has .MTS extension (or .MKT for the market share algorithm).]
[75] MR_ is replaced by EQ_ and the region indexes are dropped when running single region models.

EQRef	Constraints[75] (Indexes)	Constraint Description	GAMS Ref
9	MR_BITRDE (r,e,r,t,e,c,w)	Matches up the bi-lateral trade of electricity between two regions during a particular time-slice. Note that the name of the commodity may be different between the two regions, if desired, but the same supply step (c) must be used. [e = elc]	MMEQUA.REG
10	MR_BNDCON 1/2/3 (r,t,p)	Bound on the (total) annual output from a conversion technology [BOUND(BD)O], that is the sum over all time-slices. The 1/2/3 indicator correspond to the sense of the equation = LO/FX/UP. [p = con]	MMEQBCON
11	MR_CPT1/2/3 (r,t,p)	The capacity transfer constraint that ensures that the total capacity (R_CAP) in a period correctly reflects the remaining residual capacity from before the modeling horizon (RESID), plus all investments (R_INV) made in the current and earlier periods whose LIFE has not yet expired. The 1/2/3 corresponds to p = dmd/con/prc respectively, where for dmd the constraint is an inequality (≥, allowing for excess capacity for end-use demand devices), while for all other technologies the constraint is normally[76] an equality.	MMEQCPT
12	MR_CUM (r,s)	Limit on the total production of a commodity from a resource activity over the entire modeling horizon (CUM).	MMEQCUM
13	MR_DESEP (r,t,s)	The limit imposed on the rate at which a resource supply option can decrease between periods (DECAYr).	MMEQDES
14	MR_DETCH (r,t,p)	The limit imposed on the rate at which total installed capacity can decrease between periods (DECAY).	MMEQDET
15	MR_DEM (r,t,d)	The demand constraint that ensures that demands for useful energy services (DEMAND) are satisfied.	MMEQDEM
16	MR_ENV (r,v)	The total cumulative emissions over the entire modeling horizon. Built by summing the period emissions variables (R_EM * NYRSPER). May be limited (ENV_CUM).	MMEQENV
17	MR_EPK (r,t,e,z)	The electricity peaking constraint ensures that there is enough capacity in place to meet the highest average electricity demand during the day of any season + estimated level above that for the actual peak (moment of highest electric demand) + a reserve margin of excess capacity (ERESERV includes both), to ensure that if some plants are unavailable, the demand for electricity can still be met. [e = elc]	MMEQEPK
18	MR_GEMLIM (v)	Global (or multi-region) limit on emissions.	MMEQUA.REG
19	MR_GEMLIMT (t,v)	Global (or multi-region) limit on emissions in a period.	MMEQUA.REG
20	MR_GRSEP (r,t,s)	The limit imposed on the rate at which a resource supply option can expand between periods (GROWTHr).	MMEQGRS
21	MR_GRTCH (r,t,p)	The limit imposed on the rate at which total installed capacity can expand between periods (GROWTH).	MMEQGRT

[76] When $SET CPTEL 'L' is specified the sense of the MR_CPT constraint for processes (prc) or conversion plants (con) is changed to an equality. This is necessary when RESIDs are specified for externally load managed processes (xlm) or conversion plants (con) in order for proper calculation of the marginals.

EQRef	Constraints[75] (Indexes)	Constraint Description	GAMS Ref
22	MR_GTRD (t,g)	The balance between all the producers and consumers of the globally traded commodities.	MMEQUA. REG
23	MR_HPKW (r,t,e,z)	The low-temperature heat peaking constraint that ensures that there is enough capacity in place to meet the highest average seasonal heating demand plus estimated level above that for the actual peak (moment of highest heat demand) + a reserve margin of excess capacity (HRESERV includes both) to ensure that if some plants are unavailable, the demand for heat can still be met. [e = lth]	MMEQHPK
24	MR_LIM (r,t,p,e)	For flexible output "mixing" processes (LIMIT) the constraint that controls the levels of the individual outputs (R_LOUT, where OUT(ENC)p is maximum) from such processes. [p = prc]	MMEQLIM
25	MR_OBJ	The MARKAL total discounted system cost objective function (perfect foresight mode), for multi-region runs (otherwise EQ_PRICE (= discounted MR_PRICE) serves as the overall objective function).	MMEQUA. REG
26	MR_PBL (r,t,p)	For flexible output "mixing" processes (LIMIT) the constraint that ensures that the sum of all the outputs (R_LOUT) are in line with the overall efficiency (LIMIT) of such processes. [p = prc]	MMEQPBL
27	MR_PRICE (r)	The MARKAL objective function for each region. [In SAGE an accounting equation tracks the total discounted system cost for reporting purposes.]	MMEQPRIC
28	MR_REGELC (r,t,s)	Balance equation tying the time-sliced electricity trade variables (R_TSEPE) to the regular annual electricity variable (R_TSEP). [s = "IMP/EXP"elcp]	MMEQUA. REG
29	MR_SRM (r,t,p,z)	Seasonal reservoir management for hydro plants (p = hde).	MMEQSRM
30	MR_TENV (r,t,v)	The total amount of emissions generated in each period from all sources (resources and technologies) of a given emission indicator.	MMEQTENV
31	MR_TEZY (r,t,p,w)	The constraint that ensures that the season/time-of-day activity of a power plant (R_TEZY), that is the amount of electricity generated, is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF/AF(Z)(Y) or CF/CF(Z)(Y)), taking into consideration any scheduled maintenance (R_M). [p = ela]	MMEQTEZY
32	MR_THZ (r,t,p,z)	The constraint that ensures that the seasonal activity of a heating plant (R_THZ), that is the amount of heat generated, is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF/AF(Z)(Y) or CF/CF(Z)(Y)), taking into consideration any scheduled maintenance (R_M). [p = hpl]	MMEQHZ

EQRef	Constraints[75] (Indexes)	Constraint Description	GAMS Ref
33	MR_TXSUB (r,t,txsub)	The charging or crediting of a tax/subsidy (TSUB_COST) for commodities (TSUB_ENT) consumed/produced by technologies (TSUB_TCH). The total tax/subsidy is subtracted from the total system costs in the reports, and reported separately.	MMEQTAXS
34	MR_UTLCON (r,t,p)	The adjustment to the availability of a power plant to account for the fact that some portion of outage is due to scheduled maintenance. Normally the model will schedule all unavailability, but if the user specifies that some fraction of annual unavailability is scheduled (AF_TID ≠ 1) then this constraint is generated to adjust the amount of unavailability that the model is free to schedule. [p = con]	MMEQUCON
35	MR_UTLPRC (r,t,p)	The constraint that ensures that the annual activity of a conventional process (R_ACT) is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF/CF). [p = prc]	MMEQUPRC
not in XLS	MR_XARAT 1/2/3/4 (t,a)	The cross-region user-defined constraints that link variables of the model together for some special purpose (e.g., set market shares for a group of technologies). Only for multi-region models. [Note: the cross-region user-defined constraint is presented with the standard user-defined constraint (MR_ADRAT1/2/3/4) in the math section and the matrix, as essentially identical other than omitting the region index.]	MMEQARAT. REG

In elaborating the details of the construction of the individual equations of the model, standard mathematical terminology and expressions are employed, as summarized in Table 2-15 below. As noted in the Table, there are two extensions to standard notation, a dollar sign ($), is lifted from the GAMS language, and a '/'.

Table 2-15. Mathematical symbols used for equation specification

Symbol	Description
$\subset$	A set is a subset of another set.
$\in$	An element is a member of a set.
$\notin$	An element is not a member of a set.
$\cap$	The "anding" or intersection of two sets, where an element must exist in both sets.
$\cup$	The "oring" or union of two sets, where an element must exist in one of the listed sets.
$\sum$	Summation over a set or series of sets.
$\leq$	Less than or equal to condition.
$=$	Equal to condition.
$\geq$	Greater than or equal to condition.
$\wedge$	The "anding" or intersection of two or more logical expressions, where all expression must be true.
$\vee$	The "oring" or union of two or more logical expressions, where at least one expression must be true.

Symbol	Description
∀	A “such that” (or “for all”) condition that explains under what circumstances an equation is generated.
$	Indicates that a condition controls whether or not a particular calculation is performed.[77]
/	A shortcut employed to reduce the need to repeat identical specification of equations, for example for energy and material (e.g., ent/mat).

Before turning to the individual equations a general example is presented here to explain the syntax employed when specifying the details of the actual equations.

Equation specification header indicating the equation sequence number, the equation name (with the multi-region prefix (MR_), the indexes enumerating the specific instances of the equation (e.g., r = region, t = time period, e = energy or material). Then any overall conditions controlling the generation of the equation are specified.

$$\text{EQ\#}: MR\_ < eqname >_{r,t,e} \; \forall e \in < conditional\; criteria >$$

Multiplier applied to a part of the matrix intersections. The same XX parameter is named XXENT if it relates to energy and XXMAT if it relates to material. Note that all indexes must either appear on the equation declaration or in a ∑ expression (as shown in the next part of the example) indicating that said index is to be looped over for each qualifying element either summing or creating unique instances of the associated variable(s).

$$XX(ENT/MAT)_{r,e,t} * \Big\langle$$

Matrix intersection constructed by summing over some index(es) p (e.g., processes), according to a certain criterion that <parname> exists for the current indexes, applying coefficient parameters to a variable (whose name always has a R_ prefix) for each index of the looping set x, depending upon additional criteria.

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[77] For example, 1$(dcn) + TE(ent)$(cen) would indicate that the transmission efficiency is 1 for decentralized (dcn) plants and the user provided overall commodity efficiency (TE(ent)) if centralized (cen).

$$\left[\sum_{\substack{p \in <set> \forall \\ <pname>_{r,p,e,t} \\ p \neq '<literal>'}} \begin{matrix}\left(<pname1>_{r,p,e,t} * R\_<vname1>_{r,t,p}\right) \$ <cond1> + \\ \left(<pname2>_{r,p,e,t} * R\_<vname2>_{r,t,p}\right) \$ <cond2>\end{matrix}\right] +$$

Rest of the intersections in the left-hand-side of the equation.

Relational operator that indicates the sense of the equation, which may vary depending upon the nature of the equation (e.g., energy balance is ≥ while material balance is =).

$$\{\leq ; = ; \geq\}$$

Right-hand-side of the equation; usually 0 otherwise an expression involving constants and parameters (e.g., RESID, SEP_CUM).

$$RHS;$$

The rest of this section provides a detailed description of each of the equations of MARKAL[78]. On the equation sub-header record the equation name and index is given, along with a reference to the associated row in the MARKAL Matrix. For each equation the following information is provided on “fact” sheets:

Description: A short description of the nature of the equation.

Purpose and Occurrence: A brief explanation of what the equation is for and why it is needed, and a general description of the data circumstances and model rules that control the instances of the constraint, or not, and the variables in the constraint.

Units: The units associated with the constraint.

Type: The sense of the equation (≤, =, ≥).

[78] Note that the inclusion of any technology in an equation is conditional upon its initial availability in a period, and when running SAGE the generation of all constraints is conditional upon which period is currently being solved. These conditions are true across the board, but are **NOT** reflected in the detailed equations presented in this section.

Interpretation of the results:

Primal: Guidelines of how to interpret the primal results, or the slack.

Dual: Guidelines of how to interpret the dual results, or the shadow price.

Remarks: Some useful comments, hints or suggestions.

GAMS Routine: The GAMS source code module where the final equation code can be found.

For each equation the "fact" sheet is followed by the mathematical specification of the equation. To the fullest extent possible these detailed equation descriptions indicate the user input parameters that the matrix intersection depends upon. However, at certain times intermediate parameters are employed. The nature of the parameters can be discerned by the position of the period index (t). User input is provided by means of spreadsheet Tables where the periods run along the columns, and thus time is the last index for these parameters. However internal parameters take advantage of special preprocessor Sets that map the allowed period and technology combinations (e.g., tptch(t,p)), dependent upon when the technology first becomes available (START), in the left-most position of the parameter. Thus internal parameters have time as the 2nd index, right after the region. Note that the MARKAL Matrix depicting the model structure presents the parameters used in the code when the coefficients are created, and thus does not correspond directly to what appears in the mathematical descriptions presented here.

2.5.1 MR_ADRATn(r,t,a)/MR_XARATn(t,a)MARKAL Matrix, Row 6

Description: These specialized constraints impose user-defined relationships among the variables of the model, within and across regions respectively (e.g., set market shares for a collective group of technologies). The n = '1/2/3/4' in the equation listing does not indicate that only 4 such constraints are allowed, but rather the type of the equation (see below). Indeed, in a large model, there may be hundreds of such user-defined constraints.

Purpose and Occurrence: The user-defined constraints are unique in that their structure is completely defined by the modeler. A user-defined constraint may be composed of arbitrary linear expressions involving the variables associated with activity (R_TSEP, R_ACT, R_TEZY, R_TCZYH, R_HPL), capacity (R_CAP), or investment (R_INV). One important restriction of a user-defined constraint is that it may only involve variables with the same period index. In other words, constraints that tie variables from different time periods are not allowed. The sense and right-hand-side of a user-defined equation is specified by means of the RAT_RHS parameter. For a user-defined equation each parameter provided represents the coefficient to be applied to the associated variables when constructing the equation. For example, to equate the capacity of two technologies (e.g., scrubbers to be installed on an existing power plant) one would provide a RAT_RHS'scrubbit',t,'FX' = 0, and RAT_CAP'scrubbit',t,'p1' = 1 and RAT_CAP'scrubbit',t,'p2' = -1. The resulting equation would be:

$$\left\{\begin{array}{l}\text{MR\_ADRAT2(r,t,'scrubbit')} \;\; .. \\ \text{R\_ CAP'scrubbit' ,t,'p1' - R\_ CAP'scrubbit' ,t,'p2'} = 0\end{array}\right\}$$

This equation is generated in all time periods for which a right-hand-side is provided (RAT_RHS/XARATRHS), along with at least one variable coefficient (RAT_ACT, RAT_CAP, etc.). The cross-region constraint takes the identical form as the standard, single-region constraint, but since it only applies to multi-region models it results in lumping together the R_xxx variables from different regions. Each region involved needs to name the constraint locally and provide the appropriate data.

Units: The units of the user-defined constraints should be compatible with those of the variables composing the constraint. It is the responsibility of the user to ensure that consistent units are employed for each user-defined constraint.

Type: *Binding,* usually. The type of the equation is under user control according to the sense (b = 'LO', 'FIX', 'UP', 'NON') entered when specifying the RAT_RHS/XARATRHS parameter.

Interpretation of the results:

Primal: The level of a user-defined constraint represents the slack (difference between the RHS and LHS) if a binding constraint, or the level if a non-binding equation is specified.

Dual variable: As explained in section 3.4, the dual variable (DVR_ADRAT/DVR_XARAT) of the user-defined constraint (shadow price) indicates the amount that the cost objective function would change if the RHS of the constraint were increased by one unit. A positive value indicates an increase in the optimal cost.

Remarks: Note that in the description below the OR operator ($\vee$) is used to indicate that if RAT_ACT is provided it is assigned to the associated RAT_TEZY, RAT_TCZY, RAT_HPL parameter by the preprocessor.[79]

GAMS Routine: MMEQARAT.INC/MMEQARAT.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#1}: MR\_ADRATn_{r,t,a} \forall RAT\_RHS_{r,a,b,t}$$

where n=1/2/3/4 corresponding to equation type (b= 'LO' $\geq$, 'FX' =, 'UP'
$\leq$,
'NON' non-binding)

[79] In MARKAL, user-defined equations are always generated. In SAGE, the situation is more complicated, as will be explained in section ??.

All resource supply options for which the RAT_SEP parameter is provided are included in the associated user-defined constraint based upon their activity level.

$$\left[\sum_{s}\left(RAT\_SEP_{r,a,s,t} * R\_TSEP_{r,t,s}\right)\right]+$$

All technologies for which the RAT_INV parameter is provided are included in the associated user-defined constraint based upon the level of new investment in the technology.

$$\left[\sum_{p}\left(RAT\_INV_{r,a,p,t} * R\_INV_{r,t,p}\right)\right]+$$

All technologies for which the RAT_CAP parameter is provided are included in the associated user-defined constraint based upon the total installed capacity of the technology.

$$\left[\sum_{p}\left(RAT\_CAP_{r,a,p,t} * R\_CAP_{r,t,p}\right)\right]+$$

All technologies for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon activity level of the technology. However, as noted below there are quite a few different ways the parameter needs to be applied depending on the nature of the technology. But basically what is done is to use the internal activity parameter calculated for the technology that takes into consideration both the technical data describing the relationship between capacity and activity (e.g., CAPUNIT, AF/CF), as well as the nature of the technology (e.g., base load, externally load managed), and apply it to the appropriate variable. Note that RAT_ACT can be used in place of RAT_TEZY / RAT_TCZY / RAT_HPL time-slice input parameters for conversion plants (con).

All demand devices for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the activity derived according to the amount of installed capacity, giving consideration to demand vintaging.

$$\left[\sum_{p \in dmd}\left(\begin{array}{l} RAT\_ACT_{r,a,p,t} * CF_{r,p,t} * CAPUNIT_{r,p} * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \left\{not\left(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}\right)\right\} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array}\right) \end{array}\right) \end{array}\right)\right]+$$

All conventional (non externally load managed) processes for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the activity variable representing total output from the process.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(RAT\_ACT_{r,a,p,t} * R\_ACT_{r,t,p} \right) \right] +$$

All externally load managed processes for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the total output of all commodities as derived from the level of installed capacity of the process.

$$\left[\sum_{p \in xpr} \left(\begin{array}{l} RAT\_ACT_{r,a,p,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \sum_{e} \left(OUT(ENC/MAT)p_{r,p,e,t} * R\_CAP_{r,t,p} \right) \end{array} \right) \right] +$$

All conventional electric generation plants for which the RAT_TEZY parameter is provided for some time-slice are included in the associated user-defined constraint based upon the time-slice activity of the power plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provided), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_TEZY if the same coefficient applies to all time-slices (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_TEZY=RAT_ACT in the preprocessor). [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{w,p \in ela \\ p \notin xlm \\ except\, y='N' \wedge (bas \cup stg) \\ except((y='N' \wedge z='I') \wedge \\ PEAK\_(CON)\_TID_{r,p})}} \left(\begin{array}{l} \left(RAT\_ACT_{r,a,p,t} \vee RAT\_TEZY_{r,a,p,w,t} \right) * \\ R\_TEZY_{r,t,p,w} \end{array} \right) \right] +$$

All externally load managed electric generating plants for which the RAT_ACT parameter is provided are included in the associated user-defined constraint based upon the total output of electricity over all permitted time-slices (according to the CF(Z)(Y) provided) as derived from the level of installed capacity of the plant.

$$\left[\sum_{w,p\in xlm}\begin{pmatrix}RAT\_ACT_{r,a,p,t} * CAPUNIT_{r,p} * CF_{r,p,t} * QHR(Z)(Y)_{r,w} * \\ R\_CAP_{r,t,p}\end{pmatrix}\right]+$$

> The heat component from all coupled heat and power pass-out turbines for which the RAT_TCZY parameter is provided for some time-slice are included in the associated user-defined constraint based upon the time-slice activity of the power plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provided), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_TEZY if the same coefficient applies to all time-slices (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_TEZY=RAT_ACT in the preprocessor).

$$\left[\sum_{\substack{p\in cpd,w \\ when\ ELM_{r,p}}}\left(\left(RAT\_ACT_{r,a,p,t} \vee RAT\_TCZY_{r,a,p,w,t}\right) * R\_TCZYH_{r,t,p,w}\right)\right]+$$

> All conventional heating generation plants for which the RAT_HPL parameter is provided for some season are included in the associated user-defined constraint based upon the seasonal activity of the plant. Note that if a particular time-slice is not provided, or the plant may not operate for a time-slice (by no AF(Z)(Y) being provided), then that variable is omitted. In addition, RAT_ACT may be specified in place of RAT_HPL if the same coefficient applies to all seasons (as noted in the equation by the use of the "or" operator (∨) in the expression below; handled in the code by setting each permitted RAT_HPL=RAT_ACT in the preprocessor). [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{z,p\in hpl \\ except((z='S'\vee'I')\wedge \\ PEAK(CON)\_TID_{r,p}))}}\left\langle\left(RAT\_ACT_{r,a,p,t} \vee RAT\_HPL_{r,a,p,z,t}\right) * R\_THZ_{r,t,p,z}\right\rangle\right]$$

> The sense of the user-defined equation is determined by the RAT_RHS parameter setting such that the equation type is: 'LO' ≥, 'FX' =, 'UP' ≤, 'NON' non-binding.

$$\{\leq;=;\geq;non\!-\!binding\}$$

$$0$$

2.5.2 MR_ANNCOST(r,t) *MARKAL Matrix, Row 7*

Description: This equation tracks the total annual cost of a region in each period. In MARKAL it is a non-binding accounting equation strictly used for reporting. The annualized cost is an important model result. [80]

Purpose and Occurrence: To accumulate the total annualized system cost for each region and time period. The regional cost comprises: annual costs, annualized investment costs, and a cost representing the loss incurred when demands for energy services are elastic. As noted above, for SAGE and MACRO this value is a key component of the final objective function. This equation is generated for each region in each time period and includes all terms to which a cost is applied.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Type: *Non-binding.* The equation is constructed as an accounting equation tracking the period total annual cost for each region.

Interpretation of the results:
Primal: Is equal to the total annual cost of a region in period t.
Dual variable: Not relevant for MARKAL as this is a non-bindng constraint.

GAMS Routine: MMEQANC.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 2:} MR\_ANNCOST_{r,t}$$

Annual cost of supplying domestic resources, plus any delivery costs associated with ancillary commodities, according to the level of the resource activity.

[80] For SAGE the same expression (named MR_MTSOBJ) is made binding by equating it to a variable (R_MTSOBJ(r)), which is used to form the global cost in each period corresponding to the objective function (MR_OBJ). For MACRO the same constraint (named MR_COSTNRG) is equated to the regional energy cost variable (R_EC), which then enters the regional production function.

$$\sum_{\substack{s \in tpsep \\ x \neq 'IMP' \vee 'EXP' \\ e \notin (enu \cup \\ ecv \cup lth)}} \left[\begin{array}{l} \left\langle \begin{array}{l} COST_{r,s,t} + \\ \sum_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \end{array} \right] +$$

Annual cost of imports and exports, plus any delivery costs associated with ancillary commodities involved in the import/export process, according to the level of the resource activity. Note that if the trade is expressed as bi-lateral trade then any resource supply cost (COST) is ignored and only the transport/transaction cost applied. Also, if importing electricity then transmission and distribution O&M costs are incurred.

$$\sum_{\substack{s \in tpsep \\ x = 'IMP \vee 'EXP' \\ e \notin (enu \vee \\ ecv \vee lth)}} \left[\begin{array}{l} \left\langle \begin{array}{l} SIGN_x * \begin{pmatrix} COST_{r,s,t} \$ not\ BI\_TRDENT + \\ BI\_TRDCST_{r,s,t} \$ BI\_TRDENT \end{pmatrix} + \\ (ETRANOM_{r,e,t} + EDISTOM_{r,e,t}) \$ \begin{pmatrix} e \in elc \wedge \\ x = 'IMP' \end{pmatrix} + \\ \sum_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \end{array} \right] +$$

Annual cost of imports and exports of electricity by time slice, according to the level of activity. Any bi-lateral, time-sliced transport/transaction cost is applied here, and any other delivery or transmission and distribution costs are captured in the above expression.

$$\left[\sum_{\substack{s \subset tpsep, w \\ e \in elc \\ BI\_TRDELC}} \left(SIGN_x * BI\_TRDCSTE_{r,s,w} * R\_TSEPE_{r,t,s,w} \right) \right] +$$

Transaction cost associated with a globally traded commodity in the current region.

$$\left[\sum_{\substack{e \in ent \vee env \vee mat \\ x='IMP' \vee 'EXP' \\ G\_TRADE_e \\ t \geq TRD\_FROM}} \left(R\_GTRD_{r,t,e,x} * TRD\_COST_{r,e,t} \right) \right] +$$

Stockpile credit in the final period.

$$\left[\sum_{\substack{s \subset 'STK' \\ tpsep_{tlast,s}}} \left(-COST\_TID_{r,s} * R\_TSEP_{r,tlast,s} \right) \right] +$$

Fixed operating and maintenance costs for conventional (non externally load managed) process and conversion technologies as a function of the total installed capacity. [For externally load managed technologies the FIXOM is included when determining the total operating costs (below).]

$$\left[\sum_{p \left(\begin{array}{l} (\in tpprc \wedge \notin xpr) \vee \\ (\in tpcon \wedge \notin xlm) \end{array} \right)} \left(FIXOM_{r,p,t} * R\_CAP_{r,t,p} \right) \right] +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) processes as a function of the total process activity.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(\left\langle \begin{array}{l} VAROM_{r,p,t} + \\ \sum_{e \subset INP(ENT)p_{r,p,e,t}} \left(INP(ENT)p_{r,p,e,t} * DELIV(ENT)r_{r,p,e,t} \right) \end{array} \right\rangle * \right. \right.$$
$$\left. \left. R\_ACT_{r,t,p} \right) \right]_{+}$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) electric power plants as a function of the time-sliced generating activity. The transmission and distribution O&M costs are added on top of the plant operating costs. [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\sum_{\substack{p \in tpela, p \notin xlm, w \\ except\ p \in (bas \cup stg) \wedge \\ (z='N') \\ except\ (z='I' \vee y='N') \wedge \\ PEAK(CON)\_TID_{r,p}}} \left(\left\langle \begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} + \sum_{e \in elc} ELCOM_{r,p,e,t} \end{array} \right\rangle * \right.$$
$$\left. R\_TEZY_{r,t,p,w} \right) +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) heating plants as a function of the seasonal generating activity. The transmission O&M costs are added on top of the plant operating costs. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\sum_{\substack{p \in tphpl, p \notin xlm, z \\ except(z='I' \vee 'S') \wedge \\ PEAK(CON)\_TID_{r,p}}} \left[\begin{array}{l} \left\langle \begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} + \sum_{e \in lth} DTRANOM_{r,e,t} \end{array} \right\rangle * \\ R\_THZ_{r,t,p,z} \end{array} \right] +$$

Variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for coupled heat and power pass-out turbines as a function of the time-sliced generating activity. The transmission and distribution O&M costs for both electricity and heat are added on top of the plant operating costs. [Special conditions apply to base load plants that have only daytime variables (to force day=night generation for base load plants.]

$$\sum_{\substack{p \in tpcpd, w \\ p \in ELM_{r,p,t} \\ except(y='N' \wedge \\ e \in bas)}} \left[\begin{array}{l} \left\langle \begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ +(1 - ELM_{r,p,t}) * ELCOM_{r,p,e,t} + \sum_{e \in lth} DTRANOM_{r,e,t} + VAROM_{r,p,t} \end{array} \right\rangle * \\ \left(CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} \right) * R\_TCZYH_{r,t,p,w} \end{array} \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for demand devices as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{p \in tpdmd} \left[\left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{l} \sum_{e \subset MA(ENT)_{r,p,e,t}} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} / EFF_{r,p,t} * \\ DELIV(ENT)_{r,p,e,t} \end{array} \right) + \\ VAROM_{r,p,t} \end{array} \right] * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not \left(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t} \right) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right\rangle \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed processes as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{\substack{p \in tpprc \\ p \in xpr}} \left[\left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{l} \sum\limits_{e \subset INP(ENT)p_{r,p,e,t}} \left(INP(ENT)p_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} \end{array} \right] \end{array} \right\rangle * \\ R\_CAP_{r,t,p} \right] +$$

Fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed power plants as a function of the total installed capacity, and the operation (dictated by CF(Z)(Y)) thereof.

$$\sum_{\substack{p \in tpcon \\ p \in xlm}} \left[\left\langle \begin{array}{l} FIXOM_{r,p,t} + \sum\limits_{w} \left(CF(Z)(Y)_{r,p,w,t} * QHR_{r,w} \right) * CAPUNIT_{r,p} * \\ \left[\begin{array}{l} \sum\limits_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} + ELCOM_{r,p,e,t} \ \$ p \in ela + LTHOM_{r,p,e,t} \ \$ p \in hpl \end{array} \right] + \end{array} \right\rangle * \\ R\_CAP_{r,t,p} \right] +$$

Annualized investment costs associated with new investments still available in the current period, where t' is the vintage period in which the investment took place. The calculated investment cost (COST_INV) consists of the actual investment cost (INVCOST), as well as electricity transmission and distribution system investment costs if appropriate (see below), taking into consideration the capital recovery factor (CRF), as determined by the lifetime (LIFE) and discount rate (DISCOUNT and DISCRATE if technology based), as discussed below.

$$\left[\sum_{p} \sum_{t'=t-LIFE_{r,p}}^{t} \left(COST\_INV_{r,t,t',p} * R\_INV_{r,t',p} \right) \right] +$$

Annualized investment costs associated with residual capacity (that is capacity installed prior to the first modeling period) still available in the current period. The calculated investment cost (COST_INV) consists of the actual investment cost (INVCOST), as well as electricity transmission and distribution system investment costs if appropriate (see below), taking into consideration the capital recovery factor (CRF), as determined by the lifetime (LIFE) and discount rate (DISCOUNT and DISCRATE if technology based), as discussed below.

$$\left[\sum_{\substack{p \in RESID_{r,t,p} \\ t' = first\ period}} \left(COST\_INV_{r,t',t',p} * R\_RESID_{r,t,p} \right)\right] +$$

Emission taxes or other costs associated with the level of an emission indicator.

$$\left[\sum_{v} \left(ENV\_COST_{r,v,t} * R\_EM_{r,t,v} \right)\right] +$$

Taxes and subsidies associated with particular commodities, and the consumption and/or production thereof.

$$\left[\sum_{txs} \left(TSUB\_COST_{r,v,t} * R\_TXSUB_{r,t,txs} \right)\right] +$$

The costs associated with growth or reduction in demand as a function of the elastic demand own price elasticity and variation as represented by a step curve.

$$
\sum_{b,d,u=1}^{MED-STEP(BD)_{r,d,b,u,t}} \left\{ \begin{array}{l} R\_ELAST_{r,t,d,b,u} * MED-BPRICE_{r,d,t} * \\ \left\langle \begin{array}{l} -\left\{ \left[\begin{array}{l} 1+ \\ \left(MED-VAR_{r,d,b,t} / MED-STEP_{r,d,b,t} * (u-.5)\right) \end{array} \right]^{**} \right\} \$b='UP' + \\ \quad \left(1/MED-ELAST_{r,d,b,t}\right) \\ \left\{ \left[\begin{array}{l} 1- \\ \left(MED-VAR_{r,d,b,t} / MED-STEP_{r,d,b,t} * (u-.5)\right) \end{array} \right]^{**} \right\} \$b='LO' \\ \quad \left(1/MED-ELAST_{r,d,b,t}\right) \end{array} \right\rangle \end{array} \right\} +
$$

If the balance equation is to be formulated as a slack equality constraint, where the slack cost is provided (SLKCOST), then include the slack variable.

$$
\left[\sum_{e \in SLKCOST_{r,e,t}} \left(\begin{array}{l} SKLCOST_{r,e,t} * \\ \left(\left(R\_SKLENC_{r,t,e} + \sum_{z} R\_SLKLTH_{r,e,t,z} \right) \right) \end{array} \right) \right] +
$$

$$
\{non-binding\}
$$ [81]

Where SIGN = -1 for x='EXP'; otherwise 1

ELCOM = if $(e \in elc \wedge p \in ela)$ then $\left(\begin{array}{l} EDISTOM_{r,e,t} + \\ ETRANOM_{r,e,t} \$(p \subset cen) \end{array} \right);$

$0\$(p \in stg)$

LTHOM = if $[e \in lth \wedge p \in (p \in cpd \wedge cen)]$ and p is a back-pressure turbine (REH) then $DTRANOM_{r,e,t} / REH_{r,p}$; 0 otherwise

ELCINV = if $(e \in elc \wedge p \in ela)$ then

[81] For SAGE and MACRO the constraint is binding {=}.

$$EDISTINV_{r,e,t}\$(p \notin stg) +$$

$$ETRANINV_{r,e,t}\$(p \subset cen)$$

LTHINV = if $e \in lth$

if $p \in hpl$ then $DTRANINV_{r,e,t}$

if $p \in (p \in cpd \wedge cen)$

if $p \in REH_{r,p,t}$ then

$$\left(DTRANINV_{r,e,t} / REH_{r,p,t}\right)$$

else

$$\left(\begin{array}{l} DTRANINV_{r,e,t} * \\ ELM_{r,p,t} / CEH_{r,p,w,t} \end{array}\right)$$

otherwise

0

CRF = where $x=1/(1+DISCOUNT \text{ or } DISCRATE)$, *and then* $CRF=\{1-x\}/\{1-x^{LIFE}\}$

FRACLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
(years in last period of LIFE/NYRSPER)

FRLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
1 if current period is not the last (fractional period)
otherwise
CRF adjustment for the fraction of the period

COST_INV = $$\left\langle INVCOST_{r,p,t} + (ELCINV + LTHINV)\right\rangle * FRACLIFE_{r,p} * CRF_{r,t,p} * FRLIFE_{r,p}$$

2.5.3 MR_ARM(r,t,p) MARKAL Matrix, Row 8

Description: The **A**nnual **R**eservoir **M**anagement constraint on hydro plants.

Purpose and Occurrence: To impose a limit on the annual activity from a hydroelectric plant based upon available capacity (of the reservoir). This equation is generated for all time periods for which the maximum annual reservoir availability is specified (ARAF).

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of total annual activity from a hydro plant for which reservoir limits have been specified.

Dual variable: The dual variable (DVR_ARM) of the annual reservoir constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of annual activity from such hydro plants. A zero dual value implies that the technology has not reached the limit imposed. The value is negative when the primal equation is tight, and the system wants to use more of the technology in the current period.

Remarks: Only for non-externally load managed plants.

GAMS Routine: MMEQARM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 3}: MR\_ARM_{r,t,p} \ \forall p \in hde \wedge ARAF_{r,p,t} \wedge \left(not\ xlm_{r,p}\right) \wedge \left(not \sum_{w} AF(Z,Y)_{r,p,w,t}\right)$$

The amount of the total annual (reservoir) capacity available.

$$\left[-CAPUNIT_{r,p} * ARAF_{r,p,t} * R\_CAP_{r,t,p}\right] +$$

The total electricity generated from hydro plants subject to annual reservoir availability.

$$\sum_{w}\left(R\_TEZY_{r,t,p,w}\right)$$

$$\left\{\leq\right\}$$

$$0$$

2.5.4 MR_BAL_G/E/S(r,t,e) MARKAL Matrix, Row 9

Description: **BAL**ance equation for standard commodities (energy different from electricity or low temperature heat, and materials), ensures that production plus imports is in balance according to the demand for the commodity.

Purpose and Occurrence: The balance equation ensures the conservation of an energy or material in a region's energy system. The equation is generated for each energy carrier, other than renewable energy carriers (ern) for which no resource supply option (src = 'RNW') is provided, for example wind and solar, and for each material of the underlying Reference Energy System. It sums all sources of the commodity including production from resource options, processes and other technologies, along with imports, and ensures that this total is sufficient to cover the total amount consumed by all technologies and exports. In addition there may be movements into (accumulation) or out of (depletion) stockpiles (src = 'STK'). For energy carriers slack is permitted, as it may be the case that as a result of some rigid processes one particular commodity forces another commodity to be over produced at levels above the total amount actually needed. However, the analyst should carefully investigate any slack on the balance equations as any energy carrier for which there is slack on the balance equation will have a 0 shadow price. For materials a strict equality constraint is imposed, therefore no excess production is allowed. This equation is generated in all time periods during which some resource option or technology produces or consumes the commodity. Note that standard energy carriers do not include electricity and low-temperature heat, which are handled in separate equations that reflect the time-sliced nature of these commodities.

Units: PJ, or Bvkm, or any other unit in which the commodity is tracked.

Type: *Binding.* The equation is a greater than or equal (≥) if the commodity is an energy carrier (enc), or an equality (=) if the commodity is a material (mat).

Interpretation of the results:

Primal: The level of the balance equation represents the slack or overproduction of an energy carrier. In most cases there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of some processes that tie the fraction of one energy carrier to those of others, if one energy carrier in such a group is needed at a high level, that may result in an excess of the associated energy carriers. Under normal circumstances the user should look to redress this situation by employing more flexible output processes (e.g., LIMIT), or parallel input processes that would allow the model to optimize the commodity mix (within limits the user imposes). For materials the level of the equation is always zero, since the balance constraint is an equality.

Dual variable: For energy carriers, the dual variable (DVR_BAL) of the balance constraint (shadow price) indicates the amount that the energy system pays for each unit of the commodity consumed, and accrues for each unit produced. A zero dual value implies that the supply of the commodity exceeds its consumption. For materials, the dual value may be positive, negative or 0.

Remarks: For renewable energy carriers for which no resource supply option is provided (src = 'RNW') a non-binding accounting equation is generated that tracks the total consumption of such energy carriers. In this case the level of this row is assumed to be the fossil equivalent (feq) of the consumed renewable, but no active constraint is imposed.

GAMS Routine: MMEQBAL.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#4}: MR\_BAL\_k_{r,t,e}\ \forall e \in (enc \vee mat),$$

where k = G (≥) for standard energy carriers, k = S (=) when a slack variable is introduced, and k = E (=) for materials.

PRODUCTION - commodities entering the system from Resource Supply options and produced by Technologies as main outputs or by-products.

System-wide commodity transmission efficiency applied to the total production, reflecting overall loss and forcing higher production levels.

$$TE(ENT/MAT)_{r,e,t} \; * \quad \langle$$

Production of the current commodity from resource supply options, other than EXPorts, identified by means of the OUT(ENT)r indicator parameter.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ s \neq 'EXP'}} R\_TSEP_{r,t,s} \right] +$$

Import of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'IMP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right] +$$

Production of the current commodity from conventional (non-externally load managed and non-LIMIT) processes that deliver the current commodity according to fixed output shares.

$$\left[\sum_{\substack{p \in prc \\ p \notin (xpr \vee LIMIT_{r,p,t})}} \left(OUT(ENC)p_{r,p,e,t} * R\_ACT_{r,t,p} \right) \right] +$$

Production of the current commodity from flexible output processes (identified by means of the LIMIT parameter) that deliver the current commodity according to the level determined by the model. [The level of the R_LOUT variable for each commodity produced from a flexible process is controlled by the MR_PBL and MR_LIM equations.]

$$\left[\sum_{p \in prc \wedge LIMIT_{r,p,t}} R\_LOUT_{r,t,p,e} \right] +$$

Production of the current commodity from externally load managed processes, where the activity of the process (and thus the amount of each commodity produced) is directly determined from the level of installed capacity by means of a fixed capacity factor. [Note that flexible (LIMIT) processes may not be designated as externally load managed.]

$$\left[\sum_{p \in xpr} \left(CF_{r,p,t} * OUT(ENC)p_{r,p,e,t} * CAPUNIT_{r,p} \right) * R\_CAP_{r,t,p} \right] +$$

Annual production of the current commodity as a fixed ratio by-product of conventional (non-externally load managed) electric generation, summed over the season/time-of-day activity of power plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p\in(ela,\notin xlm),w \\ except\ y='N'\wedge p\in(bas\cup stg) \\ except\ ((y='N'+z='I')\wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(OUT(ENC)c_{r,p,e,t} * R\_TEZY_{r,t,p,w}\right)\right]+$$

Annual production of the current commodity as a by-product from externally load managed power plants, where the activity of the plant (and thus the amount of each commodity produced per unit of electricity or heat generated) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice.

$$\left[\sum_{p\in xlm}\left(\begin{array}{l} OUT(ENC)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ \sum_{z,y}\left(QHR(Z)(Y)_{r,z,y} * CF(Z)(Y)_{r,p,z,y,t}\right) * R\_CAP_{r,t,p}\end{array}\right)\right]+$$

Annual production of the current commodity as a by-product of low-temperature heat generation for conventional (non-externally load managed) facilities, summed over the seasonal activity of conventional power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p\in hpl\notin xpr,z \\ except\ ((z='S'\vee'I')\wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(OUT(ENC)c_{r,p,e,t} * R\_THZ_{r,t,p,z}\right)\right]+$$

Annual production of the current commodity as a by-product of low-temperature heat generation from coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of power plants. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd, w \\ when CEH(Z)(Y)_{r,p,w,t} \\ except(y='N' \wedge p \in bas)}} \left(OUT(ENC)c_{r,p,e,t} * R\_TCZYH_{r,t,p,w} \right) \right] +$$

Production of the current commodity as a fixed proportion of the activity of demand devices, taking into consideration demand device vintaging. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MO(ENC)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \left(R\_CAP_{r,t,p} \$ \{ not (EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \right) \end{array} \right) \right]$$

$$\rangle$$

$\{\geq ; =\}$, where ≥ for energy carriers, and = for materials and energy carriers with slack

CONSUMPTION - commodities leaving the system as Exports or consumed by Technologies according to their activity level.

EXPorts of a commodity, identified as the consumption of the commodity by means of INP(ENT)x.

$$\left[\sum_{\substack{s \forall INP(ENT)x_{r,s,e,t} \\ s='EXP'}} R\_TSEP_{r,t,s} \right] +$$

Export of a globally traded commodity.

$$\left[R\_GTRD_{r,t,e,'EXP'} \$ \begin{pmatrix} e \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right] +$$

Consumption of the current commodity as an auxiliary input required by resource supply options, according to the amount needed to produce another commodity (e.g., electricity for mining coal).

$$\left[\sum_{s \neq 'EXP'} \left(INP(ENT)r_{r,s,e,t} * R\_TSEP_{r,t,s} \right) \right] +$$

Consumption of the current commodity by conventional (non-externally load managed) processes according to the amount needed per unit of overall activity of the process.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(INP(ENT)p_{r,p,e,t} * R\_ACT_{r,t,p} \right) \right] +$$

Consumption of the current commodity by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p \in xpr} \begin{pmatrix} INP(ENT)p_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Annual consumption of the current commodity as required for a unit of electric generation from conventional (non-externally load managed) facilities, summed over the season/time-of-day activity of power plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela, w \\ except\ y='N' \wedge p \in (bas \cup stg) \\ except\ ((y='N' \wedge z='I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(INP(ENT)c_{r,p,e,t} * R\_TEZY_{r,t,p,w} \right) \right] +$$

Annual consumption of the current commodity by externally load managed power plants, where the activity of the plant (and thus the amount of each commodity required per unit of electricity or heat produced) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice.

$$\left[\sum_{p \in xlm} \left(\begin{array}{l} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * \\ \sum_{z,y} \left(QHR(Z)(Y)_{r,z,y} * CF(Z)(Y)_{r,p,z,y,t} \right) * R\_CAP_{r,t,p} \end{array} \right) \right] +$$

Annual consumption of the current commodity as required for a unit of low-temperature heat generation, summed over the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl, z \\ p \notin xlm \\ except\ ((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(INP(ENT)c_{r,p,e,t} * R\_THZ_{r,t,p,z} \right) \right] +$$

Annual consumption of the current commodity as required for a unit of low-temperature heat generation from coupled production pass-out turbines (identified by means of the ELM parameter), summed over the seasonal/time-of-day activity of power plants (as determined by CEH(Z)(Y)). [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd,w \\ when ELM_{r,p,t} \\ except(y='N' \wedge p \in bas)}} \left(INP(ENT)c_{r,p,e,t} * \left(CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} \right) * R\_TCZYH_{r,t,p,w} \right) \right] +$$

Consumption of the current commodity as required for a unit of output from demand devices. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} / EFF_{r,p,t} * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right) \right]$$

If the balance equation is to be formulated as a slack equality constraint, where the slack cost is provided (SLKCOST), then include the slack variable.

$$+R\_SLKENC_{r,t,e} \$ SLKCOST_{r,e,t}$$

2.5.5 MR_BALDH(r,t,e,z) MARKAL Matrix, Row 10

Description: **BAL**ance equation for low-temperature **D**istrict **H**eat energy carriers (lth) in each season.

Purpose and Occurrence: Like all balance equations, this one for low-temperature heat guarantees the conservation of energy throughout the energy system. The equation ensures that the production of low-temperature heat in each season is equal to or exceeds the demand for heat. It sums over all sources of low-temperature heat and ensures that this total is greater than or equal to the amount consumed by all demand devices (dmd). [Note: low-temperature heat may ONLY be consumed by demand devices]. Slack is permitted, as it may be the case owing to some rigid coupled heat and power plant (REH) that the amount of low-temperature heat generated may exceed the amount demanded in a season. However, the analyst should carefully investigate any slack on the low-temperature heat balance. This equation is generated in all time periods and for all seasons during which some demand device consumes the low-temperature heat.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding.* The equation is a greater than or equal (>=) constraint for each season.

Interpretation of the results:

Primal: The level of the low-temperature heat balance equation represents the slack or overproduction of low-temperature heat in a season. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of back-pressure turbines if electric demand forces the operation of such a plant it is possible that production may exceed the demand for low-temperature heat. Under normal circumstance the user should look to examine and possibly redress this situation by employing a flexible output coupled heat and power (CEH(Z)(Y)), or determine any other reason for the excess (e.g., lower limits forcing the operation of a power plant).

Dual variable: The dual variable (DVR_BALDH) of the low-temperature heat balance constraint (shadow price) indicates the amount that the energy system must pay per unit of low-temperature heat consumed and received per unit produced, in each season. A zero dual value implies that the supply of heat exceeds the demand by the rest of the energy system. The value is positive when the primal equation is tight, which is normally the case.

Remarks: Note that high temperature process heat is modeled as an annual energy carrier as it is for industrial use and not subject to seasonal variations.

GAMS Routine: MMEQBALH.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#5}: MR\_BALDH_{r,t,e,z} \; \forall e \in lth \wedge QHRZ_{r,z}$$

PRODUCTION – low temperature heat generated from either heating or coupled production power plants feeding the current heating grid.

Production of low temperature heat to the heat grid from conventional heating plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ except\,((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_THZ_{r,t,p,z}\right] +$$

Production of low temperature heat to the heat grid from externally load managed plants, where the activity of the plant (and thus the amount of heat produced) is directly determined from the level of installed capacity by means of a fixed capacity factor for each season. [Note that as heating plants are modeled by season (not time of day) CF(Z)(Y) should only be specified for y = 'D'.]

$$\left[\sum_{p \in (hpl \cap xlm)} \begin{pmatrix} CAPUNIT_{r,p} * \left(QHRZ_{r,z} * CF(Z)(Y)_{r,p,z,'D',t}\right) * \\ R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Production of low temperature heat to the heat grid from coupled heat and power pass-out turbines, for each time-slice (w=z,y) for which the electricity loss to heat gain ratio (CEH(Z)(Y)) is provided, as well as for which a time-sliced transmission efficiency (TRNEFF(Z)(Y)) is provided if centralized, and 1 otherwise. [Special conditions apply for base load plants that force them to operate at the same level day and night, thus only a day variable exists.]

$$\left[\sum_{\substack{p \in cpd, y \\ CEH(Z)(Y)_{r,p,z,y,t} \\ except\,(p \in bas \wedge y='N')}} \begin{pmatrix} TRNEFF(Z)(Y)_{r,p,z,y,t} \$ p \in cen * \\ R\_TCZYH_{r,t,p,z,y} \end{pmatrix}\right] +$$

Production of low temperature heat to the heat grid from coupled heat and power back-pressure turbines (as designated by REH), for each time-slice (w=z,y); for which a transmission efficiency (TRNEFF(Z)(Y)) is provided if centralized, and 1 otherwise. [Special conditions apply for base load plants that force them to operate at the same level day and night, thus only a day variable exists.]

$$\left[\sum_{\substack{p \in cpd, y \\ when\ REH_{r,p,t} \\ except\ (p \in bas \wedge y = 'N')}} \begin{pmatrix} (TRNEFF(Z)(Y)_{r,p,z,y,t} \$ p \in cen\ / REH_{r,p,t})* \\ R\_TCZYH_{r,t,p,z,y} \end{pmatrix} \right] +$$

$$\{\geq, =\}$$

where ≥ unless with slack

CONSUMPTION – of low temperature heat by demand devices, and heating grid transfers.

Consumption of low temperature heat by demand devices, taking into consideration demand device vintaging. The shape of the sector heating demand is controlled by FR(Z)(Y), and if the demand device services more than one demand sector then the output share (OUT(DM)) is taken into consideration.

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} CAPUNIT_{r,p} * MA(ENT)_{r,p,e,t} * CF_{r,p,t} / (DHDE(Z)_{r,p,z,t} * EFF_{r,p,t}) * \\ \sum_{d,y} \left(\begin{array}{l} OUT(DM)_{r,p,d,t} * FR(Z)(Y)_{r,d,z,y} * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right) \end{array} \right) \right] +$$

Transfer of low temperature heat to another conventional (non load managed) heat grid.

$$\left[\sum_{\substack{p \in hpl \\ p \notin xpr}} \left(INP(ENT)c_{r,p,e,t} / DHDE(Z)_{r,e,z,t} * R\_THZ_{r,t,e,z} \right) \right] +$$

Transfer of low temperature heat to another non load managed heat grid. [Note that as heating plants are modeled by season (not time of day) CF(Z)(Y) should only be specified for y = ‘D’.]

$$\left[\sum_{\substack{p \in hpl \\ p \in xlm}} \left(\begin{array}{l} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * \left(QHRZ_{r,z} * CF(Z)(Y)_{r,p,z,'D',t} \right) * \\ R\_CAP_{r,t,p} \end{array} \right) \right]$$

> If the balance equation is to be formulated as a slack equality constraint, where the slack cost is provided (SLKCOST), then include the slack variable.

$$+ R\_SLKLTH_{r,t,e,z} \$ SLKCOST_{r,e,t}$$

2.5.6 MR_BALE1/2(r,t,e,w) MARKAL Matrix, Row 11

Description: **BAL**ance equation for **E**lectricity energy carriers (elc) in each time-slice (w=z,y), where equation 1 corresponds to day and equation 2 to night in each season.

Purpose and Occurrence: Like all balance equations, this one for electricity guarantees the conservation of energy throughout the energy system. The equation ensures that the production (plus imports) of electricity in each time-slice is equal to or exceeds the demand (including exports) in the same time-slice. Equation 1 is for daytime constraints (w=z,'D') and equation 2 is for the night-time constraints (w=z,'N'). Slack is permitted (but very unlikely), and may occur due to some rigid coupled heat and power plant (REH) or to the existence of lower limits forcing the operation of a power plant. However, the analyst should carefully investigate any slack on the electricity balance. This equation is generated in all time periods and for all time-slices during which some resource option or technology produces or consumes the electricity.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding.* The equation is a greater than or equal (>=) constraint for each time-slice.

Interpretation of the results:

Primal: The level of the electricity balance equation represents the slack or overproduction of electricity in a time-slice. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of production/consumption. However, owing to the rigid nature of back-pressure turbines if heat demand forces the operation of such a plant it is possible that production may exceed the demand for electricity. Under normal circumstance the user should look to redress this situation by employing a flexible output coupled heat and power (CEH(Z)(Y)), or determine any other reason for the excess (e.g., base load constraint forces over-production at night, lower limits forcing the operation of a power plant).

Dual: The dual variable (DVR_BALE) of the electricity balance constraint (shadow price) indicates the amount that the energy system must pay per unit of electricity consumed, and receives per unit produced in each time-slice. A zero dual value implies that the supply of the commodity exceeds the demand for that commodity by the rest of the energy system. The value is positive when the primal equation is tight, which is normally the case.

Remarks: An important purpose of the electricity balance constraint is to bring into alignment the load shapes associated with the various demand sectors that consume electricity (DM_FR(Z)(Y)) with the fractions of the year that the power plants operate (QHR(Z)(Y)), taking into account annual and/or seasonal availability of each power plant (AF/AF(Z)(Y) or CF/CF(Z)(Y)).

GAMS Routine: MMEQBALE.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 6}: MR\_BALEk_{r,t,e,w} \ \forall \begin{pmatrix} (e \in elc) \wedge \\ QHR_{r,w} exists \end{pmatrix}$$

where k = 1 for days (w(z,'D')) and k = 2 for nights (w(z,'N'))

PRODUCTION – of electricity from imports and power plants feeding the current electricity grid.

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ s='IMP'}} \begin{array}{l} TE(ENT)_{r,e,t} * \\ \left\{ \begin{array}{l} \left(\begin{array}{l} \langle QHR(Z)(Y)_{r,w} \$ (no\, SEP\_FR_{r,s,w,t}) \vee (SEP\_FR_{r,s,w,t} \rangle * \\ R\_TSEP_{r,t,s} \end{array} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,w} \right) + \\ \left(R\_TSEPE_{r,t,s,w} \$ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,w} \right) \end{array} \right\} \end{array} \right] +$$

Production to the current electricity grid by conventional (non-externally load managed) electric generating plants for each season/time-of-day the plant is available (AF(Z)(Y), which is set from AF if only annual availability is specified). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Special conditions apply to base load and storage facilities, as well as peaking only devices, where base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only device are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela \\ p \notin xlm \\ AF(Z)(Y)_{r,p,w,t}}} \begin{array}{l} TE\_CON_{r,t,p,e} * \\ \left\langle \begin{array}{l} \left(\begin{array}{l} (QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) \$ p \subset bas \\ * R\_TEZY_{r,t,p,z,y} \end{array} \right) \$ \left(\begin{array}{l} y='D' \wedge \\ (not\, z='I' \wedge \\ PEAK\_TID_{r,p}) \end{array} \right) + \\ \left(\begin{array}{l} R\_TEZY_{r,t,p,z,y} \$ p \notin (bas \vee stg \vee PEAK\_TID_{r,p}) + \\ ((QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) * \\ (R\_TEZY_{r,t,p,z,'D'}) \$ p \in (bas \vee stg) \end{array} \right) \$ y='N' \end{array} \right\rangle \end{array} \right] +$$

Production to the current electricity grid by coupled heat and power pass-out turbines for each season/time-of-day the plant is available (AF(Z)(Y), which is set from AF if only annual availability is specified), and an electricity loss per unit of heat ratio (CEH(Z)(Y)) is provided. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Special conditions apply to base load facilities where only daytime variables (to force day=night generation) are permitted.]

$$\left[\sum_{\substack{p \in cpd \\ ELM_{r,p,t} \\ AF(Z)(Y)_{r,p,w,t}}} \left\langle \begin{array}{l} TE\_CON_{r,t,p,e} * \begin{pmatrix} CEH(Z)(Y)_{r,p,w,t} * \\ (1 - ELM_{r,p,t}) / ELM_{r,p,t} \end{pmatrix} * \\ \left[\begin{array}{l} \begin{pmatrix} ((QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) \$ p \in bas) * \\ R\_TCZYH_{r,t,p,z,y} \end{pmatrix} \$ \\ (y = 'D' \vee (y = 'N' \wedge p \notin bas)) + \\ (R\_TCZYH_{r,t,p,z,'D'} * (QHR(Z)(Y)_{r,w} / QHRZ_{r,z})) \$ \\ (y = 'N' \wedge p \in bas) \end{array} \right] \end{array} \right\rangle \right] +$$

Production of electricity from externally load managed plants and grid interchanges, where the activity of the plant is directly determined from the level of installed capacity by means of a fixed capacity factor (CF(Z)(Y)) for each time-slice.

$$\left[\sum_{p \in (ela \wedge xlm)} \begin{pmatrix} (QHRZ_{r,z} * CF(Z)(Y)_{r,p,w,t}) * CAPUNIT_{r,p} * TE\_CON_{r,t,p,e} * \\ R\_CAP_{r,t,p} \end{pmatrix} \right]$$

$$\{\geq\}$$

CONSUMPTION – of electricity by resource options and technologies, and as part of grid interchanges.

Export of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s\forall OUT(ENT)x_{r,s,e,t}\\ s='EXP'}}\left\{\begin{array}{l}\left(\left\langle\begin{array}{l}QHR(Z)(Y)_{r,w}\$(no\,SEP\_FR_{r,s,w,t})\vee\\ SEP\_FR_{r,s,w,t}\end{array}\right\rangle * \$\\ R\_TSEP_{r,t,s}\end{array}\right)\\ \left(not\sum_{r',e',c}BI\_TRDELC_{r,e,r',e',c,w}\right)+\\ \left(R\_TSEPE_{r,t,s,w}\$\sum_{r',e',c}BI\_TRDELC_{r,e,r',e',c,w}\right)\end{array}\right\}\right]+$$

Consumption of electricity as part of supplying another resource. The annual resource production electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e',t}\\ e'\neq e}}\left(QHR(Z)(Y)_{r,w}*INP(ENT)r_{r,s,e,t}*R\_TSEP_{r,t,s}\right)\right]+$$

Consumption of electricity by a conventional (non externally load managed) process. The annual process electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{p\in prc\\ p\notin xpr}}\left(QHR(Z)(Y)_{r,w}*INP(ENT)p_{r,s,e,t}*R\_ACT_{r,t,p}\right)\right]+$$

Consumption of electricity from the current grid by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p\in xpr}\left(\begin{array}{l}INP(ENT)p_{r,p,e,t}*CAPUNIT_{r,p}*CF_{r,p,t}*\\ R\_CAP_{r,t,p}\end{array}\right)\right]+$$

Transfer of electricity from the current grid to a conventional (non externally load managed) grid interchange technology, taking into consideration that base load plants only have daytime variables (to force equal day/night operation).

$$\left[\sum_{\substack{p\in \ln k\\ p\notin xlm}}\left(\begin{array}{l}\left(INP(ENT)c_{r,p,e,t}*R\_TEZY_{r,t,p,w}\right)\$\,p\notin bas*\\ \left((QHR(Z)(Y)_{r,w}/QHRZ_{r,z})*R\_TEZY_{r,t,p,z,'D'}\right)\$\,p\in bas\end{array}\right)\right]+$$

Transfer of electricity from the current grid to an externally load managed grid interchange technology for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p\in \ln k\\ p\in xlm}}\left(\begin{array}{l}INP(ENT)c_{r,p,e,t}*CAPUNIT_{r,p}*CF(Z)(Y)_{r,p,w,t}*\\ R\_CAP_{r,t,p}\end{array}\right)\right]+$$

Consumption of electricity as required for a unit of low-temperature heat generation, split into season/time-of-day based upon the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p\in hpl\\ p\notin xlm\\ except\ ((z='S'\vee'I')\wedge\\ PEAK(CON)\_TID_{r,p}))}}\left(\begin{array}{l}INP(ENT)c_{r,p,e,t}*QHR(Z)(Y)_{r,w}/QHRZ_{r,z}*\\ R\_THZ_{r,t,p,z}\end{array}\right)\right]+$$

Consumption of electricity by pumped storage facilities at night, based upon the daytime production.

$$\left[\sum_{\substack{p\in stg\\ y='N'}}\left(INP(ENT)c_{r,p,e,t}*TE(ENT)_{r,e,t}*R\_TEZY_{r,t,p,z,'D'}\right)\right]+$$

Consumption of electricity by demand devices according to the amount of electricity needed per unit output (a function of the market share of electricity to the device (MA(ENT)) and the device efficiency (EFF)) and the seasonal/time-of-day shape (DM_FR) of the demand load being serviced by the share of the output from the device to the sector (DM OUTX).

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} / EFF_{r,p,t} * CAPUNIT_{r,p} * \\ \sum_{d} \left(\begin{array}{l} DM\_FR(Z)(Y)_{r,d,w} * DM\_OUTX_{r,p,d,t} * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not (EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right) \end{array} \right) \right]$$

2.5.7 MR_BAS(r,t,e,z) MARKAL Matrix, Row 12

Description: The **BAS**e load constraint for electricity energy carriers (elc) in each season (z).

Purpose and Occurrence: The base load constraint ensures that those power plants designated as base load (bas) operate at the same level in the day/night and do not exceed a specified percentage of the highest night-time electricity demand (according to BASELOAD). The main purpose of the constraint is to ensure that enough power plants that are characterized as base load are available in each period, but do not by themselves meet all the off-peak demand for electricity. Such classes of power plants (e.g., hydro, large coal-fired, nuclear) cannot be easily started/shutdown/restarted, but instead tend to be run at full capacity almost all of the time. In MARKAL this results in a single activity variable (R_TEZY) being generated (for the day (y='D')) and the appropriate day and night share multipliers applied to ensure continuous operation (other than for maintenance). This equation is generated in all time periods and for each season during which power plants or imports, produce electricity.

Units: PJ, or any other unit in which energy is tracked.

Type: *Binding.* The equation is a less than or equal (<=) constraint for each season.

Interpretation of the results:

Primal: The level of the base load equation represents the level below the maximum percent of highest night-time demand that is met by base load power plants. If zero, then the maximum base load constraint is tight and demand is fully met only by base load plants.

Dual variable: The dual variable (DVR_BAS) of the base load constraint (shadow price) indicates the "cost" that the energy system must incur because it cannot install more base load power plants. A zero dual value implies that the amount of electricity from the base load plants is below the permitted percent of maximum night-time demand. The value is negative when the primal equation is tight. It thus represents the amount by which the objective function would be reduced if one more unit of base load power generation were permitted in the season with the highest night-time demand.

Remarks: The technique used of employing a daytime electricity generation variable (R_TEZY), and then apply the appropriate splits means that the analyst should avoid referring directly to the levels reported in the (GAMS) solution listing, but rather strictly rely on the level reported in the results Tables, that take into account the need to apply the appropriate factors to the variables. Note that non-load management power plants (nlm) are included among the plants contributing to the base load constraint, but that these plants may operate normally, not being forced to operate at equal levels day and night.

GAMS Routine: MMEQBAS.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#7}: MR\_BAS_{r,t,e,z} \ \forall \begin{pmatrix} (e \in elc) \wedge \\ QHRZ_{r,z} exists \end{pmatrix}$$

Contribution to meeting the base load requirement, that is that the night-time electric generation from base load plants does not exceed a specified percent of total night-time demand for electricity in each season.

Import of night-time electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ s='IMP' \wedge (not\ nobasie)}} \begin{matrix} TE(ENT)_{r,e,t} * (1 - BASELOAD_{r,e,t}) * \\ \left\{ \begin{matrix} \left(\left\langle \begin{matrix} QHR(Z)(Y)_{r,z,'N'} \$ (no\ SEP\_FR_{r,s,z,'N',t}) \vee \\ (SEP\_FR_{r,s,z,'N',t} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'} \$ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'N'} \right) \end{matrix} \right\} \end{matrix} \right] +$$

Night-time production sent to the current electricity grid by conventional (non-externally load managed) base load electric generating plants. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Remember, base load plants only have daytime variables from which night production is calculated based upon the length of the time-slice (QHR(Z)N).]

$$\left[\sum_{p \in bas} \left(\begin{matrix} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \left(QHR(Z)(Y)_{r,z'N'} / QHRZ_{r,z} \right) * \\ R\_TEZY_{r,t,p,z,'D'} \end{matrix} \right) \right] +$$

Night-time production sent to the current electricity grid by externally load managed base load electric generating plants, where activity is determined by the level of installed capacity. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p \in bas \\ p \in xlm}} \left(\begin{matrix} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * CAPUNIT_{r,p} * \\ CF(Z)(Y)_{r,z,'N'} * QHR(Z)(Y)_{r,z,'N'} * R\_CAP_{r,t,p} \end{matrix} \right) \right] +$$

Night-time production sent to the current electricity grid by base load coupled heat and power pass-out turbines, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON). [Remember, base load plants only have daytime variables from which night production is calculated based upon the length of the time-slice (QHR(Z)N). However only daytime CEH(Z)(Y) should be provided.]

$$\left[\sum_{\substack{p \in bas \cup cpd \\ ELM_{r,p,t}}} \begin{pmatrix} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \left(QHR(Z)(Y)_{r,z'N'} / QHRZ_{r,z}\right) * \\ \left(CEH(Z)(Y)_{r,p,z,'D',t} * (1 - ELM_{r,p,t}) / ELM_{r,p,t}\right) * R\_TCZYH_{r,t,p,z,'D'} \end{pmatrix}\right] +$$

Night-time production sent to the current electricity grid by non-load managed electric generating plants that can contribute to the base load requirements. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{p \in nlm} \left(TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * R\_TEZY_{r,t,p,z,'N'}\right)\right] +$$

Night-time production sent to the current electricity grid by non-load managed coupled heat and power pass-out turbines that can contribute to the base load requirements, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p \in nlm \cup cpd \\ ELM_{r,p,t}}} \begin{pmatrix} TE\_CON_{r,t,p,e} * (1 - BASELOAD_{r,e,t}) * \\ \left(CEH(Z)(Y)_{r,p,z,'N',t} * (1 - ELM_{r,p,t}) / ELM_{r,p,t}\right) * \\ R\_TCZYH_{r,t,p,z,'N'} \end{pmatrix}\right]$$

$$\left\{\leq\right\}$$

Electric generation by non-base load power plants at night.

Export of night-time electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s\forall OUT(ENT)r_{r,s,e,t} \\ s='EXP'\wedge(not\, nobasie)}} \begin{matrix} TE(ENT)_{r,e,t}*(1-BASELOAD_{r,e,t})* \\ \left\{ \begin{matrix} \left(\begin{matrix} \left\langle \begin{matrix} QHR(Z)(Y)_{r,z,'N'}\$ \\ (no\,SEP\_FR_{r,s,z,'N',t})\vee \\ (SEP\_FR_{r,s,z,'N',t} \end{matrix} \right\rangle * \\ R\_TSEP_{r,t,s} \end{matrix} \right) \$\left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) + \\ \left(R\_TSEPE_{r,t,s,z,'N'}\$ \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'N'} \right) \end{matrix} \right\} \end{matrix} \right] +$$

Night-time production sent to the current electricity grid by conventional (non-externally load managed) non-base load electric generating plants. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE CON).

$$\left[\sum_{\substack{p\in ela \\ p\notin bas}} \left(\begin{matrix} TE\_CON_{r,t,p,e}*BASELOAD_{r,e,t}*\left(QHR(Z)(Y)_{r,z,'N'}/QHRZ_{r,z}\right)* \\ R\_TEZY_{r,t,p,z,'N'} \end{matrix} \right)\right] +$$

Night-time production sent to the current electricity grid by externally load managed non-base load electric generating plants, where activity is determined by the level of installed capacity. Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE CON).

$$\left[\sum_{\substack{p\in ela \\ p\notin bas \\ p\in xlm}} \left(\begin{matrix} TE\_CON_{r,t,p,e}*BASELOAD_{r,e,t}*CAPUNIT_{r,p}* \\ CF(Z)(Y)_{r,p,z,'N',t}*QHR(Z)(Y)_{r,z,'N'}*R\_CAP_{r,t,p} \end{matrix} \right)\right] +$$

Night-time production sent to the current electricity grid by non-base load coupled heat and power pass-out turbines at night in each season, taking into consideration the electricity loss per unit of heat ratio (CEH(Z)(Y)). Electricity sent to the grid is subject to any grid transmission efficiency if the plant is centralized (TE_CON).

$$\left[\sum_{\substack{p \in cpd \\ p \notin bas \\ ELM_{r,p,t}}} \left(\begin{array}{l} TE\_CON_{r,t,p,e} * BASELOAD_{r,e,t} * \\ \left(QHR(Z)(Y)_{r,z'N'} / QHRZ_{r,z}\right) * \\ \left(CEH(Z)(Y)_{r,p,z,'N',t} * (1 - ELM_{r,p},t) / ELM_{r,p,t}\right) * \\ R\_TCZYH_{r,t,p,z,'N'} \end{array}\right)\right]$$

2.5.8 MR_BITRD(r,e,r,t,e,c) MARKAL Matrix, Row 13

Description: The **BI**-lateral **TR**a**D**e constraint matches up the trade in a commodity between two regions. Note that the name of the commodity may be different between the two regions, if desired, but the same cost step (trade route) must be used.

Purpose and Occurrence: The bi-lateral trade constraint ensures that the amount of a commodity delivered from a source region equals the amount received by the destination region. It includes provisions to convert the units (REG_XCVT) if necessary. Note that the individual export and import supply options must also be specified in each region as part of the list of resource options (srcencp). This equation is generated in each time period from which both resource options first become available in their respective regions, and the mapping set indicating the pairing of the importing/exporting of the commodity between the regions is provided (bi_trdent).

Units: PJ, or any other unit in which a commodity is tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the bi-lateral trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual variable: The dual variable (DVR_BITRD) of the bi-lateral trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of trade. A zero dual value implies that there is no cost associated with producing and delivering the commodity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: This bi-lateral trade constraint is an annual constraint. Electricity may be traded by season/time-of-day if desired by means of the bi-lateral electricity trade constraint (MR_BITRDE). Note that to force trade between regions one of the resource variables in the region (R_TSEP) can be bounded.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#8}: MR\_BITRD_{r,e,t,r',e',c} \ \forall \left(\begin{array}{l} BI\_TRDENT_{r,e,t,r',e',c} \wedge \\ \left(TPSEP_{r,t,"EXP",e,c} \wedge TPSEP_{r',t,"IMP",e',c} \right) \end{array} \right)$$

Where BI_TRDENT = mapping of permitted bi-lateral trade options
TPSEP = 2-tuple indicating the periods (from START) that a resource supply option is available in a region

> Imports to region r', possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left[REG\_XCVT_{r,e} * R\_TSEP_{r,t,"EXP",e,c} \right] -$$

> Exports from region r, possibly with unit conversion applied for the commodity. [Note that while the commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left[REG\_XCVT_{r',e'} * R\_TSEP_{r',t,"IMP",e',c} \right]$$

$$\{=\}$$

$$0$$

2.5.9 MR_BITRDE(r,e,r,t,e,c,w) MARKAL Matrix, Row 14

Description: The **BI**-lateral **TR**a**D**e of **E**lectricity constraint matches up the trade in an electricity commodity between two regions in each time-slice. Note that the name of the electricity commodity may be different between the two regions, if desired, but the same cost step (trade route) must be used.

Purpose and Occurrence: The bi-lateral trade of electricity constraint ensures that the amount of electricity delivered from a source region in each time-slice equals the amount received into the destination region. It includes provisions to convert the units (REG_XCVT) if necessary. Note that the individual export and import supply options must also be specified in each region as part of the list of resource options (srcencp). This equation is generated in each time period from which both exchange options first become available in their respective regions, and the mapping set (bi_trdelc) indicates that the pairing of the importing/exporting of electricity between the regions is permitted in the time-slice.

Units: PJ, or any other unit in which electricity is tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the bi-lateral electricity trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual variable: The dual variable (DVR_BITRDE) of the bi-lateral electricity trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of that trade. A zero dual value implies that there is no cost associated with producing and delivering the electricity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: This bi-lateral electricity trade constraint is a time-sliced constraint. Electricity may also be traded annually if desired by means of the conventional commodity trade constraint (MR_BITRD). Note that there is currently no way to bound the amount of electricity exchanged between regions for a particular time-slice, unless distinct time-slice based resource options (srcencp with SEP_FR for only 1 season/time-of-day for each possible exchange) are specified in each region. The commodity conversion factor (REG_XCVT) needs to be used to make any necessary adjustments in the time-slice fractions if different between two regions. Note that this constraint works in tandem with the MR_REGELC constraint, which hooks the time-slice based bi-lateral electric trade variables (R_TSEPE) with the traditional annual resource variable (R_TSEP).

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 9}: MR\_BITRDE_{r,e,t,r',e',c,w} \;\forall \left(\begin{array}{l} e \in elc \wedge \\ BI\_TRDELC_{r,e,t,r',e',c,w} \wedge \\ \left(\begin{array}{l} TPSEP_{r,t,\text{"}EXP\text{"},e,c} \wedge \\ TPSEP_{r',t,\text{"}IMP\text{"},e',c} \end{array} \right) \end{array} \right)$$

Where bi_trdelc = mapping of permitted bi-lateral electric trade options, including permitted time-slices

tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

Exports from region r, possibly with unit conversion applied for the electricity commodity. [Note that while the electricity commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEPE_{r,t,"EXP",e,c,w} \right\rfloor -$$

Imports to region r', possibly with unit conversion applied for the electricity commodity, and subject to any transmission losses. [Note that while the electricity commodities may have different names in each region, the cost index (c) must be the same in both regions.]

$$\left\lfloor REG\_XCVT_{r,e} * R\_TSEPE_{r,t,"IMP",e,c,w} \right\rfloor$$

$$\{=\}$$

$$0$$

2.5.10 MR_BNDCON1/2/3(r,t,p) MARKAL Matrix, Row 15

Description: Annual **B**ou**ND** on the output from a **CON**version technology.

Purpose and Occurrence: The annual bound constraint imposes a limit on the total output from conversion technologies for all relevant time-slices. In the case of electric generation and coupled heat and power plants the (up to 6) season/time-of-day time-slices are summed, for heating plants only for the winter season. The equation is generated for each period for which the user specifies an output limit (BOUND(BD)O) on a conversion technology.

Units: PJ, or any other unit in which activity of power plants is tracked.

Type: *Binding.* The type of the equation is a function of the sense (b = 'LO', 'FIX', 'UP') specified by the user when providing the bound parameter.

Interpretation of the results:

Primal: The level indicates any slack on a constraint imposing a production limit above/below a minimum/maximum permitted level.

Dual variable: The dual variable (DVR_BNDCON) of the annual conversion bound constraint represents the amount by which the objective function would be changed if the bound were increased by one unit. A zero dual value implies that the annual output from a power plant is below the permitted maximum or above the required minimum level. The value is negative when the limit is a maximum, positive when a lower limit is imposed, and the constraint is tight.

Remarks: The only way that the user can limit the level of production in an individual time-slice (or collection less than the entire year, e.g., a season) is to apply a user-defined constraint using the appropriate time-sliced based parameters (RAT_TCZY, RAT_TEZY, RAT_HPL). Note also that if the technology is externally load managed (xlm), that results in only a capacity variable (R_CAP) being generated, then the annual activity bound is applied directly to R_CAP properly taking into consideration the capacity factor(s) (CF/CF(Z)(Y)), unit (CAPUNIT) and time-slices (QHR(Z)(Y)).

GAMS Routine: MMEQBCON.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 10}: MR\_BCONn_{r,t,p} \ \forall \left(\begin{array}{l} p \in con \wedge p \notin xlm \wedge \\ \sum_{b} BOUND(BD)O_{r,p,b,t} \end{array} \right)$$

where n indicates the nature of the constraint such that n = 1 for b='LO' (≥), n = 2 for b='FX' (=), and n = 3 for b='UP' (≤).

Total annual electricity generation from standard power plants, summed over the season/time-of-day activity of power plants when the plant is available (AF(Z)(Y)). [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{p \in ela, w \\ except \ y='N' \wedge p \in (bas \cup stg) \\ AF(Z)(Y)_{r,p,w,t} \\ except \ ((y='N' \vee z='I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_TEZY_{r,t,p,w} \right] +$$

Total annual production of low-temperature heat generation from coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of power plants. As total production is bounded the total electricity output and this hybrid heat term are summed. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd, w \\ ELM_{r,p,t} \\ except(y='N' \wedge p \in bas)}} \begin{pmatrix} CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} \\ * R\_TCZYH_{r,t,p,w} \end{pmatrix}\right] +$$

Total annual production of low temperature heat from heating plants. [Special conditions apply to peaking plants, where only daytime winter variables are generated.]

$$\left[\sum_{\substack{p \in hpl, z \\ except((z='I' \vee 'S') \wedge \\ PEAK(CON)\_TID_{r,p}))}} R\_THZ_{r,t,p,z}\right]$$

$$\{\leq ; = ; \geq\}$$

Limit on total annual production from the conversion plant.

$$BOUND(BD)O_{r,p,b,t}$$

2.5.11 MR_CPT1/2/3(r,t,p) MARKAL Matrix, Row 16

Description: **C**a**P**acity **T**ransfer of a technology across time periods, accounting for all existing residual capacity and new investments made up to the present time period that remain available.

Purpose and Occurrence: The capacity transfer relation ensures that the existing capacity of each technology in each time period is the result of investments in the current and previous time periods within its lifetime, plus the residual capacity from investments made before the initial model year and still active. Its main purpose is to properly monitor the turnover of the capital stock. *This is an inter-temporal equation and does not contribute to the Reference Energy System of the model.* This equation is generated in all time periods beginning from the START period for each technology for which the LIFE or residual capacity (RESID) has been specified. If LIFE is not specified, or is less-than-or-equal to 1, and no RESID exists, then the MR_CPT equation is not generated, as there is no carryover of capital stock between periods.

Units: Gw, or PJ/a, or Bvkm/a, or any other unit in which the capacity of a technology is specified (e.g., parameters INVCOST, RESID, FIXOM, etc).

Type: *Binding.* The equation is a less than or equal (≤) if the technology is a demand device (dmd), see Interpretation of Results below. The relation is an equality (=) if the technology is a conversion plant (con) or a process (prc).

Interpretation of the results:

Primal: The level of capacity transfer constraint in the solution output (primal or slack value) is the value of the left hand side (LHS) of the CPT equation, that is the difference between the capacity in place in time period t minus the new capacities built / purchased in previous time periods still in operation. In processes and power plants the level always has to be equal to the residual capacity (RESID), which is the right hand side of the constraint (RHS); demand devices may have a LHS that may be lower than the RHS. This may happen when the RESID is larger than required by demands, or when the demand fluctuates up and down between successive time periods, etc.

Dual variable: The dual variable (DVR_CPT) of the capacity transfer constraint in the solution (shadow price) indicates the annuity to be paid in the time period t to pay back the investment cost incurred to build a new unit of capacity of the technology p in time period t. In the case of end-use technologies, the dual value is zero when there is excess capacity at some period, and positive if some new investment is made by the model in the current time period. For process and conversion technologies, the value may be positive or negative.

Remarks: The user should omit the lifetime parameter of a technology (and thereby avoid generating extra relations) in cases where (a) there are no competing technologies, e.g., a demand category such as air transport that might have only one technology (jet) to satisfy the demand; or where (b) the technology is not a real device but acts only to transfer energy carriers, e.g., a demand technology that supplies oil to the petrochemicals industry or a process technology that mixes fuels (such technologies are usually referred to as *dummy technologies*).

GAMS Routine: MMEQCPT.INC, FRACLIFE.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 11}: MR\_CPTn_{r,t,p} \;\forall\; LIFE_{r,p} \;exists$$

where n indicates the nature of the constraint such that n = 1 for demand devices (dmd) where the constraint is (≥), n = 2 for conversion plants and n = 3 for processes where the constraint is (=).

Total installed capacity in place in the current period.

$$\left[R\_CAP_{r,t,p}\right] -$$

Sum of all investments made in earlier periods that have not yet reached their technical lifetimes (LIFE), and thus are still available. [Note that an earlier investment may only be available for part of the period if the lifetime is not a multiple of the number of years per period (NYRSPER). This is handled by the CPT_INV coefficient as discussed further below.]

$$\left[\sum_{u=t-LIFE}^{t}\left(CPT\_INV_{r,u,p} * R\_INV_{r,u,p}\right)\right]$$

$$\{\geq;=\}$$

The residual capacity installed prior to the start of the modeling horizon that is still available in the current period.

$$RESID_{r,p,t}$$

Where *t,u* are time periods: t is the current period and u is the period in which the investment was originally made;

CPT_INV = 1 for u = t-LIFE+1, FRACLIFE for the last period (that means investments made t-LIFE periods earlier, where it may be a fraction of the period if LIFE is not a multiple of the period length (NYRSPER)), and

FRACLIFE = 1 except for the last period where it is (no. of years the technology exists in the period/NYRSPER).

2.5.12 MR_CUM(r,s) MARKAL Matrix, Row 17

Description: The **CUM**ulative or total limit on the production of a commodity from a resource activity over the entire modeling horizon (CUM).

Purpose and Occurrence: The cumulative resource constraint limits the total production from a coalmine or oil/gas well, or is a contracted maximum for imports/exports. It is most often used to indicate total proven reserves.

Occurrence: The equation is generated for each resource option (srcencp) for which the user specifies a cumulative limit (CUM).

Units: PJ, or any other unit in which a commodities are tracked.

Type: *Binding.* The type of the equation is a less than or equal to (≤).

Interpretation of the results:

Primal: The level indicates the cumulative production from a resource supply option.

Dual variable: The dual variable (DVR_CUM) of the cumulative resource constraint (shadow price) indicates how desirable is an additional unit of the resource option. A zero dual value implies that the limit has not been reached at period t (yet). The value is negative when the imposed limit is reached and represents the amount by which the cost objective function would be reduced if one more unit of production were permitted.

Remarks: In a clairvoyant model like MARKAL the model can optimize the use of limited reserves over the entire horizon. This contrasts with the situation for a time-stepped model like SAGE (see PART III).

Note that since the variables in the model represent annual production, the resource variable must be multiplied by the number of years in each period when monitoring the cumulative amount.

GAMS Routine: MMEQCUM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#12}: MR\_CUM_{r,s} \ \forall \begin{pmatrix} CUM_{r,s}\ exists, \\ except\ src = 'STK' \end{pmatrix}$$

Each annual resource supply option is multiplied by the number of years per period to determine the total cumulative amount supplied over the modeling horizon.

$$\sum_{t} NYRSPER * R\_TSEP_{r,s,t}$$

$$\{\leq\}$$

Cumulative limit imposed by the user.

$$CUM_{r,s}$$

Where NYRSPER is the number of years in each period.

2.5.13 MR_DESEP(r,t,s) MARKAL Matrix, Row 18

Description: The inter-temporal **DE**cay constraint on resource supply (**S**rc**E**nc**P**) options.

Purpose and Occurrence: To impose a limit on the rate at which a resource supply option can decrease (DECAYr) between periods. This equation is generated for all time periods for which the decay rate parameter (DECAYr) is provided.

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a greater than or equal (≥) constraint.

Interpretation of the results:

Primal: The level of the resource decay equation indicates how much reduction in a resource supply option has occurred between two periods.

Dual variable: The dual variable (DVR_DESEP) of the resource decay constraint (shadow price) indicates the amount that the energy system would be willing to pay for one less unit of decay in the resource supply option. A zero dual value implies that the supply of the resource has not reached the limit imposed on the rate of decay between two periods. The value is negative when the primal equation is tight, and the system wants to use less of the resource in the current period.

Remarks:

GAMS Routine: MMEQDES.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#13}: MR\_DESEP_{r,t,s} \ \forall DECAYr_{r,s,t}$$

Supply of the resource in the previous period, with annual decay rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\left[-(DECAYr_{r,s,t}**NYRSPER)*R\_TSEP_{r,t-1,s}\right]+$$

Supply of the resource in the current period.

$$R\_TSEP_{r,t,s}$$

$$\{\geq\}$$

$$0$$

2.5.14 MR_DETCH(r,t,p) *MARKAL Matrix, Row 19*

Description: The inter-temporal **DE**cay constraint on a **T**e**CH**nology.

Purpose and Occurrence: To impose a limit on the rate at which a technology capacity can decrease (DECAY) between periods. This equation is generated for all time periods for which the decay rate parameter (DECAY) is provided.

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a greater than or equal (≥) constraint.

Interpretation of the results:

Primal: The level of a technology decay equation indicates how much reduction in a technology capacity level has occurred between two periods.

Dual variable: The dual variable (DVR_DETCH) of the technology decay constraint (shadow price) indicates the amount that the energy system would be willing to pay for one less unit of decay in the technology level. A zero dual value implies that the technology has not reached the limit imposed on the rate of decay between two periods. The value is negative when the primal equation is tight, and the system wants to use less of the technology in the current period.

Remarks:

GAMS Routine: MMEQDET.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#14}: MR\_DETCH_{r,t,p} \ \forall DECAY_{r,p,t}$$

Technology capacity in the previous period, with annual decay rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\left[-(DECAY_{r,p,t} ** NYRSPER) * R\_CAP_{r,t-1,p}\right] +$$

Technology capacity in the current period.

$$R\_CAP_{r,t,p}$$

$$\{\geq\}$$

$$0$$

2.5.15 MR_DEM(r,t,d) *MARKAL Matrix, Row 20*

Description: The **DEM**and constraint that ensures that all sector demands for useful energy services (DEMAND) are satisfied.

Purpose and Occurrence: The demand constraint ensures that for each energy service, the total output of energy service from demand devices to the sector they serve meets or exceeds the level of the demand for that energy service. In addition, MARKAL employs price elastic demands, that may rise/fall (relative to the base case level) in response to changes in the implicit price of the service. See section 1.5 for a detailed discussion on elastic demands. The equation is generated for each demand sector (dm) in each period for which a demand for energy services (DEMAND) has been provided by the user. Also, the elastic demand variables (R_ELAST) are generated only for the demand sectors for which the appropriate input data is provided, in particular the number of step-wise approximation blocks (MED_STEP) to be used for estimating the demand curve, and the elasticities (MED_ELAST).

Units: PJ, bvmt, or any other unit in which a demand for energy services is specified.

Type: *Binding.* The type of the equation is equal to or greater than (≥).

Interpretation of the results:

Primal: The level indicates the final level of useful energy services delivered to a demand sector. The final level is both a function of the output of the individual demand devices servicing the sector, as well as the movement up or down of the demand level from the reference case level in response to the own price elasticities (MED_ELAST) provided by the analyst.

Dual variable: The dual variable (DVR_DEM) of the demand constraint represents the cost of meeting the last unit of demand services for the sector, i.e. the market price of the energy service. A zero dual value implies that the output of the various demand devices exceeds the final level, and such a situation should be examined and resolved by the analyst. Normally the value is negative and thus represents the amount by which the objective function would be reduced if one less unit of demand were required.

Remarks: The analyst controls the activation of the elastic demand formulation by means of the model variant selected at run time (Elastic Demand 'YES'), and the inclusion of the required elastic data.

GAMS Routine: MMEQDEM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#15}: MR\_DEM_{r,t,d} \ \forall DEMAND_{r,d,t}$$

Useful energy demand provided to the current demand sector according to any fractional output shares (OUT(DM)), taking into consideration demand device vintaging. Remember, there are no activity variables for demand devices, rather activity is directly derived from the total installed capacity based upon the capacity factor.

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} OUT(DM)_{r,p,d,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not (EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t}) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right) \right] -$$

The growth in demand owing to price relaxation, where each step of the approximation of the producer/consumer curve is “climbed” in sequence (j). The own price elasticity (change in cost per unit change in demand) and variation associated with each step of the curve is reflected in the objective function.

$$\left[\sum_{j \$ \left(\begin{array}{l} (MED\_STEP_{r,d,'UP',t} \\ \wedge MED\_ELAST_{r,d,'UP',t}) \end{array} \right)} R\_ELAST_{r,t,'UP',d} \right] +$$

The reduction in demand owing to price pressure, where each step of the approximation of the producer/consumer curve is “descended” in sequence (j). The own price elasticity (change in cost per unit change in demand) and variation associated with each step of the curve is reflected in the objective function.

$$\left[\sum_{j \$ \left(\begin{array}{l} (MED\_STEP_{r,d,'LO',t} \\ \wedge MED\_ELAST_{r,d,'LO',t}) \end{array} \right)} R\_ELAST_{r,t,'LO',d} \right]$$

$$\{ \geq \}$$

Reference demand for energy services.

$$DEMAND_{r,d,t}$$

2.5.16 MR_ENV(r,v) MARKAL Matrix, Row 21

Description: The total cumulative emissions or other **ENV**ironmental indicators (env) over the entire modeling horizon. If desired the level may be capped by means of the ENV_CUM input parameter.

Purpose and Occurrence: The cumulative emissions equation monitors the total production of an environmental indicator from all resource options and technologies. The equation is generated for each emission or other environmental indicator option. The equation is generated regardless of whether or not the user (srcencp) specifies a cumulative limit (CUM).

Units: Tons, kton or any other unit in which emission indicators are tracked.

Type: *Binding.* The type of the equation is a less than or equal to (≤), but with an unlimited (infinity) right-hand-side value if no limit is provided by the user.

Interpretation of the results:

Primal: The level indicates cumulative production from an emissions indicator over the entire modeling horizon.

Dual variable: The dual variable (DVR_ENV) of the cumulative emissions equation (shadow price) indicates how much the energy system would want to pay in order to relax the emission limit by one unit. A zero dual value implies that the limit has not been reached (yet). In particular, this is the case if no limit has been specified by the user. The value is negative when the imposed limit is reached and represents the amount by which the objective function would be reduced if one more unit of emission were permitted.

Remarks: In a clairvoyant model like MARKAL the model can optimize the pattern of emissions over the entire horizon while respecting an overall cumulative emission cap. This contrasts with the situation for a time-stepped model like SAGE (see PART III).

Note that since the variables in the model represent annual production, the resource variable must be multiplied by the number of years in each period when monitoring the cumulative amount.

GAMS Routine: MMEQENV.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#16}: MR\_ENV_{r,v}$$

Total period emissions determined by multiplying the annual emissions by the number of years per period (NYRSPER) are summed over the entire modeling horizon.

$$\left[\sum_{t} NYRSPER * R\_EM_{r,t,v} \right]$$

$$\{\leq\}$$

Cumulative emissions limit imposed by the user, otherwise non-binding (+INF).

$$CUM_{r,v}$$

Where NYRSPER is the number of years in each period,
CUM is either +INF if not provided, or the cumulative emissions limit provided by the user.

2.5.17 MR_EPK(r,t,e,z) MARKAL Matrix, Row 22

Description: The **E**lectricity **P**ea**K**ing constraint ensures that there is enough capacity in place to meet the electricity demand during the day of the season with the highest demand.

Purpose and Occurrence: The electricity peaking constraint ensures that there is enough capacity in place to meet the electricity demand during the day of the season with the highest demand taking into consideration an estimate of the level above that demand that corresponds to the actual peak moment plus a reserve margin of excess capacity (ERESERV includes both), to ensure that if some capacity is unavailable the highest demand for electricity (including unforeseen capacity reductions) can still be met. This equation is generated for all time periods and for the 'W'inter and 'S'ummer seasons (z), during which some resource option or technology produces or consumes the electricity.

Units: PJ, or any other unit in which electricity is tracked. Note that although the constraint is thought of in terms of capacity, it is actually modeled in energy units, based upon consumption and production arising from the associated capacity.

Type: *Binding.* The equation is a greater than or equal (>=) constraint for each season.

Interpretation of the results:

Primal: The level of the electricity peaking equation represents the slack or excess capacity over and above the amount required based upon the season in which the highest demand occurs, plus the electricity reserve (ERESERV) margin provided by the analyst. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of capacity above the minimum level required. If there is slack the user should check the residual capacity levels (RESID) to ensure that they properly reflect the level in place prior to the modeling period, and review the value of the ERESERV parameter (remembering that is has a dual role of estimating both how much above the level of highest average electric demand the peak moment is, plus the traditional requirement for extra 'just in case' capacity).

Dual: A zero dual value implies that the available capacity exceeds the requirements imposed by the constraint. The value is negative when the primal equation is tight, which is normally the case, and indicates how much the objective function would be lowered if the reserve constraint was one unit lower.

Remarks: As already noted, the electricity peaking constraint is highly dependent upon the reserve margin (ERESERV), that is usually substantially above that of a traditional utility reserve margin.

GAMS Routine: MMEQEPK.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 17}: MR\_EPK_{r,t,e,z} \ \forall \begin{pmatrix} e \in elc \wedge z \neq 'I' \wedge \\ QHRZ_{r,z} exists \end{pmatrix}$$

PEAK Contribution – imports and peak technology capacity credit towards the peaking requirement, as well as grid interchanges.

Import of electricity, either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice. The contribution is subject to transmission losses as well as the reserve margin requirements.

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e,t} \\ s='IMP'}} \begin{array}{c} PEAKDA(SEP)_{r,s,e,t} * TE(ENT)_{r,e,t} / (1+ERESERV_{r,e,t}) * \\ \left\{ \begin{array}{l} \left(\left\langle \begin{array}{l} 1\$(no\ SEP\_FR_{r,s,z,'D',t}) \vee \\ (SEP\_FR_{r,s,z,'D',t} / QHR(Z)(Y)_{r,z,'D'} \end{array} \right\rangle * R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'D'} \right) + \\ \left(\begin{array}{l} \left\langle R\_TSEPE_{r,t,s,z,'D'} / QHR(Z)(Y)_{r,z,'D'} \right\rangle \$ \\ \sum_{r',e',c} BI\_TRDELC_{r',e',r,e,c,z,'D'} \end{array} \right) \end{array} \right\} \end{array} \right] +$$

Deliverable capacity from non-peaking (PEAK_TID) power plants, and grid exchange link technologies, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements. The contribution is subject to transmission losses as well.

$$\left[\sum_{\substack{p \in ela \\ p \notin PEAK\_TID_{r,p}}} \begin{pmatrix} TE(ENT)_{r,e,t} * PEAK(CON)_{r,p,t} / \\ (1+ERESERV_{r,e,t}) * CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Contribution from peaking only power plants, where the amount to credit to the peak in a season is a function of the activity of the plant with the explicitly specified peak duration factor (PD(Z)D), as well as the capacity credit factor (PEAK). The contribution is subject to transmission losses as well as the reserve margin requirements.

$$\left[\sum_{\substack{p \in ela \\ p \in PEAK(CON)\_TID_{r,p}}} \begin{pmatrix} TE(ENT)_{r,e,t} * PEAK(CON)_{r,p,t} / \\ \left(QHR(Z)(Y)_{r,z,'D'} * (1+ERESERV_{r,e,t}) * PD(Z)D_{r,p,t}\right) * \\ R\_TEZY_{r,t,p,z,'D'} \end{pmatrix}\right]$$

$$\{\geq\}$$

CONSUMPTION – of electricity by resource options and technologies, and as part of grid interchanges.

Export of electricity occurring during peak times (PEAKDA(SEP)), either annually with seasonal distribution (either the standard QHR(Z)(Y) splits or fractions specifically related to the importing over this electricity interchange (SEP_FR)) applied, or via bi-lateral trade by time-slice. Efficiency losses are applied. Note that if SEP_FR is specified for only some time-slice(s) it is assumed that no transfer is permitted during those missing. Similarly, any missing time-slice related to bi-lateral trade also implies that no trade is permitted (in that direction) during the time-slice.

$$\left[\sum_{\substack{s \forall OUT(ENT)x_{r,s,e,t} \\ s='EXP'}} \begin{matrix} PEAKDA(SEP)_{r,s,t} * \\ \left\{ \begin{matrix} \left(\left\langle \begin{matrix} 1\$(no\ SEP\_FR_{r,s,z,'D',t}) \vee \\ SEP\_FR_{r,s,z,'D',t} / QHR(Z)(Y)_{r,z,'D'} \end{matrix} \right\rangle * R\_TSEP_{r,t,s} \right) \$ \\ \left(not \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'D'} \right) + \\ \left(\begin{matrix} \left\langle R\_TSEPE_{r,t,s,z,'D'} / QHR(Z)(Y)_{r,z,'D'} \right\rangle \$ \\ \sum_{r',e',c} BI\_TRDELC_{r,e,r',e',c,z,'D'} \end{matrix} \right) \end{matrix} \right\} \end{matrix} \right] +$$

Consumption of electricity during peak time as part of supplying another resource. The annual resource production electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{s \forall OUT(ENT)r_{r,s,e',t} \\ e' \neq e}} \begin{pmatrix} PEAKDA(SEP)_{r,s,t} * INP(ENT)r_{r,s,e,t} * \\ R\_TSEP_{r,t,s} \end{pmatrix} \right] +$$

Consumption of electricity by a conventional (non externally load managed) process. The annual process electricity needs are apportioned into the current time-slice by QHR(Z)(Y).

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \begin{pmatrix} QHR(Z)(Y)_{r,w} * INP(ENT)p_{r,s,e,t} * \\ R\_ACT_{r,t,p} \end{pmatrix} \right] +$$

Consumption of electricity from the current grid by externally load managed processes, where the activity of the process (and thus the amount of each commodity consumed) is directly determined from the level of installed capacity.

$$\left[\sum_{p \in xpr} \begin{pmatrix} INP(ENT)p_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Transfer of electricity from the current grid to a conventional (non externally load managed) grid interchange technology, taking into consideration that base load plants only have daytime variables (to force equal day/night operation).

$$\left[\sum_{\substack{p \in lnk \\ p \notin xlm}} \begin{pmatrix} \left(INP(ENT)c_{r,p,e,t} * R\_TEZY_{r,t,p,e,w}\right) \$ p \notin bas * \\ \left((QHR(Z)(Y)_{r,w} / QHRZ_{r,z}) * R\_TEZY_{r,t,p,z,'D'}\right) \$ p \in bas \end{pmatrix}\right] +$$

Transfer of electricity from the current grid to externally load managed grid interchange technologies for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p \in lnk \\ p \in xlm}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p,e,t} * CF(Z)(Y)_{r,p,w,t} * \\ R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Consumption of electricity as required for a unit of low-temperature heat generation, split into season/time-of-day based upon the seasonal activity of power plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ except\ ((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * \left(QHR(Z)(Y)_{r,w} / QHRZ_{r,z}\right) * \\ R\_THZ_{r,t,p,z} \end{pmatrix}\right] +$$

Consumption of electricity by pumped storage facilities at night, based upon the daytime production.

$$\left[\sum_{\substack{p \in stg \\ y='N'}} \left(INP(ENT)c_{r,p,e,t} * TE(ENT)_{r,e,t} * R\_TEZY_{r,t,p,z,'D'} \right) \right] +$$

Consumption of electricity by demand devices, taking into consideration demand device vintaging, shaped according to the season/time-of-day profile of the demand via DM_FR(Z)(Y).

$$\left[\sum_{p \in dmd} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \sum_{d} \left\langle \begin{array}{l} \left(DM\_FR(Z)(Y)_{r,d,w} * DMD\_OUTX_{r,p,d,t} \right) / \\ \left(QHR(Z)(Y)_{r,z,'D'} * EFF_{r,p,t} \right) * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \{ not \left(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t} \right) \} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right\rangle \end{array} \right) \right]$$

2.5.18 MR_GEMLIM(v) MARKAL Matrix, Row 23

Description: The **G**lobal **EM**issions **LIM**it constraint.

Purpose and Occurrence: To impose a limit on the total cumulative emissions that can be generated over the entire modeling horizon by all regions producing (or consuming) the emission indicator. This equation is generated for each emissions indicator for which a limit (GEMLIM) is provided.

Units: Tons, or thousand tons or any other unit in which emissions are tracked.

Type: *Binding.* The equation is a less than or equal to (≤) constraint, with all emissions accumulated from each region in a total emissions variable (R_EM) in all time period.

Interpretation of the results:

Primal: The level of the emission equation represents the slack (level below the limit (upper)) imposed of the total emissions from all regions in all periods.

Dual variable: The dual variable (DVR_GEMLIM) of the global cumulative emission constraint (shadow price) is 0 unless a limit on the cumulative emissions is imposed (on the sum of the region emissions variables R_TENV) and reached, in which case it represents the endogenous price of this emission.

Remarks: Only relevant for multi-region models.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\#}18: MR\_GEMLIM_{v} \ \forall GEMLIM_{v}$$

Annual (net) emissions from each region in each time period, summed over all regions and time periods.

$$\sum_{r,t} R\_EM_{r,t,v} * NYRSPER$$

$$\{\leq\}$$

$$GEMLIM_{v}$$

2.5.19 MR_GEMLIMT(t,v) MARKAL Matrix, Row 24

Description: The **G**lobal **EM**issions **LIM**it constraint for a **T**ime-period.

Purpose and Occurrence: To impose a limit on the total emissions that can be generated in a period by all regions producing (or consuming) the emission indicator. This equation is generated for each emissions indicator for which a limit is provided for a period (GEMLIMT).

Units: Tons, or thousand tons or any other unit in which emissions are tracked.

Type: *Binding.* The equation is a less than or equal to (≤) constraint, with all emissions accumulated from each region in a total emissions variable (R_EM) for each indicator in this time period.

Interpretation of the results:

Primal: The level of the emission equation represents the slack (level below the limit (upper)) imposed on the total emissions from all regions in a period.

Dual variable: The dual variable (DVR_GEMLIMT) of the global period emission constraint (shadow price) is 0 unless a limit on the total annual emissions is imposed (on the sum of the region emissions variables R_TENV) and reached. It thus represents the endogenous price of this emission in a period.

Remarks: Only relevant for multi-region models.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 19}: MR\_GEMLIMT_{v,t} \ \forall GEMLIMT_{v,t}$$

Annual (net) emissions from each region in each time period, summed over all regions.

$$\sum_{r} R\_EM_{r,t,v}$$

$$\{\leq\}$$

$$GEMLIMT_{v,t}$$

2.5.20 MR_GRSEP(r,t,s) MARKAL Matrix, Row 25

Description: The inter-temporal **GR**owth constraint on the expansion of a resource supply (**S**rc**E**nc**P**) option between periods.

Purpose and Occurrence: To impose a limit on the rate at which a resource supply option can expand (GROWTHr) between periods. The growth constraint needs a non-zero starting point (GROWTH_TIDr) from which to expand if the activity is zero initially. This equation is generated for all time periods for which the growth rate parameter (GROWTHr) is provided.

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the resource growth equation indicates how much growth in a resource supply option has occurred between two periods.

Dual variable: The dual variable (DVR_GRSEP) of the resource growth constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of growth in the resource supply option. A zero dual value implies that the supply of the resource has not reached the limit imposed on the rate of growth between two periods. The value is negative when the primal equation is tight, and the system desires more of the resource in the current period.

Remarks: The initial potential level of production from the resource option (GROWTH_TIDr) needs to be provided as a maximum starting level or "seed" in case the resource supply option has not yet been tapped.

GAMS Routine: MMEQGRS.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#20}: MR\_GRSEP_{r,t,s} \;\forall GROWTHr_{r,s,t}$$

Supply of a resource in the previous period, with annual growth rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\left[-(GROWTHr_{r,s,t}**NYRSPER)*R\_TSEP_{r,t-1,s}\right]+$$

Supply of the resource in the current period.

$$R\_TSEP_{r,t,s}$$

$$\{\le\}$$

Initial permitted level for a resource to "seed" the growth constraint. Needed if the resource option was not used in the previous period.

$$GROWTH\_TIDr_{r,s}$$

2.5.21 MR_GRTCH(r,t,p) MARKAL Matrix, Row 26

Description: The inter-temporal **GR**owth constraint on the expansion of total installed capacity of a **TeCH**nology between periods.

Purpose and Occurrence: To impose a limit on the rate at which total installed capacity can expand (GROWTH) between periods. The growth constraint needs a non-zero starting point (GROWTH_TID) from which to expand if the technology has not yet been deployed. This equation is generated for all time periods for which the growth rate parameter (GROWTH) is provided.

Units: GW, PJa, or any other unit in which capacity is tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the technology growth equation indicates how much growth in a technology has occurred between two periods.

Dual variable: The dual variable (DVR_GRTCH) of the technology growth constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of growth in the total installed capacity of a technology. A zero dual value implies that the increase in capacity between two periods has not reached the level imposed by the growth limit. The value is negative when the primal equation is tight, and the system desires more of the technology in the current period.

Remarks: The initial capacity level of the technology (GROWTH_TID) needs to be provided as a maximum starting level or "seed" in case the technology has not yet been deployed.

GAMS Routine: MMEQGRT.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#21}: MR\_GRTCH_{r,t,p} \ \forall GROWTH_{r,p,t}$$

Total installed capacity in the previous period, with annual growth rate adjusted to a period rate according to the number of years in the period (NYRSPER) to establish the current limit.

$$\left[-(GROWTH_{r,p,t} ** NYRSPER) * R\_CAP_{r,t-1,p}\right] +$$

Total installed capacity in the current period.

$$R\_CAP_{r,t,p}$$

$$\{\leq\}$$

Initial permitted level for a technology to "seed" the growth constraint. Needed if the technology was not installed in the previous period.

$$GROWTH\_TID_{r,p}$$

2.5.22 MR_GTRD(r,g) *MARKAL Matrix, Row 27*

Description: The **G**lobal **TR**a**D**e of a commodity (g = enc, mat or env) among all regions in the
model.

Purpose and Occurrence: To allow for trading in a globally available commodity between all regions in the model. The equation ensures the balance between all producers ('EXP') and consumers ('IMP') of each globally traded commodity. This equation is generated in each time period beginning from the initial trading period (TRD_FROM) and for which some region produces or consumes the commodity.

Units: PJ (energy), Mton (materials or emissions), M$ (permits) or any other unit in which a commodity is tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the global trade constraint indicates the amount by which the trade is out-of-balance in a period. If non-zero the constraint, and thus model, is infeasible.

Dual variable: The dual variable (DVR_GTRD) of the global trade constraint (shadow price) indicates the amount that the energy system has to pay for a unit of that trade. A zero dual value implies that there is no cost associated with producing and delivering the traded commodity, which is highly unlikely. The value is negative when the primal equation is tight, and indicates how much lower the objective function would be if one more unit of trade was permitted.

Remarks: Flows into individual regions may be limited by the TRD_BND parameter.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 22}: MR\_GTRD_{t,g} \ \forall\, t \geq TRD\_FROM_{g}$$

Imports (SIGN = +1) and Exports (SIGN = -1) of a globally traded commodity into/from each region. A region based conversion factor (REG_XCVT) may be applied, defaulted to 1.

$$\sum_{r,ie}\left(SIGN_{ie} * REG\_XCVT_{r,e} * G\_TRD_{r,t,g,ie}\right)$$

$$\{=\}$$

$$0$$

2.5.23 MR_HPKW(r,t,e,z) MARKAL Matrix, Row 28

Description: The low-temperature **H**eat (lth) **P**ea**K**ing constraint ensures that there is enough capacity in place to meet the heating demand during the **W**inter, or the season set by the analyst for the heating/cooling grid (heatcool).

Purpose and Occurrence: The heat peaking constraint ensures that there is enough capacity in place to meet the heating demand during the season designated by the analyst. It takes into consideration an estimate of the level above the demand that corresponds to the actual peak moment plus a reserve margin of excess capacity (HRESERV includes both), to ensure that if some plants are unavailable the highest demand for heat can still be met. This equation is generated for all time periods and for the heat/cool seasons (z=heatcool), during which some demand device consumes heat.

Units: PJ, or any other unit in which heat is tracked. Note that although the constraint is thought of in terms of capacity, it is actually modeled in energy units.

Type: *Binding.* The equation is a greater than or equal (>=) constraint for the (heatcool) season.

Interpretation of the results:

Primal: The level of the low-temperature heat peaking equation represents the slack or excess capacity over and above the amount required based upon the highest demand, plus the heating reserve (HRESERV) margin provided by the analyst. Under normal circumstances there would be no slack as the energy system should incur a cost for an additional unit of capacity above the minimum level required. If there is slack the user should check the residual capacity levels (RESID) to ensure that they properly reflect the level in place prior to the modeling period, and review the value of the HRESERV parameter (remembering that is has a dual role of estimating both how much above the level of highest average heat demand the peak moment is, plus the traditional requirement for extra 'just in case' capacity).

Dual variable: A zero dual value implies that the available capacity exceeds the requirements imposed by the constraint. The value is negative when the primal equation is tight, which is normally the case, and indicates how much the objective function would be lowered if the reserve constraint was one unit lower.

Remarks: As already noted, the low-temperature heat peaking constraint is highly dependent upon the reserve margin (HRESERV), which is usually substantially above that of a traditional utility reserve margin. Only demand devices (dmd) can consume low-temperature heat. Note also that the analyst can set the season in which the peaking constraint is to be modeled so that the "heating" grid may be used as a "cooling" grid in hot climates (e.g., Middle East).

GAMS Routine: MMEQHPK.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 23}: MR\_HPKW_{r,t,e} \;\forall \begin{pmatrix} e \in lth \wedge z \in HEATCOOL_{r,e} \wedge \\ \sum_{p \in dmd} MA(ENT)_{r,p,e,t} \end{pmatrix}$$

Contribution from conventional (non externally load managed and non peaking only) heating plants and grid interchange technologies, where the amount to credit to the peak in a season is a function of the installed capacity of the plant taking into consideration the capacity credit factor (PEAK) as well.

$$\left[\sum_{\substack{p \in hpl \\ p \notin xlm \\ p \notin PEAK(CON)\_TID_{r,p}}} \begin{pmatrix} PEAK(CON)_{r,p,t} / (1 + HRESERV_{r,e,t}) * \\ CAPUNIT_{r,p} * R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Contribution from peaking only heating plants, where the amount to credit to the peak in a season is a function of the activity of the plant with the explicitly specified peak duration factor (PD(Z)D), as well as the capacity credit factor (PEAK).

$$\left[\sum_{\substack{p \in hpl \\ p \in PEAK(CON)\_TID_{r,p}}} \begin{pmatrix} PEAK(CON)_{r,p,t} / \\ \left(QHRZ_{r,z} * (1 + HRESERV_{r,e,t}) * PD(Z)D_{r,p,t} \right) * \\ R\_THZ_{r,t,p,z} \end{pmatrix} \right] +$$

Contribution from coupled heat and power pass-out turbine plants, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements, as well as the distribution efficiency.

$$\left[\sum_{\substack{p \in cpd \\ ELM_{r,p}}} \begin{pmatrix} PEAK(CON)_{r,p,t} * CAPUNIT_{r,p} * ELM_{r,p} * TRNEFF(Z)(Y)_{r,p,z,'D',t} / \\ \left(CEH(Z)(Y)_{r,p,z,'D',t} * (1 + HRESERV_{r,e,t}) * PD(Z)D_{r,p,t} \right) * \\ R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Contribution from coupled heat and power back-pressure turbine plants, taking into consideration the reserve margin requirements and the amount of capacity to be credited to the peaking requirements, as well as the distribution efficiency.

$$\left[\sum_{\substack{p \in cpd \\ REH_{r,p}}} \begin{pmatrix} PEAK(CON)_{r,p,t} * CAPUNIT_{r,p} * TRNEFF(Z)(Y)_{r,p,z,'D',t} / \\ \left(REH(Z)(Y)_{r,p} * \left(1 + HRESERV_{r,e,t}\right)\right) * R\_CAP_{r,t,p} \end{pmatrix}\right]$$

$$\{\geq\}$$

Transfer of heat from the current grid to conventional (non externally load managed) grid interchange technologies.

$$\left[\sum_{\substack{p \in hlk \\ p \notin xlm}} \left(INP(ENT)c_{r,p,e,t} * R\_THZ_{r,t,p,z}\right)\right] +$$

Transfer of heat from the current grid to externally load managed grid interchange technologies for each time-slice for which a capacity factor (CF(Z)(Y)) is provided.

$$\left[\sum_{\substack{p \in \ln k \\ p \in xlm}} \begin{pmatrix} INP(ENT)c_{r,p,e,t} * CAPUNIT_{r,p} * CF(Z)(Y)_{r,p,w,t} * \\ R\_CAP_{r,t,p} \end{pmatrix}\right] +$$

Consumption of electricity by demand devices according to the amount of electricity needed per unit output (a function of the market share of electricity to the device (MA(ENT)) and the device efficiency (EFF)) and the seasonal/time-of-day shape (DM_FR) of the demand load being serviced by the share of the output from the device to the sector (DM_OUTX), as well as the loss on the heat grid.

$$\left[\sum_{p \in dmd} \left(\sum_{d,y} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ \left(DM\_FR(Z)(Y)_{r,d,z,y} * DM\_OUTX_{r,p,d,t}\right) / \\ \left(QHRZ_{r,z} * EFF_{r,p,t} * DHDE(Z)_{r,z,t}\right) * \\ \left(R\_CAP_{r,t,p} \$ \left\{ not\left(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t} \right) \right\} + \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \right) \end{array} \right) \right) \right]$$

2.5.24 MR_LIM(r,t,p,e) MARKAL Matrix, Row 29

Description: For flexible output "mixing" process (**LIM**IT) the constraint that controls the levels of the individual outputs according to the maximum level provided by the analyst for each commodity produced (OUT(ENC)p) from such processes.

Purpose and Occurrence: For flexible output "mixing" processes this constraint ensures that the output of the individual commodities (R_LOUT) does not exceed their maximum permitted share (OUT(ENC)p). This equation is generated for all time periods for which the flexible process (LIMIT) produces the commodity (OUT(ENC)p).

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a less than or equal (<=) constraint.

Interpretation of the results:

Primal: The level of the limit equation indicates how much below the maximum permitted production of a commodity a process is actually outputting.

Dual variable: The dual variable (DVR_LIM) of the limit constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of the commodity from the flexible process. A zero dual value implies that this commodity is not driving the actual operation of the process. The value is negative when the primal equation is tight, and the system desires more of the commodity from the process. It may be worth examining the maximum output share for such limited commodities to determine whether a bit more of the most desired commodity/commodities can be squeezed out of the flexible process.

Remarks: This equation works in tandem with the overall limit constraint (MR_PBL) that ensures that the total output from a flexible process does not exceed its overall efficiency (LIMIT). Note that the output share (OUT(ENC)p) for all non-LIMIT processes fixes the share, and thus the sum of all OUT(ENC)p is the overall efficiency of the process. But for these flexible processes the individual shares (OUT(ENC)p) represent the maximum and thus they may sum to more than 1.

GAMS Routine: MMEQLIM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 24}: MR\_LIM_{r,t,p,e} \ \forall \left(\begin{array}{l} LIMIT_{r,p,t} \wedge \\ OUT(ENC)_{r,p,e,t} \end{array} \right)$$

Level of the current commodity from a flexible limit process.

$$R\_LOUT_{r,t,p,e} -$$

Maximum share of the total output from a conventional (non externally load managed) limit process of the current commodity.

$$\left[OUT(ENC)p_{r,p,e,t} * R\_ACT_{r,t,p} \right] \$(not\ p \in xpr) -$$

Maximum share of the total output from an externally load managed limit process of the current commodity.

$$\left[\begin{array}{l} OUT(ENC)p_{r,p,e,t} * CAPUNIT_{r,p} * CF_{r,p,t} * \\ R\_CAP_{r,t,p} \end{array} \right] \$(p \in xpr)$$

$$\{\leq\}$$

$$0$$

2.5.25 MR_OBJ *MARKAL Matrix, Row 30*

Description: The multi-region **OBJ**ective function of Standard MARKAL. As discussed in section 1.3, the **OBJ**ective of MARKAL is to minimize the discounted total system cost of all regions together over the entire horizon, expressed in common monetary units. The total discounted cost is the addition of the regional total discounted costs (see PRICE(r)). By using the total discounted cost as objective function, the perfect foresight MARKAL model can implement optimal trade-offs of costs incurred in different periods, in contrast with time-stepped models that optimize each period's cost separately, as for instance the SAGE model (see PART III).

Purpose and Occurrence: The objective function expression is equated to the variable R_TOTobjZ, which in turn is minimized by the optimizer. For details pertaining to the cost components, see MR_PRICE(r), representing the total discounted system cost for each region. This equation is generated only once.

Units: Million 2000 US$, or any other unit in which costs are tracked. Note that a regional monetary conversion factor (REG_XMONY) may be applied if different monetary units are used for the various regions.

Type: *Binding.* The equation is actually constructed as a balance equation that equates (=) the total cost to a single variable across all regions (R_TOTobjZ) that is minimized by the solver.

Interpretation of the results:

Primal: The value of the minimum cost for all regions together, expressed in a single monetary unit, as a discounted present value in a user chosen year.

Dual variable: The undiscounted dual value is equal to 1.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 25}: MR\_OBJ$$

Regional producer/consumer surplus, with optional monetary conversion factor applied. [For MARKAL R_objZ is the regional minimized total discounted system cost.]

$$\sum_{r}\left(REG\_XMONY_{r} * R\_TOTOBJ_{r}\right)$$

$$\{=\}$$

Objective function variable to be minimized by the solver.

$$\boldsymbol{R\_TOTobjZ}$$

2.5.26 MR_PBL(r,t,p) *MARKAL Matrix, Row 31*

Description: For flexible output "mixing" **P**rocesses this **B**alance constraint **L**imits the sum of all the outputs (OUT(ENC)p) from such a process to ensure that they are compatible with the overall efficiency of such processes (LIMIT).

Purpose and Occurrence: To ensure that the sum of all outputs from a flexible "mixing" process respects the overall efficiency of that process. For such processes a variable is created for each individual output commodity (R_LOUT), limited to a maximum share (OUT(ENC)p) of the process activity (R_ACT), which are then summed here. This equation is generated for all time periods for which the flexible process (LIMIT) produces the commodity (OUT(ENC)p).

Units: PJ, or any other unit in which commodities are tracked.

Interpretation of the results:

Primal: The level of the process balance equation indicates how much below the maximum combined output from a flexible process the total output is.

Dual variable: The dual variable (DVR_PBL) of the process balance constraint (shadow price) indicates the cost pressure on the operation of a flexible plant. A zero dual value rarely occurs unless there is no cost associated with the commodities consumed by the process. The value may be positive or negative.

Type: *Binding.* The equation an equality (=) constraint.

Remarks: This equation works in tandem with the individual flexible output commodity limit constraint (MR_LIM) that ensures that no commodity output rises above the maximum output of the commodity (OUT(ENC)p).

GAMS Routine: MMEQPBL.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 26}: MR\_PBL_{r,t,p} \; \forall \left(p \in prc \wedge LIMIT_{r,p,t} \right)$$

Overall efficiency of a flexible limit process applied to the total activity of conventional processes or the activity derived from the capacity for externally load managed processes.

$$\left[\begin{array}{l} LIMIT_{r,p,t} * \\ \left(\begin{array}{l} R\_ACT_{r,t,p} \$(p \notin xpr) \vee \\ \left[\left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * R\_CAP_{r,t,p} \right] \$ p \in xpr \end{array} \right) \end{array} \right] -$$

Sum over all the commodities produced from a flexible process.

$$\sum_{e}\left(R\_LOUT_{r,t,p,e}\right)$$

$$\{=\}$$

$$0$$

2.5.27 MR_PRICE(r) *MARKAL Matrix, Row 32*

Description: The regional equation corresponding to the discounted total system cost of each region over the entire horizon. This is the addition of (properly discounted) annualized total period's costs of a given region.

Occurrence: An equation is generated for each region.

Units: The monetary units used in each region.

Type: *Binding.* The cost expression is equalized to the variable ???

Interpretation of results:
Primal: the value of the total global discounted cost for each region at optimal.
Dual: of no interest

GAMS Routine: MMEQPRIC.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 27:} MR\_PRICE_{r}$$

Scale factor to adjust the magnitude of the objective function.

$$GobjZscal * \langle$$

Discounted cost of supplying domestic resources, plus any delivery costs associated with ancillary commodities, according to the level of the resource activity.

$$\sum_{\substack{t,s \in tpsep \\ x \neq 'IMP' \vee 'EXP' \\ e \notin (enu \cup \\ ecv \cup lth)}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} COST_{r,s,t} + \\ \sum\limits_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \end{array} \right] +$$

Discounted cost of imports and exports, plus any delivery costs associated with ancillary commodities involved in the import/export process, according to the level of the resource activity. Note that if the trade is expressed as bi-lateral trade then any resource supply cost (COST) is ignored and only the transport/transaction cost applied. Also, if importing electricity then transmission and distribution O&M costs are incurred.

$$\sum_{\substack{t,s \in tpsep \\ e \notin (enu \vee \\ ecv \vee lth)}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} SIGN_x * \begin{pmatrix} COST_{r,s,t} \$ not\ BI\_TRDENT + \\ BI\_TRDCST_{r,s,t} \$ BI\_TRDENT \end{pmatrix} + \\ (ETRANOM_{r,e,t} + EDISTOM_{r,e,t}) \$ \begin{pmatrix} e \in elc \wedge \\ x = 'IMP' \end{pmatrix} + \\ \sum\limits_{e' \subset INP(ENT)r_{r,s,e',t}} \begin{pmatrix} INP(ENT)r_{r,s,e',t} * \\ DELIV(ENT)r_{r,s,e',t} \end{pmatrix} \end{array} \right\rangle * \\ R\_TSEP_{r,t,s} \end{array} \right] +$$

Discounted cost of imports and exports of electricity by time slice, according to the level of activity. Any bi-lateral, time-sliced transport/transaction cost is applied here, and any other delivery or transmission and distribution costs are captured in the above expression.

$$\left[\sum_{\substack{t,s\subset tpsep,w \\ x='IMP'\vee 'EXP' \\ e\in elc \\ BI\_TRDELC}} \begin{pmatrix} PRI\_DF_{r,t} * SIGN_{x} * BI\_TRDCSTE_{r,t,w} * \\ R\_TSEPE_{r,t,s,w} \end{pmatrix}\right]^{+}$$

Discounted transaction cost associated with a globally traded commodity in the current region.

$$\left[\sum_{\substack{e\in(ent\vee env\vee mat)\wedge \\ G\_TRADE_{e} \\ x='IMP'\vee 'EXP' \\ t\geq TRD\_FROM}} \begin{pmatrix} PRI\_DF_{r,t} * TRD\_COST_{r,e,t} * \\ R\_GTRD_{r,t,e,x} \end{pmatrix}\right]^{+}$$

Stockpile credit in the final period.

$$\left[\sum_{\substack{s\subset 'STK' \\ tpsep_{tlast,s}}} \begin{pmatrix} -PRI\_DF_{r,tlast} * COST\_TID_{r,s} * \\ R\_TSEP_{r,tlast,s} \end{pmatrix}\right]^{+}$$

Discounted fixed operating and maintenance costs for conventional (non-externally load managed) process and conversion technologies as a function of the total installed capacity. [For externally load managed technologies the FIXOM is included when determining the total operating costs (below).]

$$\left[\sum_{t,p\$\left(\begin{array}{l} (\in tpprc \wedge \notin xpr) \vee \\ (\in tpcon \wedge \notin xlm) \end{array} \right)} \left(\begin{array}{l} PRI\_DF_{r,t} * FIXOM_{r,p,t} * \\ R\_CAP_{r,t,p} \end{array} \right) \right] +$$

Discounted variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non-externally load managed) processes as a function of the total process activity.

$$\left[\sum_{\substack{t,p \in prc \\ p \notin xpr}} \left(\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} VAROM_{r,p,t} + \\ \sum_{e \subset INP(ENT)p_{r,p,e,t}} \left(INP(ENT)p_{r,p,e,t} * DELIV(ENT)r_{r,p,e,t} \right) \end{array} \right\rangle * \\ R\_ACT_{r,t,p} \end{array} \right) \right] +$$

Discounted variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non externally load managed) electric power plants as a function of the time-sliced generating activity. The transmission and distribution O&M costs are added on top of the plant operating costs. [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\sum_{\substack{t,p \in tpela, p \notin xlm, w \\ except\ p \in (bas \cup stg) \wedge \\ (z='N') \\ except (z='I' \vee y='N') \wedge \\ PEAK(CON)\_TID_{r,p}}} \left(\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + VAROM_{r,p,t} + \sum_{e \in elc} ELCOM_{r,p,e,t} \right\rangle * \\ R\_TEZY_{r,t,p,w} \end{array} \right) +$$

Discounted variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for conventional (non-externally load managed) heating plants as a function of the seasonal generating activity. The transmission O&M costs are added on top of the plant operating costs. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\sum_{\substack{t,p \in tphpl, p \notin xlm, z \\ except (z='I' \vee 'S') \wedge \\ PEAK(CON)\_TID_{r,p}}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + VAROM_{r,p,t} + \sum_{e \in lth} DTRANOM_{r,e,t} \right\rangle * \\ R\_THZ_{r,t,p,z} \end{array} \right] +$$

Discounted variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for coupled heat and power pass-out turbines as a function of the time-sliced generating activity. The transmission and distribution O&M costs for both electricity and heat are added on top of the plant operating costs. [Special conditions apply to base load that have only daytime variables (to force day=night generation for base load plants.]

$$\sum_{\substack{t,p \in tpcpd,w \\ p \in ELM_{r,p,t} \\ except(y='N' \wedge \\ e \in bas)}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ + (1 - ELM_{r,p,t}) * ELCOM_{r,p,e,t} + \sum_{e \in lth} DTRANOM_{r,e,t} + VAROM_{r,p,t} \end{array} \right\rangle * \\ \left(CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} \right) * R\_TCZYH_{r,t,p,w} \end{array} \right] +$$

Discounted fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for demand devices as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{t,p \in tpdmd} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{l} \sum_{e \subset MA(ENT)_{r,p,e,t}} \left(\begin{array}{l} MA(ENT)_{r,p,e,t} / EFF_{r,p,t} * \\ DELIV(ENT)_{r,p,e,t} \end{array} \right) + \\ VAROM_{r,p,t} \end{array} \right] * \\ \left(\begin{array}{l} R\_CAP_{r,t,p} \$ \left\{ not \left(EFF\_I_{r,p,t} \vee EFF\_R_{r,p,t} \right) \right\} + \\ \left(\begin{array}{l} R\_INV_{r,t,p} \$ EFF\_I_{r,p,t} + \\ R\_RESIDV_{r,t,p} \$ EFF\_R_{r,p,t} \end{array} \right) \end{array} \right) \end{array} \right\rangle \end{array} \right] +$$

Discounted fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed processes as a function of the total installed capacity, and the operation (dictated by CF) thereof.

$$\sum_{\substack{t,p \in tpprc \\ p \in xpr}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} FIXOM_{r,p,t} + \left(CAPUNIT_{r,p} * CF_{r,p,t} \right) * \\ \left[\begin{array}{l} \sum_{e \subset INP(ENT)p_{r,p,e,t}} \left(\begin{array}{l} INP(ENT)p_{r,p,e,t} * \\ DELIV(ENT)_{r,p,e,t} \end{array} \right) + \\ VAROM_{r,p,t} \end{array} \right] \end{array} \right\rangle * \\ R\_CAP_{r,t,p} \end{array} \right] +$$

Discounted fixed and variable operating and maintenance costs, plus any delivery costs associated with the consumption of commodities, for externally load managed power plants as a function of the total installed capacity, and the operation (dictated by CF(Z)(Y)) thereof.

$$\sum_{\substack{t,p \in tpcon \\ p \in xlm}} \left[\begin{array}{l} PRI\_DF_{r,t} * \\ \left\langle \begin{array}{l} FIXOM_{r,p,t} + \sum_{w} \left(CF(Z)(Y)_{r,p,w,t} * QHR_{r,w} \right) * CAPUNIT_{r,p} * \\ \left[\begin{array}{l} \sum_{e \subset INP(ENT)c_{r,p,e,t}} \left(INP(ENT)c_{r,p,e,t} * DELIV(ENT)_{r,p,e,t} \right) + \\ VAROM_{r,p,t} + ELCOM_{r,p,e,t} \; \$ p \in ela + LTHOM_{r,p,e,t} \; \$ p \in hpl \end{array} \right] + \end{array} \right\rangle * \\ R\_CAP_{r,t,p} \end{array} \right] +$$

Discounted investment costs associated with new investments. The calculated discounted investment cost (PRI_INV) consists of the actual investment cost (INVCOST), as well as electricity transmission and distribution system investment costs if appropriate (see below), taking into consideration the capital recovery factor (CRF), as determined by the lifetime (LIFE) and discount rate (DISCOUNT and DISCRATE if technology based). In addition, the salvage value of the technology is accounted for as well. See below.

$$\left[\sum_{t,p \in tptch} \left(PRI\_INV_{r,t,p} * R\_INV_{r,t,p} \right) \right] +$$

Discounted emission taxes or other costs associated with the level of an emission indicator.

$$\left[\sum_{t,v} \left(PRI\_DF_{r,t} * ENV\_COST_{r,v,t} * R\_EM_{r,t,v} \right) \right] +$$

Discounted taxes and subsidies associated with particular commodities, and the consumption and/or production thereof.

$$\left[\sum_{t,txs} \left(PRI\_DF_{r,t} * TSUB\_COST_{r,v,t} * R\_TXSUB_{r,t,txs} \right) \right] +$$

Discounted costs associated with growth or reduction in demand as a function of the elastic demand own price elasticity and variation as represented by a step curve.

$$\sum_{b,d,u=1,t}^{MED-STEP(BD)_{r,d,b,u,t}} \left\{ \begin{array}{l} PRI\_DF_{r,t} * MED-BPRICE_{r,d,t} * \\ \left\langle \begin{array}{l} -\left\{ \begin{array}{l} \left[\begin{array}{l} 1+ \\ \left(MED-VAR_{r,d,b,t} / MED-STEP_{r,d,b,t} * (u-.5) \right) \end{array} \right] ** \\ \left(1 / MED-ELAST_{r,d,b,t} \right) \end{array} \right\} \$b='UP' + \\ \left\{ \begin{array}{l} \left[\begin{array}{l} 1- \\ \left(MED-VAR_{r,d,b,t} / MED-STEP_{r,d,b,t} * (u-.5) \right) \end{array} \right] ** \\ \left(1 / MED-ELAST_{r,d,b,t} \right) \end{array} \right\} \$b='LO' \end{array} \right\rangle * \\ R\_ELAST_{r,t,d,b,u} \end{array} \right\} +$$

If the balance equation is to be formulated as a slack equality constraint, where the slack cost is provided (SLKCOST), then include the slack variable discounted.

$$\left[\sum_{t,e \in SLKCOST_{r,e,t}} \left(\begin{array}{l} PRI\_DF_{r,t} * SLKCOST_{r,e,t} * \\ \left(R\_SLKENC_{r,t,e} + \sum_{z} R\_SLKLTH_{r,e,t,z} \right) \end{array} \right) \right]$$

$$\rangle$$

$$\{=\}$$

$$\boldsymbol{R\_objZ(r)}$$

Where SIGN = -1 for x='EXP'; otherwise 1

ELCOM = if $\left(e \in elc \wedge p \in ela\right)$ then

$$\left(\begin{array}{l} EDISTOM_{r,e,t} + \\ ETRANOM_{r,e,t} \$ (p \subset cen) \end{array}\right);$$

$$0\$\left(p \in stg\right)$$

LTHOM = if $\left[e \in lth \wedge p \in \left(p \in cpd \wedge cen\right)\right]$ and p is a back-pressure turbine (REH) then

$DTRANOM_{r,e,t} / REH_{r,p}$; 0 otherwise

ELCINV = if $\left(e \in elc \wedge p \in ela\right)$ then

$$EDISTINV_{r,e,t} \$ (p \notin stg) +$$

$$ETRANINV_{r,e,t} \$ (p \subset cen)$$

LTHINV = if $e \in lth$

if $p \in hpl$ then $DTRANINV_{r,e,t}$

if $p \in \left(p \in cpd \wedge cen\right)$

if $p \in REH_{r,p,t}$ then

$$\left(DTRANINV_{r,e,t} / REH_{r,p,t}\right)$$

else

$$\left(\begin{array}{l} DTRANINV_{r,e,t} * \\ ELM_{r,p,t} / CEH_{r,p,w,t} \end{array}\right)$$

otherwise

0

CRF = where *x=1/(1+DISCOUNT or DISCRATE), and then CRF={1-x}/{1-x^LIFE}* ($CRF = \{1-x\}/\{1-x^{LIFE}\}$)

FRACLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
(years in last period of LIFE/NYRSPER)

FRLIFE = 1 if LIFE is a multiple of the number of years per period (NYRSPER),
otherwise
1 if current period is not the last (fractional period)

otherwise

CRF adjustment for the fraction of the period

$$\text{PRI\_DF}_t = \left\{ \begin{array}{l} \left\langle \sum_{yrs=1}^{NYRSPER} \left[(1+DISCOUNT)**(1-ord(yrs)) \right] \right\rangle / \\ \left\langle (1+DISCOUNT)** \left(-STARTYRS + NYRSPER*(ord(t)-1)\right) \right\rangle \end{array} \right\}$$

$$\text{PRI\_DISC}_{t,p} = \left\{ \begin{array}{l} CRF_{r,t,p} * FRACLIFE_{r,p} * \\ (1/(1+DISCOUNT))** \\ (-STARTYRS + NYRSPER*(ord(t)-1)) \end{array} \right\}$$

$$\text{INV\_INV}_{t,p} = \left\langle INVCOST_{r,p,t} + (ELCINV + LTHINV) \right\rangle$$

The salvage component of the investment variable accounts for stranded capacity still in place at the end of the modeling horizon, as well as any sunk material (and nuclear fuel). It must be adjusted for any technology-based discount (hurdle) rate.

$$\text{SALV\_INV}_{t,p} \,\$ \left(\begin{array}{l} ord(t) + LIFE_p - \\ card(t) - 1 \end{array} \right) =$$

$$
\left\{
\begin{array}{l}
INV\_INV_{t,p} + \\
\left[\sum_{\substack{e \in SAL\_REL_{e,t}\$ \\ [(ord(t)+LIFE_{p}- \\ card(t))>0]}} \left\langle \begin{array}{l} SAL\_REL_{e,t} * \\ \left(\begin{array}{l} OUT(MAT)\_TIDc_{p} + \\ OUT(MAT)\_TIDp_{p} + \\ MO(MAT)\_TID_{p} \end{array} \right) * \\ \left(\begin{array}{l} (1+DISCOUNT)** \\ (-NYRSPER*LIFE_{p}) \end{array} \right) \end{array} \right\rangle \right] + \\
\left[\sum_{e \in SAL\_SNK_{e,t}} \left\langle \begin{array}{l} SAL\_SNK_{e,t} * \\ \left(\begin{array}{l} INP(ENC)\_TID_{p} + \\ INP(MAT)\_TIDc_{p} + \\ INP(MAT)\_TIDp_{p} + \\ MA(ENC)\_TID_{p} + \\ MA(MAT)\_TID_{p} \end{array} \right) \end{array} \right\rangle * \left(\begin{array}{l} (1+DISCOUNT)** \\ NYRSPER \end{array} \right) \$ e \in enu \right] +
\end{array}
\right\} *
$$

$$
\left[1 - \left(\begin{array}{l} \left(\begin{array}{l} 1+DISCOUNT\ \$not\ DISCRATE_{p} + \\ DISCRATE_{p} \end{array} \right) ** \\ \left(-NYRSPER * \left(ord(t) + LIFE_{p} - card(t) - 1\right)\right) \end{array} \right) \right] /
$$

$$
\left[\begin{array}{l} \left(\begin{array}{l} 1+DISCOUNT\ \$not\ DISCRATE_{p} + \\ DISCRATE_{p} \end{array} \right) ** \\ \left(NYRSPER * \left(card(t) + 1 - ord(t)\right)\right) \end{array} \right] /
$$

$$
\left[1 - \left(\begin{array}{l} \left(\begin{array}{l} 1+DISCOUNT\ \$not\ DISCRATE_{p} + \\ DISCRATE_{p} \end{array} \right) ** \\ \left(-NYRSPER * LIFE_{p}\right) \end{array} \right) \right] /
$$

$$
PRI\_INV_{t,p} = \left\langle \begin{array}{l} PRI\_DISC_{t,p} * \\ \left(INV\_INV_{t,p} - SALV\_INV_{t,p}\right) \end{array} \right\rangle
$$

2.5.28 MR_REGELC(r,t,s) *MARKAL Matrix, Row 33*

Description: The **REG**ional **EL**e**C**tricity trade constraint maps the time-sliced import or export of electricity by a region, to the conventional annual electricity import/export variable (R_TSEP).

Purpose and Occurrence: In order to link bi-lateral time-sliced electricity trade into the parts of MARKAL that are not time-slice dependent (e.g., the objective function, emissions accounting) the individual season/time-of-day trade variables (R_TSEPE) are summed to the annual electricity import/export variable (R_TSEP). This equation is generated in each time period from the first period the resource option is available.

Units: PJ, or any other unit in which electricity is tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the regional electricity trade constraint indicates the amount by which the time-sliced electricity trade does not match up with the associated annual import/export variable. If non-zero the constraint, and thus model, is infeasible. This may be due to some conflicting limits in the regions involved imposed on the annual import/export variables.

Dual variable: The dual variable (DVR_REGELC) of the regional electricity time-sliced trade constraint (shadow price) indicates the pressure on the system to supply one more/less unit of electricity trade. A zero dual value implies that there is no cost associated with producing and delivering the commodity, which is highly unlikely. The value may be positive or negative.

Remarks: Note that this constraint works in tandem with the MR_BITRDE constraint that links the time-sliced electric trade variables between the two regions involved. This constraint ties the bi-lateral time-sliced electricity trade variables (R_TSEPE) to the annual variable (R_TSEP). The former appear directly in the time-sliced electricity constraints (MR_BALE1/2, MR_BAS, MR_EPK) instead of the latter when used.

GAMS Routine: MMEQUA.REG

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 28}: MR\_REGELC_{r,t,s} \ \forall s \in tpsep \wedge$$

$$\left[\begin{array}{l} s \subset \left('IMP' \vee 'EXP + e = elc + c\right) \wedge \\ \sum_{r',e',w} \left(BI\_TRDELC_{r,e,r',e',c,w} \vee BI\_TRDELC_{r',e',r,e,c,w}\right) \end{array}\right]$$

Standard annual resource supply option corresponding to electricity import/export.

$$R\_TSEP_{r,t,s} -$$

Bi-lateral, time-slice electricity trade summed over each time-slice.

$$\sum_{w}\left(R\_TSEPE_{r,t,s,w}\right)$$

$$\{=\}$$

$$0$$

Where bi_trdelc = bi-lateral time-sliced electric trade options
tpsep = 2-tuple indicating the periods (from START) that a resource supply options is available in a region

2.5.29 MR_SRM(r,t,p,z) MARKAL Matrix, Row 34

Description: The **S**easonal **R**esevoir **M**anagement constraint on hydro plants.

Purpose and Occurrence: To impose a limit on the activity from a hydroelectric plant based upon available capacity (of the reservoir) in a season. This equation is generated for all time periods and each season for which the maximum seasonal reservoir availability is specified (SRAF).

Units: PJ, or any other unit in which commodities are tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of total seasonal activity from a hydro plant for which reservoir limits have been specified.

Dual variable: The dual variable (DVR_ARM) of the seasonal reservoir constraint (shadow price) indicates the amount that the energy system would be willing to pay for one more unit of seasonal activity from such hydro plants. A zero dual value implies that the technology has not reached the limit imposed. The value is negative when the primal equation is tight, and the system wants to use more of the technology in the current period.

Remarks: Only for non-externally load managed plants.

GAMS Routine: MMEQSRM.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#29}: MR\_SRM_{r,t,p,z} \ \forall \begin{pmatrix} p \in hde \wedge SRAF_{r,p,z,t} \ \wedge \\ \left(not\,xlm_{r,p}\right) \wedge \left(not \sum_{w} AF(Z,Y)_{r,p,w,t} \right) \end{pmatrix}$$

The amount of the seasonal (reservoir) capacity available.

$$\left[-CAPUNIT_{r,p} * SRAF_{r,p,z,t} * QHRZ_{r,z} * R\_CAP_{r,t,p} \right] +$$

The day ('D'), plus when not base load the night ('N'), electricity generation from hydro plants subject to seasonal reservoir availability.

$$\begin{pmatrix} R\_TEZY_{r,t,p,z,'D'} + \\ R\_TEZY_{r,t,p,z,'N'} \$ \left(not\ bas\right) \end{pmatrix}$$

$$\{\leq\}$$

$$0$$

2.5.30 MR_TENV(r,t,v) MARKAL Matrix, Row 35

Description: The **T**otal amount of an **ENV**ironmental indicator (emissions) generated in each period from all sources (resources and technologies).

Purpose and Occurrence: The environmental equation tracks all sources of emissions from resources and technologies and accumulates the total level of emissions in each period. For each source of emissions the analyst associates the emissions rate with some aspect of the technology, that is overall activity of resource options or technologies (ENV_SEP, ENV_ACT) or time-sliced activity for conversion technologies (ENV_TEZY, ENV_TCZY, ENV_HPL) if desired, installed capacity (ENV_CAP), or new investment (ENV_INV). The model then applies the emission rate directly to the associated variable (R_TSEP, R_ACT, R_TEZY, R_TCZYH, R_HPL, R_CAP, R_INV) respectively. The net sum of all the emissions is accumulated in variable (R_EM) for each period. This equation is generated in all time periods during which some resource option or technology produces the environmental indicator.

Units: Tons, or thousand tons or any other unit in which emissions are tracked.

Type: *Binding.* The equation is an equality (=) constraint, with all emissions accumulated in a total emissions variable (R_EM) for each indicator in each period. Note that to limit total emissions in a period a bound is applied to the R_EM variable.

Interpretation of the results:

Primal: The interpretation of the level of the emission equation depends upon whether a limit is imposed upon the total emissions variable or not. If so, it represents the slack (level above/below) the limit (lower/upper) imposed on the total emissions variable. If there is no limit imposed then the constraint will always balance to 0.

Dual variable: The dual variable (DVR_TENV) of the emission constraint (shadow price) is normally 0, unless a limit on emissions is imposed (on variable R_TENV) and reached, in which case it represents the endogenous price of this emission.

Remarks: When using the standard technology-based emissions coefficients for activity related emissions the analyst must remember that the parameter is applied to the output variable of the technologies. This being the case any efficiency loss associated with the technology needs to be embedded in the parameter to properly account for emissions based upon the amount of the source commodity consumed, while taking into consideration the nature of the way the commodity is used where appropriate. An alternative is to employ the commodity-based emission accounting parameter (ENV_ENT), that uses the internally calculated balance equation coefficient (see MR_BAL for the components of the BAL_x parameter referenced here) and applies that to the ENV_ENT entry for each technology. Note that ENV_ACT takes precedence over ENV_ENT when explicitly specified. Also, for exports or processes that serve as emission sinks (e.g., sequestration) or are emission reduction options (e.g., a scrubber for which the emissions have already been included in the emission accounting constraint), the analyst needs to enter a negative emissions indicator.

GAMS Routine: MMEQTENV.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#30:} MR\_TENV_{r,t,v}$$

Annual production of the current emission indicator as a function of the level of activity of resource supply options, including consumption where exports are subtracted from the balance.

$$\left[\sum_{s \forall ENV\_SEP_{r,s,v,t}} \left(SIGN_x * ENV\_SEP_{r,s,v,t} * R\_TSEP_{r,t,s} \right) \right] +$$

Annual production of the current emission indicator as a function of the level of activity of resource supply options, where exports are subtracted from the balance, according to ENV_ENT. Note that SIGNx is embedded in the balance coefficient.

$$\left[\sum_{\substack{s \forall ENV\_SEP_{r,s,v,t,t'} \wedge \\ e \in OUT(ENT)r_{r,s,e,t}}} \begin{pmatrix} ENV\_ENT_{r,v,e,t} * \\ BAL\_TSEP_{r,t,t',s} * \\ R\_TSEP_{r,t,s} \end{pmatrix} \$ \left(not\, ENV\_SEP_{r,s,v,t} \right) \right] +$$

Annual production of the current emission indicator as a function of the consumption of a commodity as part of a resource supply options according to ENV_ENT.

$$\left[\sum_{\substack{s \forall ENV\_SEP_{r,s,v,t} \wedge \\ e \in INP(ENT)r_{r,s,e,t}}} \begin{pmatrix} ENV\_ENT_{r,v,e,t} * \\ BAL\_SENT_{r,t,s,e} * \\ R\_TSEP_{r,t,s} \end{pmatrix} \$ \left(not\, ENV\_SEP_{r,s,v,t} \right) \right] +$$

Import or export of a globally traded emissions (or permit).

$$\left[\left(\sum_{ie} SIGN_{ie} * R\_GTRD_{r,t,v,ie} \right) \$ \begin{pmatrix} v \in g\_trade \wedge \\ t \geq TRD\_FROM \end{pmatrix} \right] +$$

All technologies for which the ENV_ACT parameter is provided are included in the associated emissions constraint based upon activity level of the technology. However, as noted below there are quite a few different ways the parameter needs to be applied depending on the nature of the technology. But basically what is done is to use the internal activity parameter calculated for the technology that takes into consideration both the technical data describing the relationship between capacity and activity (e.g., CAPUNIT, AF/CF), as well as the nature of the technology (e.g., baselaod, externally load managed), and apply it to the appropriate variable.

Production of the current emission indicator based upon activity from conventional (non-externally load managed and non-LIMIT) processes.

$$\left[\sum_{\substack{p \in prc \\ p \notin xpr}} \left(ENV\_ACT_{r,p,v,t} * R\_ACT_{r,t,p} \right) \right] +$$

Production of the current emission indicator based upon activity from conventional (non-externally load managed and non-LIMIT) processes, according to the ENV_ENT algorithm. [Note, the LIMIT criteria only applies for output flows (OUT(ENT)p).]

$$\left[\sum_{\substack{p\in prc,\, p\notin xpr \wedge t'\wedge \\ e\in\begin{pmatrix} INP(ENT)p_{r,p,e,t}\vee \\ OUT(ENC)p_{r,p,e,t}\end{pmatrix}}} \left(\begin{matrix} ENV\_ENT_{r,v,e,t} * BAL\_ACT_{r,t,t'p,e} * \\ \begin{pmatrix} R\_ACT_{r,t,p}\$(not\ LIMIT_{r,p,t})\vee \\ R\_LOUT_{r,t,p,e}\$LIMIT_{r,p,t}\end{pmatrix}\end{matrix}\right)\$\left\{not\begin{pmatrix} ENV\_ACT_{r,p,v,t}\vee \\ ENV\_ENTXio_{r,p,v}\end{pmatrix}\right\}\right]+$$

Production of the current emission indicator based upon the level of activity from externally load managed processes, where the activity of the process (and thus the amount of each emission produced) is directly determined from the level of installed capacity by means of a fixed capacity factor.

$$\left[\sum_{p\in xpr}\left(\begin{matrix}\left(ENV\_ACT_{r,p,v,t} * CAPUNIT_{r,p} * CF_{r,p,t}\right) \\ * R\_CAP_{r,t,p}\end{matrix}\right)\right]+$$

Production of the current emission indicator based upon the level of activity from externally load managed technologies, where the activity of the process (and thus the amount of each emission produced) is directly determined from the level of installed capacity by means of a fixed capacity factor, according to the ENV ENT algorithm.

$$\left[\sum_{\substack{t'\wedge \\ \begin{pmatrix} p\in xpr\wedge \\ e\in\begin{pmatrix} INP(ENT)p_{r,p,e,t}\vee \\ OUT(ENC)p_{r,p,e,t}\end{pmatrix}\end{pmatrix}\vee \\ \begin{pmatrix} p\in xlm\wedge \\ e\in\begin{pmatrix} INP(ENT)p_{r,p,e,t}\vee \\ OUT(ENC)p_{r,p,e,t}\end{pmatrix}\end{pmatrix}}} \begin{pmatrix} ENV\_ENT_{r,v,e,t} * \\ BAL\_CAP_{r,t,t'p,e} * \\ R\_CAP_{r,t,p}\end{pmatrix}\$\left\{not\begin{pmatrix} ENV\_ACT_{r,p,v,t}\vee \\ ENV\_ENTXio_{r,p,v}\end{pmatrix}\right\}\right]+$$

Annual production of the current emission indicator as a function of electric generation from conventional (non-externally load managed) power plants, summed over the season/time-of-day activity of the plants. [Special conditions apply to base load and storage facilities, as well as peaking only devices: base load and storage plants only have daytime variables (to force day=night generation for base load plants, and night consumption of electricity to be a function of the daytime dispatch of the storage plants) and peaking only devices are only permitted to operate during the day in the Summer/Winter.]

$$\left[\sum_{\substack{w, p \in ela, p \notin xlm \\ except\, y='N' \wedge p \in (bas \cup stg) \\ except((y='N' + z='I') \wedge \\ PEAK(CON)\_TID_{r,p})}} \left(ENV\_ACT_{r,p,v,t} * R\_TEZY_{r,t,p,w} \right) \right] +$$

Annual production of the current emission indicator as a function of electric generation from conventional (non-externally load managed) power plants, summed over the season/time-of-day activity of the plants, according to the ENV_ENT algorithm.

$$\left[\sum_{\substack{p \in ela, p \notin xlm \wedge t' \wedge w \\ e \in \begin{pmatrix} INP(ENT)c_{r,p,e,t} \vee \\ OUT(ENC)c_{r,p,e,t} \end{pmatrix}}} \begin{pmatrix} ENV\_ENT_{r,v,e,t} * BAL\_TEZY_{r,t,t'p,e} * \\ R\_TEZY_{r,t,p,w} \end{pmatrix} \$ \left\{ not \begin{pmatrix} ENV\_ACT_{r,p,v,t} \vee \\ ENV\_ENTXio_{r,p,v} \end{pmatrix} \right\} \right] +$$

Annual production of the current emission indicator as a function of generation from externally load managed power plants, where the activity of the plant (and thus the amount of each emission indicator produced per unit of electricity or heat generated) is directly determined from the level of installed capacity by means of a fixed capacity factor for each time-slice (CF(Z)(Y)).

$$\left[\sum_{p \in xlm, w} \begin{pmatrix} ENV\_ACT_{r,p,v,t} * CAPUNIT_{r,p} * QHR(Z)(Y)_{r,w} * CF(Z)(Y)_{r,p,z,w,t} * \\ R\_CAP_{r,t,p} \end{pmatrix} \right] +$$

Annual production of the current emission indicator as a function of the amount of heat generation for conventional (non-externally load managed) heating plants, summed over the seasonal activity of the plants. [Special conditions apply to peaking only devices, where only Winter operation is permitted.]

$$\left[\sum_{\substack{p \in hpl, p \notin xlm, z \\ except\ ((z='S' \vee 'I') \wedge \\ PEAK(CON)\_TID_{r,p}))}} \left(ENV\_ACT_{r,p,v,t} * R\_THZ_{r,t,p,z} \right) \right] +$$

Annual production of the current emission indicator as a function of heat generation from conventional (non-externally load managed) heating plants, summed over the seasonal activity of the plants, according to the ENV_ENT algorithm.

$$\left[\sum_{\substack{p \in hpl, p \notin xlm \wedge t' \wedge z \\ e \in \begin{pmatrix} INP(ENT)c_{r,p,e,t} \vee \\ OUT(ENC)c_{r,p,e,t} \end{pmatrix}}} \begin{pmatrix} ENV\_ENT_{r,v,e,t} * BAL\_THZ_{r,t,t'p,e} * \\ R\_THZ_{r,t,p,z} \end{pmatrix} \$ \left\{ not \begin{pmatrix} ENV\_ACT_{r,p,v,t} \vee \\ ENV\_ENTXio_{r,p,v} \end{pmatrix} \right\} \right] +$$

Annual production of the current emission indicator according to the activity of coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of the plants. [Special conditions apply to base load plants, where only daytime variables are generated in order to force equal day/night production.]

$$\left[\sum_{\substack{p \in cpd, w \\ when\ CEH(Z)(Y)_{r,p,w,t} \\ except\ (y='N' \wedge p \in bas)}} \left(ENV\_ACT_{r,p,v,t} * R\_TCZYH_{r,t,p,w} \right) \right] +$$

Annual production of the current emission indicator according to the activity of coupled production pass-out turbines (identified by means of the CEH(Z)(Y) parameter for each time-slice), summed over the seasonal/time-of-day activity of the plants, according to the ENV_ENT algorithm.

$$\left[\sum_{\substack{p\in cpd,t'\wedge w\\ e\in\begin{pmatrix}INP(ENT)c_{r,p,e,t}\vee\\ OUT(ENC)c_{r,p,e,t}\end{pmatrix}}}\begin{pmatrix}ENV\_ENT_{r,v,e,t}*BAL\_TCZY_{r,t,t'p,e}*\\ R\_TCZYH_{r,t,p,w}\end{pmatrix}\$\left\{not\begin{pmatrix}ENV\_ACT_{r,p,v,t}\vee\\ ENV\_ENTXio_{r,p,v}\end{pmatrix}\right\}\right]+$$

Production of the current emission indicator as a function of the total level of installed capacity for a technology.

$$\left[\sum_{p}\left(ENV\_CAP_{r,p,v,t}*R\_CAP_{r,t,p}\right)\right]+$$

Production of the current emission indicator as a function of the total level of new investment in a technology.

$$\left[\sum_{p}\left(ENV\_INV_{r,p,v,t}*R\_INV_{r,t,p}\right)\right]+$$

Production of the current emission indicator as a function of the activity of demand devices, taking into consideration demand device vintaging, according to the ENV_ENT algorithm. [Note that demand devices do not have separate activity and capacity variables, thus output is determined directly from the capacity variable according to the capacity factor and capacity-to-activity unit conversion multiplier.]

$$\left[\sum_{\substack{p\in dmd,\\ e\in\begin{pmatrix}MA(ENT)_{r,p,e,t}\vee\\ MO(ENC)_{r,p,e,t}\end{pmatrix}}}\left\langle\begin{matrix}ENV\_ENT_{r,p,v,t}*BAL\_CAP_{r,t,p,e}*\\ \begin{pmatrix}R\_CAP_{r,t,p}\$\left\{not\left(EFF\_I_{r,p,t}\vee EFF\_R_{r,p,t}\right)\right\}+\\ \begin{pmatrix}R\_INV_{r,t,p}\$EFF\_I_{r,p,t}+\\ R\_RESIDV_{r,t,p}\$EFF\_R_{r,p,t}\end{pmatrix}\end{pmatrix}\end{matrix}\right\rangle\right]+$$

Application of any GWP factor for GHG accounting that relates one emission indicator to another.

$$\left[\sum_{v'\in gwp}\left(ENV\_GWP_{r,v',v,t}*EM_{r,v}\right)/ENV\_SCAL_{r}\right]+$$

The annual total of the current emission indicator, optionally scaled. Note that the variable may be bounded (EM_BOUND) to cap emissions in a region.

$$\left(R\_EM_{r,v} / ENV\_SCAL_{r}\right)$$

$$\{=\}$$

$$0$$

Where ENV_SCAL = scale parameter applied to all emissions to scale the model and marginals reported.

2.5.31 MR_TEZY(r,t,p,w) MARKAL Matrix, Row 36

Description: The **T**ime-slice **E**lectric generation utilization constraint that ensures that the season(**Z**)/time-of-day(**Y**) activity of a power plant (R_TEZY), that is the amount of electricity generated, is in line with the total installed capacity (R_CAP), according to the availability of the technology (AF/AF(Z)(Y)), taking into consideration any scheduled maintenance (R_M).

Purpose and Occurrence: The electric generation utilization equation oversees the basic operation of a power plant. As such it ensures that the production of electricity in each time-slice from all the installed capacity is in accordance with the availability of the technology at each point of time. The determination of the availability is dependent upon the nature of the technology (e.g., conventional or externally load managed (xlm)), and whether the analyst defines the availability annually or by time-slice (AF/AF(Z)(Y) or CF/CF(Z)(Y)), and if unavailability can be scheduled by the model or some portion of the availability is pre-scheduled (AF_TID $\neq$1) and thus removes some of the flexibility for the model to schedule downtime. This equation is generated in all time periods and time-slices for which a technology is available.

Units: PJ, or any other unit in which heat is tracked.

Type: *Binding.* The equation is a less than or equal ($\leq$) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under utilization of a power plant in a time-slice and period. As conventional power plants normally operate at different levels at various times of the day it would not be surprising to see slack with respect to the utilization of the capacity, particularly at night.

Dual variable: The dual variable (DVR_TEZY) of the utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more unit of production from a power plant in a time-slice. A zero dual value implies that the plant is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of power plant modeled, conventional plants where a maximum availability factor is provided (by means of AFs), and externally load managed plants where the operation is fixed with respect to the total capacity installed (by means of CFs). These AF/CFs are then applied along with the time-slices (QHR(Z)(Y)) to determine the actual amount of electricity produced by each power plant in each time-slice. Since CF-based plants have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_TCZYH, R_TEZY) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQTEZY.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#31}: MR\_TEZY_{r,t,p,w} \ \forall \left\{ \begin{matrix} AF(Z)(Y)_{r,p,w,t} \wedge \\ (p \in ela \wedge p \notin xlm \wedge except) \wedge \\ \left(\begin{matrix} [p \in (bas \cup stg) \wedge y = 'N'] \vee \\ [PEAK(CON)\_TID_{r,p} \wedge (y = 'N')] \end{matrix} \right) \end{matrix} \right\}$$

Electric production from conventional electric generating plants.

$$\left[R\_TCZYH_{r,t,p,w} \right] -$$

Electric production from coupled heat and power pass-out turbines, where the amount of electricity is dependent upon the level of heat output.

$$\left[\left(\begin{matrix} CEH(Z)(Y)_{r,p,w,t} / ELM_{r,p,t} \ * \\ R\_TCZYH_{r,t,p,w} \end{matrix} \right) \$ \left(p \in cpd \wedge ELM_{r,p,t} \right) \right] -$$

Available capacity from plants for which availability is specified by season/time-of-day, as opposed to annually, apportioned to the current time-slice.

$$\left[\left\langle \begin{array}{l} CAPUNIT_{r,p} * \begin{pmatrix} QHR(Z)(Y)_{r,w} \$ p \notin bas \vee \\ QHRZ_{r,z} \$ p \in bas \end{pmatrix} * \\ AF(Z)(Y)_{r,p,w,t} * R\_CAP_{r,t,p} \end{array} \right\rangle \$ \left(not\ AF_{r,p,t} \right) \right] -$$

Available capacity from plants for which annual availability is specified, and perhaps scheduled outage.

$$\left[\left\langle \begin{array}{l} CAPUNIT_{r,p} * \begin{pmatrix} QHR(Z)(Y)_{r,w} \$ p \notin bas \vee \\ QHRZ_{r,z} \$ p \in bas \end{pmatrix} * \\ \left[\left(1 - AF_{r,p,t}\right) * \left(1 \$ (not\ AF\_TID_{r,p}) + AF\_TID_{r,p}\right) \right] * \\ R\_CAP_{r,t,p} \end{array} \right\rangle \$ AF_{r,p,t} \right] -$$

Available capacity from plants for which annual availability is specified, and where a fraction of outage is scheduled (AF_TID ≠1). Here adding in the scheduled portion of the outage.

$$\left[\left\langle \begin{array}{l} \begin{pmatrix} \left(QHR(Z)(Y)_{r,p,w} / QHRZ_{r,p,z}\right) \$ p \notin bas \vee \\ 1 \$ p \in bas \end{pmatrix} * \\ R\_M_{r,t,p,z} \end{array} \right\rangle \$ \begin{pmatrix} AF\_TID_{r,p} \neq 1 \wedge \\ AF_{r,p,t} \end{pmatrix} \right] -$$

Available capacity from peaking only plants (PEAK(CON)_TID) according to the peak operation fraction for the current time-slice.

$$\left[\begin{pmatrix} CAPUNIT_{r,p} * QHR(Z)(Y)_{r,w} * PD(Z)D_{r,p,z,t} * \\ R\_CAP_{r,t,p} \end{pmatrix} \$ PEAK(CON)\_TID_{r,p} \right]$$

$$\{\leq\}$$

$$0$$

2.5.32 MR_THZ(r,t,p,z) MARKAL Matrix, Row 37

Description: The **T**ime-slice **H**eat generation utilization constraint that ensures that the seasonal (**z**) activity of a heating plant (R_THZ), that is the amount of heat generated, is in line with the total installed capacity (R_CAP), according to the availability of the technology (AF/AF(Z)(Y)), taking into consideration any scheduled maintenance (R_M).

Purpose and Occurrence: The heat generation utilization equation oversees the basic operation of a power plant.
As such it ensures that the production of heat in each season from all the installed capacity is in accordance with the availability of the technology. The determination of the availability is dependent upon the nature of the technology (e.g., conventional or externally load managed (xlm)), whether the analyst defines the availability annually or by time-slice (AF/AF(Z)(Y) or CF/CF(Z)(Y)), and whether or not unavailability can be scheduled by the model or some portion of the availability is pre-scheduled (AF_TID≠1) and thus removes some of the flexibility for the model to schedule downtime. This equation is generated in each season and time period from which a technology first becomes available. Note that for peaking only heating plants (PEAK(CON)_TID) the constraint is only generated in the heating/cooling (heatcool, default 'W'inter) primary season.

Units: PJ, or any other unit in which heat is tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under-utilization of a heating plant in a season and period.

Dual variable: The dual variable (DVR_THZ) of the utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more unit of production from a heating plant in a season. A zero dual value implies that the plant is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of power plant modeled, conventional plants where a maximum availability factor is provided (by means of AFs), and externally load managed plants where the operation is fixed with respect to the total capacity installed (by means of CFs). These AF/CFs are then applied along with the seasonal time-slices (QHRZ) to determine the actual amount of heat produced by each power plant in each season. Since CF-based plants have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_TCZYH, R_THZ) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQTHZ.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 32}: MR\_THZ_{r,t,p,z} \;\forall \begin{pmatrix} AF(Z)(Y)_{r,p,z,'D',t} \wedge \left(p \in hpl \wedge p \notin xlm\right) \\ except\left[PEAK(CON)\_TID_{r,p} \wedge (z='I' \vee 'S')\right] \end{pmatrix}$$

Heat production from conventional heating plants.

$$\left[R\_THZ_{r,t,p,z}\right] -$$

Available capacity from plants for which availability is specified by season/time-of-day, as opposed to annually, apportioned to the current time-slice.

$$\left[\begin{pmatrix} CAPUNIT_{r,p} * QHRZ_{r,z} * AF(Z)(Y)_{r,p,z,'D',t} * \\ R\_CAP_{r,t,p} \end{pmatrix} \$\left(not\; AF_{r,p,t}\right)\right] -$$

Available capacity from plants for which annual availability, and perhaps scheduled outage is specified.

$$\left[\left\langle \begin{matrix} CAPUNIT_{r,p} * QHRZ_{r,z} * \\ \left[1-\left(1-AF_{r,p,t}\right) * AF\_TID_{r,p}\right] \$ AF\_TID_{r,p} * \\ R\_CAP_{r,t,p} \end{matrix} \right\rangle \$\, AF_{r,p,t}\right] -$$

Available capacity from plants for which annual availability is specified, and where a fraction of outage is scheduled (AF_TID ≠1). Here adding in the scheduled portion of the outage.

$$\left[R\_M_{r,t,p,z}\; \$\left(AF\_TID_{r,p} \neq 1 \wedge AF_{r,p,t}\right)\right] -$$

Available capacity from peaking only plants (PEAK_TID) according to the peak operation fraction for the current time-slice.

$$\left[\begin{pmatrix} CAPUNIT_{r,p} * QHR(Z)(Y)_{r,w} * PD(Z)D_{r,p,'W',t} * \\ R\_CAP_{r,t,p} \end{pmatrix} \$ PEAK\_TID_{r,p}\right]$$

$$\{\leq\} 0$$

2.5.33 MR_TXSUB(r,txsub) MARKAL Matrix, Row 38

Description: The charging or crediting of a tax/subsidy (TSUB_COST) for commodities (TSUB_ENT) consumed/produced by technologies (TSUB_TCH). The total tax/subsidy is subtracted from the total system costs in the reports, and reported separately.

Purpose and Occurrence: To allow a tax (+TSUB_COST) or subsidy (-TSUB_COST) to be associated with a commodity entering or leaving a technology. For example, to give a credit to green power the "green" electricity leaving each such generating facility would be listed. This equation is generated in each time period when at least one tax or subsidy cost is provided.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of the tax or subsidy accumulated from all the commodities and technologies associated with the tax/subsidy. Note that the tax/subsidy is applied to the level of incoming and/or outgoing commodities listed for a particular tax/subsidy.

Dual variable: The dual variable (DVR_TXSUB) of the tax/subsidy constraint (shadow price) indicates the amount that the energy system would be benefit if one less unit of the tax was generated, or lose if one less unit of the subsidy was available. A zero dual value is not possible unless no tax/subsidy is produced. The value is negative when the primal equation is tight and a tax is imposed, indicating how much the objective function would decrease, and positive if a subsidy is available indicating how much more direct cost would be incurred without the subsidy.

Remarks: The level of the tax subsidy is included in the overall objective function, but split out from the total energy system costs as they are considered direct energy system expenditures.

GAMS Routine: MMEQTAXS.INC

MATHEMATICAL DESCRIPTION

As the MR_TXSUB equation is built up directly from the three primary balance equations:

- MR_BAL_G/E/S for energy and materials respectively;
- MR_BALE1/2 for electricity; and
- MR_BALDH for heat.

rather than reproducing those entire equations here these equations are simply referred to in this equation specification, with the TSUB_TCH multiplier applied as appropriate.

$$\text{EQ\#33}: MR\_TXSUB_{r,t,txs} \; \forall TSUB\_COST_{r,txs,t}$$

Tax/subsidy component that involves energy, other than electricity or heat, or material from or to resource supply options. Refer to EQ#2 for MR_BALcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \notin elc \\ s \in TSUB\_SEP_{r,txs,s,t}}} \left(MR\_BALcoef * TSUB\_SEP_{r,txs,s,t} \right) \right] +$$

Tax/subsidy component that involves electricity from or to resource supply options. Refer to EQ#4 for MR_BALEcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \in elc \\ s \in TSUB\_SEP_{r,txs,s,t}}} \left(MR\_BALEcoef * TSUB\_SEP_{r,txs,s,t} \right) \right] +$$

Tax/subsidy component that involves regular energy, other than electricity or low-temperature heat, or material from technologies. Refer to EQ#2 for MR_BALcoef coefficient details.

$$\left[\sum_{\substack{e \in TSUB\_ENT_{r,txs,e} \\ e \notin (elc \vee lth) \\ p \in TSUB\_TCH_{r,txs,p,t}}} \left(MR\_BALcoef * TSUB\_TCH_{r,txs,p,t} \right) \right] +$$

Tax/subsidy component that involves electricity from power plants. Refer to EQ#4 for MR_BALEcoef coefficient details.

$$\left[\sum_{\substack{e \in elc \\ e \in TSUB\_ENT_{r,txs,e} \\ p \in TSUB\_TCH_{r,txs,p,t} \\ p \in ela}} \left(MR\_BALEcoef * TSUB\_TCH_{r,txs,p,t} \right) \right] +$$

Tax/subsidy component that involves low-temperature heat from power plants. Refer to EQ#3 for MR_BALDHcoef coefficient details.

$$\left[\sum_{\substack{e \in lth \\ e \in TSUB\_ENT_{r,txs,e} \\ p \in TSUB\_TCH_{r,txs,p,t} \\ p \in (hpl \cup cpd)}} \left(MR\_BALDHcoef * TSUB\_TCH_{r,txs,p,t} \right) \right] -$$

The total accumulated amount of the tax/subsidy from all sources (commodity/technology combinations. The variable enters the objective function with the (+/-) TSUB_COST applied.

$$R\_TAXSUB_{r,txs,t}$$

$$\{=\}$$

$$0$$

2.5.34 MR_UTLCON(r,t,p) MARKAL Matrix, Row 39

Description: The adjustment to the availability of a power plant to account for the fact that some portion of outage is due to scheduled maintenance. Normally the model will schedule all unavailability, but if the user specifies that some fraction of annual unavailability is scheduled [AF_TID ≠ 1] then this constraint is generated to adjust the amount of unavailability that the model is free to schedule.

Purpose and Occurrence: The conversion plant utilization equation ensures that the sum of all scheduled maintenance in all seasons is in line with the portion of the capacity that is unavailable due to scheduled outages, such as fuel reloading or maintenance. This equation is generated in each time period from which a technology first becomes available if scheduled maintenance is specified for the plant (AF_TID ≠ 1) and time-sliced availability factors (AF(Z)(Y)) are not provided.

Units: PJ, or any other unit in which process activity is tracked.

Type: *Binding.* The equation is a greater than or equal (≥) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or excess scheduled maintenance owing to idling of a conversion plant in a period.

Dual variable: The dual variable (DVR_UTLCON) of the conversion plant utilization maintenance constraint (shadow price) indicates the amount that the energy system would have to pay if the scheduled maintenance was increased by one unit. A zero dual value implies that the plant operation is not limited by its scheduled outage. The value is positive when the primal equation is tight.

Remarks: The MR_UTLCON equation is only generated when the AF_TID ≠ 1, thus the equation is eliminated when the model is left to schedule all unavailability.

GAMS Routine: MMEQUCON.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#34}: MR\_UTLCON_{r,t,p} \ \forall \begin{pmatrix} p \in con \wedge AF_{r,p,t} \wedge \\ (AF\_TID_{r,p} \neq 1) \end{pmatrix}$$

Total scheduled maintenance in all seasons.

$$\left[\sum_{z}\left(R\_M_{r,t,p,z}\right)\right] -$$

The portion of the capacity that is unavailable due to scheduled outage.

$$\left[\begin{array}{l}\left\langle\left(1-AF_{r,p,t}\right)*\left(1-AF\_TID_{r,p}\right)\right\rangle*CAPUNIT_{r,p}*\\R\_CAP_{r,t,p}\end{array}\right]$$

$$\{\geq\}$$

$$0$$

2.5.35 MR_UTLPRC(r,t,p) MARKAL Matrix, Row 40

Description: The constraint that ensures that the annual activity of a conventional process (R_ACT) is in line with the total installed capacity (R_CAP) according to the availability of the technology (AF).

Purpose and Occurrence: The process utilization equation oversees the basic operation of a process (prc). As such it ensures that the annual activity for all the installed capacity is in accordance with the availability of the process. The determination of the actual availability of a process is dependent upon the nature of technology (e.g., conventional or externally load managed (xpr)). This equation is generated in each time period from which a technology first becomes available.

Units: PJ, or any other unit in which process activity is tracked.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of the utilization equation represents the slack or under utilization of a process in a period.

Dual variable: The dual variable (DVR_UTLPRC) of the process utilization constraint (shadow price) indicates the amount that the energy system would benefit from permitting one more units of production from a process. A zero dual value implies that the process is operating below its maximum permitted output level. The value is negative when the primal equation is tight.

Remarks: With respect to utilization there are basically two types of processes modeled, conventional processes where a maximum availability factor is provided (by means of AFs), and externally load managed processes where the operation is fixed with respect to the total capacity installed (by means of CF). Since CF-based processes have no flexibility with respect to their operation relative to available capacity no utilization equation is generated for them, but rather the appropriate CF*R_CAP expression is substituted for the activity variable (R_ACT) directly in the other constraints where necessary (e.g., balance, emissions, etc.).

GAMS Routine: MMEQUPRC.INC

MATHEMATICAL DESCRIPTION

$$\text{EQ\#35}: MR\_UTLPRC_{r,t,p} \ \forall \left(p \in prc \wedge p \notin xpr \right)$$

Annual activity of a process.

$$R\_ACT_{r,t,p} -$$

Total amount of available capacity of a process.

$$\left(CAPUNIT_{r,p} * AF_{r,p,t} \right) * R\_CAP_{r,t,p}$$

$$\left\{ \leq \right\}$$

2.6 Lumpy Investment

As discussed in section 1.9 there are times when for certain technologies only discrete increments of new investments are to be permitted. This is the case when a "build or not" issue arises for an equipment of large size or when the cost of a technology is heavily dependent upon its size. Some examples include nuclear power plants, natural gas pipelines, long-distance electric transmission lines, etc.

In order to represent the yes/no/ nature of such decisions Mixed Integer Programming (MIP) is employed. As MIP problems are much more difficult to solve than standard LP problems the Lumpy Investment (LI) feature should be applied to only those situations where it is deemed necessary to model investments as discrete decisions.

In this section we provide the data and modeling details associated with modeling Lumpy Investments (LI) in MARKAL. To this end the next three subsections will address the Sets and Parameters, Variables, and Equations related to Lumpy Investment.

2.6.1 Sets and Parameters

Like all other aspects of MARKAL the user describes the LI components of the energy system by means of the Sets, Switches and Parameters described in this section. Table

2-16 below describes the collection of user input parameters associated with the Lumpy Investment feature.

Besides the basic data described in Table 2-16 the user controls whether or not to activate the LI feature by means of the $SET LUMPYINV 'YES' switch. This switch is provided by the data handling system (ANSWER or VEDA-FE) when the user indicates that LI's are to be included in a run. This permits the easy exclusion of the feature if the user does not want to perform a MIP solve without having to remove the LI data.

Table 2-16. Definition of user input parameters

Input Parameter (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
INV_BLOCK (p,t)	INV_BLK	*INV_BIN* *INV_INT* *INV_SOS*	• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Required to model lumpy investments in new capacity for a technology. • One of the parameters INV_BIN / INV_INT / INV_SOS must also be specified for the technology to indicate which of three lumpy investment modeling options is desired.	The size of the "lump" for investment in new capacity for a non-resource technology. Specified where the technology is such that the level of investment in new capacity (R_INV) must be at this specific size, or possibly an integer multiple of this size, or zero. • Three different lumpy investment modeling options are available, according to which of the indicators INV_BIN /INV_INT/INV_SOS is specified. • The requirement that investment in new capacity (R_INV) must be at the lump size, or an integer multiple of the lump size, is achieved via the MR_INVBLKn equations in which R_INV is linked to INV_BLOCK by *binary* variable R_INVBIN, or by *integer* variable R_INVINT.
INV_BIN (p)		*INV_BLK*	• Indicator. • [0,1]; no default.	• Provided to model lumpy investments for a technology where investment in new capacity in each period (R_INV) is restricted to be INV_BLOCK, or zero. • INV_BLOCK parameter must also be specified for the technology.	An indicator (always 1) that investment in new capacity in a specified non-resource technology is lumpy and in each period is restricted to be either INV_BLOCK, or zero. Thus INV_BIN represents the strictly yes/no build or not option. • This is achieved by the equation MR_INVBLK2 in which investment in new capacity for the technology (R_INV) is set equal to INV_BLK * R_INVBIN, where R_INVBIN is a *binary* variable (takes the value 0 or 1).

INV_INT (p)		*INV_BLK*	• Indicator. • [0,1]; no default.	• Provided to model lumpy investments for a technology where investment in new capacity in each period (R_INV) is restricted to be an integer multiple of INV_BLOCK, or zero. • INV_BLOCK parameter must also be specified for the technology.	An indicator (always 1) that investment in new capacity in a specified non-resource technology is lumpy and in each period is restricted to be either an integer multiple of INV_BLOCK, or zero. Thus INV_INT permits only incremental investments of the same size, or none. • This is achieved by the equation MR_INVBLK1 in which investment in new capacity for the technology (R_INV) is set equal to INV_BLK * R_INVINT, where R_INVINT is an *integer* variable (takes the value 0 or 1 or 2 or 3 or …).
INV_SOS (p)		*INV_BLK*	• Indicator. • [0,1]; no default.	• Provided to model lumpy investments for a technology where investment in new capacity in each period (R_INV) is restricted to be INV_BLOCK or zero, and where investment is allowed in at most one period. • INV_BLOCK parameter must also be specified for the technology.	An indicator (always 1) that investment in new capacity in a specified non-resource technology is lumpy and over the entire modeling horizon is restricted to be either INV_BLOCK or zero. Thus the lumpy investment is permitted to occur in at most one period. • The "INV_BLOCK or zero" restriction is achieved by the equation MR_INVBLK2 in which investment in new capacity for the technology (R_INV) is set equal to INV_BLK * R_INVBIN, where R_INVBIN is a *binary* variable (takes the value 0 or 1). • The requirement that investment is allowed in at most one period is achieved by the equation MR_INVSOS.

2.6.2 Variables

The variables that are used to model the Lumpy Investment feature in MARKAL are presented in Table 2-17 below. Since the primary role of the variables and equations used to model LI is to control the standard MARKAL investment variable (R_INV), LI is rather self-contained. That is the R_INV variable links the LI decisions to the rest of the model.

Table 2-17. Model variables

VAR Ref	Variable (Indexes)	Variable Description
LIV.1	R_INVBIN (r,t,p)	*Binary* variable (takes the value 0 or 1) associated with a technology where investment in new capacity (R_INV) is modeled as lumpy by providing the INV_BLOCK and INV_BIN (and perhaps INV_SOS) parameters. If R_INVBIN takes the value 1, then investment in new capacity is equal to the lump size INV_BLOCK; if it takes the value 0, then investment in new capacity is zero.
LIV.2	R_INVINT (r,t,p)	*Integer* variable (takes the value 0 or 1 or 2 or 3 or …) associated with a technology where investment in new capacity is modeled as lumpy by providing the INV_BLOCK and INV_INT parameters. If R_INVINT takes the (non-negative integer) value *n*, then investment in new capacity is *n* times the lump size INV_BLOCK. (This includes the case where R_INVINT takes the value *n*=0, in which case investment in new capacity is zero.)

2.6.2.1 R_INVBIN(r,t,p)

Description: *Binary* variable (takes the value 0 or 1) associated with a technology where investment in new capacity is modeled as lumpy by providing the INV_BLOCK and INV_BIN or INV_SOS parameters.

Purpose and Occurrence: As noted above, this binary variable is generated for each (non-resource) technology where the user has indicated, by specifying the INV_BLOCK and INV_BIN (and perhaps INV_SOS) parameters, that investment in new capacity is to be modeled as lumpy; and where in each period, such investment is to be either at the lump size INV_BLOCK or zero.

This binary variable is generated for each appropriate technology in all time periods beginning from the period that the technology is first available (START). The requirement that investment in new capacity in a period (R_INV) must be at the lump size, or zero, is achieved by the equation MR_INVBLK2 in which investment in new capacity for the technology (R_INV) is set equal to INV_BLOCK * R_INVBIN. In addition, if the user has specified the INV_SOS parameter to indicate that investment in new capacity is allowed in at most one period, then this binary variable also occurs in the equation MR_INVSOS.

Units: None. This is a binary variable that takes the value 0 or 1.

Bounds: This binary variable may be indirectly bounded by specifying a bound (IBOND(BD)) on the level of investment in new capacity (R_INV). In each period, the equation MR_INVBLK2 sets the investment level R_INV = INV_BLOCK * R_INVBIN. So if an upper/lower/fixed bound is specified for R_INV (via IBOND(BD)), the indirect upper/lower/fixed bound that applies in each period to R_INVBIN is [IBOND(BD) ÷INV_BLOCK]. The user is cautioned to consider carefully the consequences of specifying bounds for the R_INV variable. For example, if R_INV is subject to a *fixed* positive bound IBOND(FX) ≠ INV_BLOCK, this would imply that R_INVBIN is subject to a *fixed* bound ≠ 0 or 1, and so create an infeasibility, since binary variable R_INVBIN must take the value 0 or 1. Similarly, a lower bound on R_INV that is larger than INV_BLOCK would also create an infeasibility, and an upper bound lower than INV_BLOCK would force the R_INV variable to be equal to 0.

2.6.2.2 R_INVINT(r,t,p)

Description: *Integer* variable (takes the value 0 or 1 or 2 or 3 or …) associated with a technology where investment in new capacity is modeled as lumpy by providing the INV_BLOCK and INV_INT parameters.

Purpose and Occurrence: As noted above, this integer variable is generated for each (non-resource) technology where the user has indicated, by specifying the INV_BLOCK and INV_INT parameters, that investment in new capacity is to be modeled as lumpy; and where in each period, such investment is to occur in (non-negative integer) multiples of the lump size INV_BLOCK. This includes the possibility that investment in a period is zero.

This integer variable is generated for each appropriate technology in all time periods beginning from the period that the technology is first available (START). The requirement that investment in new capacity in a period (R_INV) must be a multiple of the lump size is achieved by the equation MR_INVBLK1 in which investment in new capacity for the technology (R_INV) is set equal to INV_BLOCK * R_INVINT.

Units: None. This is an integer variable that takes non-negative integer values (0, 1, 2, 3, …).

Bounds: By default, an upper bound of 10 is applied to this integer variable (assuming INV_BLOCK > 0; if INV_BLOCK = 0 in some period, an upper bound of zero is applied in this period), meaning that investments can be at most 10 increments of INV_BLOCK. Where an *upper* bound (IBOND(UP)) on the level of investment in new capacity (R_INV) is specified by the user, the default upper bound of 10 for R_INVINT is replaced by the integer part of the quantity IBOUND(UP) / INV_BLOCK. The reason for setting a default upper bound on R_INVINT is that the MIP optimizer requires an upper bound on every integer variable.

As noted above, an explicit upper bound on R_INVINT is set whenever an *upper* bound (IBOND(UP)) is specified for the investment variable R_INV. But note that R_INVINT is subject to an indirect lower/fixed bound whenever lower/fixed bound is specified for R_INV. In each period, the effect of the equation

MR_INVBLK1 is that R_INV = INV_BLOCK * R_INVINT. So if a lower/fixed bound of IBOND(BD) is specified for R_INV, the indirect lower/fixed bound that applies in each period to R_INVINT is Integer-lower (IBOND(BD) ÷ INV_BLOCK).

It is good practice for the user to specify lower and/or upper bounds on R_INV that are as tight as possible, so as to reduce the optimization effort of the MIP algorithm. However, the user is cautioned to consider carefully the consequences of specifying positive values for IBOND(BD) that are not integer multiples of INV_BLOCK. For example, if R_INV is subject to a *fixed* positive bound of IBOND(FX) that is *not an integer multiple* of INV_BLOCK, this would imply that R_INVINT is subject to a *fixed* bound that is not an integer, and so create an infeasibility.

2.6.3 Equations

The equations that are used to model the Lumpy Investment feature in MARKAL are presented in Table 2-18 below. Since the primary role of the variables and equations used to model LI is to control the standard MARKAL investment variable (R_INV), LI is rather self-contained. That is the R_INV variable links the LI decisions to the rest of the model.

Table 2-18. Model constraints

EQRef	Constraints (Indexes)	Constraint Description	GAMS Ref
LIE.1	MR_INVBLK1 (r,t,p)	The constraint for a lumpy investment technology that ensures that investment in new capacity in a period (R_INV) is either an integer multiple (INV_INT) of the lump size (INV_BLOCK), or zero.	MMEQLUMP.LIV
LIE.2	MR_INVBLK2 (r,t,p)	The constraint for a lumpy investment technology that ensures that investment in new capacity in a period (R_INV) is either equal (INV_BIN) to the lump size (INV_BLOCK), or zero.	MMEQLUMP.LIV
LIE.3	MR_INVSOS (r,p)	The constraint for a lumpy investment technology that ensures that investment in new capacity occurs in at most one period.	MMEQLUMP.LIV

2.6.3.1 MR_INVBLK1(r,t,p)

Description: The constraint for a lumpy investment technology that ensures that investment in new capacity in a period (R_INV) is either an integer multiple (INV_INT) of the lump size (INV_BLOCK), or zero.

Purpose and Occurrence: To ensure that new investments (R_INV) only occur as whole increments of the lump size by requiring that investment in new capacity is either zero or a positive integer multiple of the lump size. This is achieved by setting R_INV equal to INV_BLOCK * R_INVINT, where R_INVINT is an *integer* variable (takes the value 0, 1, 2, 3, …). This equation is generated in each time period beginning from the period in which a technology is first available (START) when investment lump size (INV_BLOCK) along with parameter INV_INT to indicate that investment is restricted to be an integer multiple of the lump size, or zero is provided for a technology. Note that when parameter INV_BIN or INV_SOS are specified, equation MR_INVBLK1 will **not** be generated.

Units: Technology capacity units.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero if a feasible solution to the MIP is to occur.

Dual variable: The dual variable (DVR_INVBLK1) of this constraint in the MIP solution (shadow price) is of little interest: a dual value represents the cost impact of an infinitesimal change in the RHS of a constraint, but such an infinitesimal change is ruled out for this constraint due to the integer nature of the R_INVINT variable.

Remarks: The analyst controls the activation of the lumpy investment formulation by means of the model variant selected at run time (Lumpy Investment switch $SET LUMPYINV 'YES' as set by the data handling system) and the inclusion of the required lumpy investment data.

GAMS Routine: MMEQLUMP.LIV

MATHEMATICAL DESCRIPTION

$$\text{EQ\#LIE.1: } \boldsymbol{MR\_INVBLK1_{r,t,p}} \ \forall \left(\boldsymbol{INV\_INT_{r,p}} \wedge \boldsymbol{not} \left(\begin{matrix} \boldsymbol{INV\_BIN_{r,p}} \vee \\ \boldsymbol{INV\_SOS_{r,p}} \end{matrix} \right) \right)$$

Lump size multiplied by *integer* variable R_INVINT, in the current period.

$$INV\_BLK_{r,p,t} * R\_INVINT_{r,t,p} -$$

Investment in new capacity in the current period.

$$R\_INV_{r,t,p}$$

$$\{=\}$$

2.6.3.2 MR_INVBLK2(r,t,p)

Description: The constraint for a lumpy investment technology that ensures that investment in new capacity in a period (R_INV) is either equal to the lump size (INV_BLOCK), or zero.

Purpose and Occurrence: To ensure that new investments (R_INV) only occur equal to the lump size by requiring that investment in new capacity is either zero or the lump size. This is achieved by setting R_INV equal to the lump block size (INV_BLOCK) times a yes/no binary variable (R_INVBIN), where R_INVBIN takes on the value 0 or 1. This equation is generated in each time period beginning from the period in which a technology is first available (START) when investment lump size (INV_BLOCK) is specified, along with parameter INV_BIN or INV_SOS to indicate that investment is restricted to be an integer multiple of the lump size, or zero is provided for a technology. Note that where parameter INV_SOS is specified, equation MR_INVSOS will also be generated to ensure that investment in new capacity occurs in at most one period.

Units: Technology capacity units.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:
Primal: The level of this constraint must be zero in a feasible solution to the MIP.
Dual variable: The dual variable (DVR_INVBLK2) of this constraint in the MIP solution (shadow price) is of little interest: a dual value represents the cost impact of an infinitesimal change in the RHS of a constraint, but such an infinitesimal change is ruled out for this constraint due to the integer nature of the R_INVBIN variable.

Remarks: The analyst controls the activation of the lumpy investment formulation by means of the model variant selected at run time (Lumpy Investment switch $SET LUMPYINV 'YES' as set by the data handling system) and the inclusion of the required lumpy investment data.

GAMS Routine: MMEQLUMP.LIV

MATHEMATICAL DESCRIPTION

$$\text{EQ\#LIE.2:}\ MR\_INVBLK2_{r,t,p}\ \forall \begin{pmatrix} INV\_BIN_{r,p} \vee \\ INV\_SOS_{r,p} \end{pmatrix}$$

Lump size multiplied by *binary* variable R_INVBIN, in the current period.

$$INV\_BLK_{r,p,t} * R\_INVBIN_{r,t,p} -$$

Investment in new capacity in the current period.

$$R\_INV_{r,t,p}$$

$$\{=\}$$

2.6.3.3 MR_INVSOS(r,p)

Description: The constraint for a lumpy investment technology that ensures that investment in new capacity occurs in at most one period. This constraint is only available in the context of lumpy investment, and where investment in new capacity in a period is required to be either zero or equal to the lump size (INV_SOS). It is not available where investment in new capacity can be a multiple (>1) of the lump size.

Purpose and Occurrence: To ensure that new investments (R_INV) only occur once over the entire modeling horizon. Investment in new capacity that is restricted to being yes/no in nature occurs in the period when the binary variable R_INVBIN takes on a value of 1. The requirement that investment in new capacity occurs in at most one period is achieved by constraining the sum over all periods of the R_INVBIN *binary* variables to be less than or equal to 1. If the sum of the R_INVBIN variables is zero, then there is no investment in new capacity in any period; if this sum is 1 then exactly one of the R_INVBIN variables takes the value 1 and there is an investment in new capacity (at the lump size) in the corresponding period. This equation is generated for each technology for which investment lump size (INV_BLOCK) is specified, along with an indication that such investment is only to occur once (INV_SOS) to indicate both that investment is restricted to the lump size or zero, and that investment is allowed in at most one period.

Units: None.

Type: *Binding.* The equation is a less than or equal (≤) constraint.

Interpretation of the results:

Primal: The level of this constraint must be either 0 or 1 in a feasible solution to the MIP, with the values of 0 and 1 corresponding to the situations where investment does not occur in any period, and where it occurs in exactly one period, respectively.

Dual variable: The dual variable (DVR_INVSOS) of this constraint in the MIP solution (shadow price) is of little interest: a dual value represents the cost impact of an infinitesimal change in the RHS of a constraint, but such an infinitesimal change is ruled out for this constraint due to the integer nature of the R_INVBIN variable.

Remarks: The analyst controls the activation of the lumpy investment formulation by means of the model variant selected at run time (Lumpy Investment switch $SET LUMPYINV 'YES' as set by the data handling system) and the inclusion of the required lumpy investment data.

GAMS Routine: MMEQLUMP.LIV

MATHEMATICAL DESCRIPTION

$$\text{EQ\#LIE.3: } MR\_INVSOS_{r,p} \ \forall \left(INV\_SOS_{r,p} \right)$$

Sum over all periods beginning from the period that a technology is first available (START) of the *binary* variables R_INVBIN.

$$\left[\sum_{t \geq START} \left(R\_INVBIN_{r,t,p} \right) \right]$$

$$\left\{ \leq \right\}$$

2.7 The Damage Cost Option

As discussed in Section 1.8 MARKAL has facilities to permit the assessment of environmental externalities by means of two approaches to determine the impact or cost of damages arising from emissions. As noted in Chapter 1 there are two approaches that are available in Standard MARKAL:

3. the environmental damages are computed ex-post ($SET DAMAGE 'OBJLP'), without feedback into the optimization process, and
4. the environmental damages are part of the objective function ($SET DAMAGE 'OBJNLP') and therefore taken into account in the optimization process.

In order to model environmental damages in MARKAL the standard model formulation is in the first case essentially untouched, and in the latter has an augmented non-linear objective function. The data requirements and model adjustments related to the damage options are presented in the rest of this section. Also, the standard MARKAL-ANSWER result tables T01 and T02 have entries added to report on the total and period-wise damage costs.

Note that owing to the non-linear nature of the modified objective function that endogenizes the damages, the damage option may not be activated with certain other Standard MARKAL options, as presented in Table 2-19 below.

Table 2-19. Valid MARKAL-EV/Standard MARKAL variant combinations

MARKAL Model Variant	$SET DAMAGE ‘OBJLP’	$SET DAMAGE ‘OBJNLP’
Basic Standard MARKAL*	Yes	Yes
MARKAL-MACRO	Yes (via COSTNRG)	Not Implemented
MARKAL-STOCHASTIC	Yes	Not Implemented
MARKAL-ETL/LI	Yes	Not Implemented
MARKAL-MICRO	Not Implemented	Not Implemented

* Includes elastic demands and multi-regional features

2.7.1 Sets, Switches and Parameters

Like all other aspects of MARKAL the user describes the environmental damages to be calculated or incorporated into the optimization by means of a Switch, a Set and the Parameters described in this section. Table 2-20 and Table 2-21 below describe the User Input data, and the Matrix Coefficient and Internal Model Sets and Parameters, respectively, that are associated with the damage option.

Table 2-20. Definition of damage user input data

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
DAMAGE			• Switch. • ['NO', 'OBJLP', 'OBJNLP']; default 'NO'.	• Set to 'OBJLP' or 'OBJNLP' to activate the damage option.	The DAMAGE switch indicates whether and which damage option is to be applied. • To simply include direct cost of emissions in the objective function and report the calculated damage cost estimate based upon the Standard MARKAL, include the line $SET DAMAGE 'OBJLP' in the .GEN file for the model run. This results in the damage estimates being reported in Tables T01 and T02. • To incorporate the estimate of damage cost into the model's objective function, include the line $SET DAMAGE 'OBJNLP' in the .GEN file for the model run. This results in a non-linear problem, and the damage estimates being reported in Tables T01 and T02.
EV_B *(v)*		*EV_COEF*	• Fraction. • [0-1]; no default.	• Provided for any emission that is to be subjected to damages.	The elasticity of the damage with respect to an emission. • Controls whether or not to include an emission in the damage estimate. • Used to determine the environmental externalities coefficient (ENV_COEF) for each emission subject to damages.
EV_COST *(v)*		*EV_COEF*	• Base year money units per emission unit, e.g., 2000M$/ton. • [0-any]; no default.	• Provided for any emission that is to be subjected to damages.	Marginal social cost of an emission. • Used to determine the environmental externalities coefficient (ENV_COEF) for each emission subject to damages.
EV_RLEV *(v)*		*EV_COEF*	• Emission unit, e.g., ton.	• Provided for any emission that is to	Reference level of an emission, from a previous model run.

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
			• [0-any]; no default.	be subjected to damages.	• Used to determine the environmental externalities coefficient (ENV_COEF) for each emission subject to damages.
EVDAMINT *(v,t)*			• Indicator. • [0-1]; default = 0.	• Indicator of the periods in which an emission is to be subjected to damages.	Period control indicator to turn damage accounting on/off for an emission in a period.

Table 2-21. Damage-specific matrix coefficient and internal model parameters

Matrix Controls & Coefficients (indexes)	Type	Description & Calculations
EV_COEF *(v,t)*	I	The externality coefficient corresponding to the damage cost for an emission in each period, calculated as $EV\_COEF_{v,t} = \left[EV\_COST_v / \left(EV\_B_v * EV\_RLEV_v ** \left(EV\_B_v - 1\right)\right)\right]$

2.7.2 Variables

There are no new variables introduced to support the environmental damage option.

2.7.3 Equation

As noted earlier, only the objective function is affected by the activation of the damage option.

2.7.3.1 MR_PRICE(r)

Description: Total system cost augmented with the damage estimates.

Purpose and Occurrence: To include an estimate of damage costs associated with emissions in the model formulation.

This equation is generated once, as the objective function.

GAMS Routine: MMPRIC.MEV

MATHEMATICAL DESCRIPTION

$$\text{EQ\#}27: MR\_PRICE_r$$

All the basic objective function terms.

• • •

The damage cost added as either a direct cost with variable R_EM having no exponent (when 'OBJLP') or as a damage function with variable R_EM having EV_B as exponent (when 'OBJNLP'), for each emission and period.

$$\sum_{\substack{v\$EV\_B_{r,v} \\ t\$EVDAMINT_{r,v,t}}} \begin{bmatrix} PRI\_DF_{r,t} * EV\_COEF_{r,v,t} * \\ R\_EM_{r,t,v} ** EV\_B_{r,v} \$'OBJNLP' \end{bmatrix}$$

2.8 The Endogenous Technological Learning (ETL) option

As discussed in section 1.10 there are situations in which the rate at which a technology's unit investment cost changes over time is a function of cumulative investment in the technology. In these situations, technological learning is called endogenous.

In order to model Endogenous Technological Learning (ETL) in MARKAL Mixed Integer Programming (MIP) is employed. As has already been noted in the section on modeling of Lumpy Investments, MIP problems are much more difficult to solve than standard LP problems, and so the ETL feature should be applied only where it is deemed necessary to model a limited number of technologies as candidates for Endogenous Technological Learning.

In this section we provide the data and modeling details associated with modeling Endogenous Technological Learning (ETL) in MARKAL. To this end the next three subsections will address the Sets, Parameters, Variables, and Equations related to the Endogenous Technological Learning option, including the special clustered learning ETL option where a component common to several technologies learns, thereby benefiting all the related (clustered) technologies.

2.8.1 Sets, Switches and Parameters

Like all other aspects of MARKAL the user describes the ETL components of the energy system by means of a Set and the Parameters and Switches described in this section. Table 2-22 and Table 2-23 below describe the User Input Parameters, and the Matrix Coefficient and Internal Model Sets and Parameters, respectively, that are associated with the Endogenous Technological Learning option. Note that the special clustered learning ETL option requires one additional User Input Parameter (ETL-CLUSTER), and two additional Matrix Coefficient/Internal Model Parameters (CLUSTER and NTCHTEG).

Besides the basic data described in Table 2-22 the user controls whether or not the ETL component is activated by means of the $SET ETL 'YES' switch. This switch is provided by the data handling system (ANSWER or VEDA-FE) when the user indicates that the ETL option is to be included in a run. This permits the easy exclusion of the feature if the user does not want to perform a MIP solve without having to remove the ETL data.

> Note that ETL may not be used in conjunction with MARKAL-MACRO, MICRO, or Damage, as these formulations employ non-linear optimization (NLP) which cannot be combined with MIP.

Table 2-22. Definition of ETL user input parameters

Input Parameter (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
ETL-CUMCAP0 (p)	CCAP0	*PAT* *CCOST0*	• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Required, along with the other ETL input parameters, for each learning technology (TEG).	The initial cumulative capacity (starting point on the learning curve) for a (non-resource) technology that is modeled as one for which endogenous technology learning (ETL) applies. Learning only begins once this level of installed capacity is realized. • The CCAP0 parameter appears as the right-hand-side of the cumulative capacity definition constraint (MR_CUINV). • Note that if the RESID parameter is specified for an ETL technology, then its value in the START period should match the value of ETL-CUMCAP0, otherwise an infeasibility will occur.
ETL-CUMCAPMAX (p)	CCAPM	*CCOSTM*	• Units of capacity (e.g., GW, PJa). • [open]; no default	• Required, along with the other ETL input parameters, for each learning technology (TEG).	The maximum cumulative capacity (ending point on the learning curve) for a (non-resource) technology that is modeled as one for which endogenous technology learning (ETL) applies. • The parameter CCAPM does not appear in any of the ETL constraints, but its value affects the values of a number of internal parameters that directly contribute to one or more of the ETL constraints.
ETL-INDIC (p)	TEG	*ETL-CUMCAP0* *ETL-CUMCAPMAX* *ETL-INVCOST0* *ETL-NUMSEG* *ETL-PROGRATIO*	• Indicator. • [1]; no default.	• Required to identify the learning technologies. • For each TEG the other ETL input parameters are required.	An indicator (always 1) that a non-resource technology is modeled as one for which endogenous technology learning (ETL) applies. • The set TEG, constructed from ETL-INDIC, controls the generation of the ETL constraints. Each of the ETL constraints is generated only for those technologies that are in set TEG.
ETL-INVCOST0	SC0	*PAT*	• Base year	• Required, along	The investment cost corresponding to the starting point

Input Parameter (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
(p)			monetary units per unit of capacity (e.g., 2000 M$/GW or PJa). • [open]; no default.	with the other ETL input parameters, for each learning technology (TEG)..	on the learning curve for a (non-resource) technology that is modeled as one for which endogenous technology learning (ETL) applies. • The parameter SC0 does not appear in any of the ETL constraints, but its value affects the values of a number of internal parameters that directly contribute to one or more of the ETL constraints.
ETL-NUMSEG (p)	SEG	*ALPH* *BETA* *CCAPK* *CCOSTK*	• Number of steps. • [1-9]; no default.	• Required, along with the other ETL input parameters, for each learning technology (TEG)..	The number of segments to be used in approximating the learning curve for a (non-resource) technology that is modeled as one for which endogenous technology learning (ETL) applies. • The SEG parameter appears in all of the ETL constraints that are related to piecewise linear approximation of the learning curve (MR_CC, MR_COS, MR_EXPE1, MR_EXPE2, MR_LA1, MR_LA2).
ETL-PROGRATIO (p)	PRAT	*CCAPK* *CCOST0* *CCOSTM* *PAT* *PBT*	• Decimal fraction. • [0-1]; no default.	• Required, along with the other ETL input parameters, for each learning technology (TEG)..	The “progress ratio” for a (non-resource) technology that is modeled as one for which endogenous technology learning (ETL) applies. The progress ratio, which is referred to as the learning rate, is defined as the ratio of the change in unit investment cost each time cumulative investment in an ETL technology doubles. That is, if the initial unit investment cost is SC0 and the progress ratio is PRAT, then after cumulative investment is doubled the unit investment cost will be PRAT * SC0. • The parameter PRAT does not appear in any of the ETL constraints, but its value affects the values of a number of internal parameters (ALPH, BETA, CCAPK, CCOST0) that directly contribute to one or more of the ETL constraints.

Input Parameter (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
ETL-CLUSTER (p,p)	CLUSTER NCLUSTER		• Decimal fraction. • [0-1]; no default.	• Provided to model clustered endogenous technology learning. • Each of the learning parameters must also be specified for the key learning technology.	The "cluster mapping and coupling factor" for a (non-resource) technology that is modeled as a clustered technology is associated with a key learning technology to which endogenous technology learning (ETL) applies. Clustered technologies use the key ETL technology, and are subject to learning via the key technology. • The first index of the ETL-CLUSTER parameter is a key learning technology. • The second index of the ETL-CLUSTER parameter is a clustered technology that is associated with this key learning technology. • In general there may be several clustered technologies each of which is associated with the same key learning technology, and hence there may be several instances of the ETL-CLUSTER parameter each of which has the same key learning technology as its first index. • The numerical value of the ETL-CLUSTER parameter indicates the extent of coupling between the clustered technology and the key learning technology to which it is associated.

Table 2-23. ETL-specific matrix coefficient and internal model parameters[82]

Matrix Controls & Coefficients (indexes)	Type	Description & Calculations
ALPH (k,p)	I	ALPH are the intercepts on the vertical axis of the line segments in the piecewise linear approximation of the cumulative cost curve. They are calculated in MMCOEF.ML from the starting and ending points of the cumulative cost curve, its assumed form, the number of segments used in its piecewise linear approximation, and the choice of successive interval lengths on the *vertical* axis to be such that each interval is twice as wide as the preceding one. The parameter ALPH occurs in the ETL equation MR_COS that defines the piecewise linear approximation to the cumulative cost curve.
BETA (k,p)	I	BETA are the slopes of the line segments in the piecewise linear approximation of the cumulative cost curve. They are calculated in MMCOEF.ML from the starting and ending points of the cumulative cost curve, its assumed form, the number of segments used in its piecewise linear approximation, and the choice of successive interval lengths on the *vertical* axis to be such that each interval is twice as wide as the preceding one. The parameter BETA occurs in the ETL equation MR_COS that defines the piecewise linear approximation to the cumulative cost curve.
CCAP0 (p)	A	CCAP0 (user input parameter ETL-CUMCAP0) is the initial cumulative capacity (starting point on the learning curve). The parameter CCAP0 occurs in the ETL equation MR_CUINV that defines cumulative capacity in each period.
CCAPK (k,p)	I	CCAPK are the break points on the horizontal axis in the piecewise linear approximation of the cumulative cost curve. They are calculated in MMCOEF.ML from the starting and ending points of the cumulative cost curve, its assumed form, the number of segments used in its piecewise linear approximation, and the choice of successive interval lengths on the *vertical* axis to be such that each interval is twice as wide as the preceding one. The parameter CCAPK occurs in the ETL equations MR_LA1 and MR_LA2 whose role is to ensure that variable R_LAMB(r,t,k,p) lies in the k^{th} interval, i.e., between CCAPK(k-1,p) and CCAP(k,p), when its associated binary variable R_DELTA(r,t,k,p) = 1.
CCOST0 (p)	I	CCOST0 is the initial cumulative cost (starting point on the learning curve). It is calculated in MMCOEF.ML from the initial cumulative capacity (ETL-CUMCAP0) and corresponding initial investment cost (user input parameter ETL-INVCOST0) and the progress ratio (user input parameter ETL-PROGRATIO). The parameter CCOST0 occurs in the ETL equation MR_IC1 that defines first period investment costs (prior to discounting).

[82] Parameters that occur in the ETL-specific equations but that also occur in non-ETL equations (e.g., TCH_LIFE) are not listed in this table.

Matrix Controls & Coefficients (indexes)	Type	Description & Calculations
SEG (p)	A	SEG (user input parameter ETL-NUMSEG) is the number of segments in the cumulative cost curve. The parameter SEG occurs in all of those ETL equations that are related to the piecewise linear approximation of the cumulative cost curve.
TEG (p)	S	TEG is the set of (non-resource) technologies to which endogenous technology learning (ETL) applies. It is derived directly from the user input indicator parameter ETL-INDIC. Each of the ETL equations has set TEG as an index.
CLUSTER (p,p)	I	The "cluster mapping and coupling factor" CLUSTER (user input parameter ETL-CLUSTER) is only relevant when modeling clustered endogenous technology learning. The parameter CLUSTER occurs in the special ETL cluster equation MR_CLU that defines investment in new capacity (R_INV) in the key learning technology as the weighted sum of investments in new capacity of the clustered technologies that are attached to the key technology. (The weights used are the numeric values of the CLUSTER parameter.)
NTCHTEG (p)	I	The parameter NTCHTEG is only relevant when modeling clustered endogenous technology learning. If TEG is an ETL technology, then NTCHTEG(TEG) is the number of clustered technologies that are attached to key technology TEG. NTCHTEG is calculated in MMCOEF.ML from the "cluster mapping and coupling factor" (CLUSTER). It occurs in the special ETL cluster equation MR_CLU.
SAL_ETLINV (r,t,p)	I	This parameter credits back to the regional objective function (MR_PRICE) the discounted salvage cost of learning technology investments that remain available past the end of the modeling horizon. [Note that this parameter does not exist in the code, but rather the expression is explicitly written in MMEQTEG.ML. See MR_SV in the equation section.]

2.8.2 Variables

The variables that are used to model the Endogenous Technological Learning option in MARKAL are presented in Table 2-24 below. As is the case with the modeling of lumpy investments, the primary role of the variables and equations used to model ETL is to control the standard MARKAL investment variable (R_INV) and the associated dynamic cost of these investments, so ETL is rather self-contained. That is the R_INV variable links the ETL decisions to the rest of the model, and the R_IC investment cost variable determines the associated contribution to the regional objective function (MR_PRICE). Note that the special clustered learning ETL option does not require any additional variables, as compared with the modeling of endogenous technology learning when there are no clusters.

Table 2-24. ETL-specific model variables

VAR Ref	Variable (Indexes)	Variable Description
ETLV.1	R_CCAP (r,t,p)	The cumulative investment in capacity for an ETL technology. This variable represents the initial cumulative capacity (ETL-CUMCAP0) plus investments in new capacity made up to and including the current period. This variable differs from the total installed capacity for a technology (R_CAP) in that it includes *all* investments in new capacity made up to and including the current period, whereas the latter only includes investments that are still available (i.e. whose life has not expired yet).
ETLV.2	R_CCOST (r,t,p)	The cumulative cost of investment in capacity for an ETL technology. The cumulative cost is interpolated from the piecewise linear approximation of the cumulative cost curve.
ETLV.3	R_DELTA (r,t,p,k)	*Binary* variable (takes the value 0 or 1) used for an ETL technology to indicate in which interval of the piecewise linear approximation of the cumulative cost curve the cumulative investment in capacity (R_CCAP) lies. A value of 1 for this variable for exactly one interval k indicates that R_CCAP lies in the k^{th} interval.
ETLV.4	R_IC (r,t,p)	The portion of the cumulative cost of investment in capacity for an ETL technology (R_CCOST) that is incurred in period t, and so subject to the same discounting that applies to other period t investment costs. This variable is calculated as the difference between the cumulative costs of investment in capacity for periods t and t-1, and enters the regional objective function (MR_PRICE), adjusted for any salvage (see R_SV_INV)
ETLV.5	R_LAMBD (r,t,p,k)	*Continuous* variable used for an ETL technology to represent the portion of cumulative investment in capacity (R_CCAP) that lies in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve. For a given ETL technology and given time period, ETL model constraints involving this variable and the associated binary variable R_DELTA ensure that R_LAMBD is positive for exactly one interval k.
ETLV.6	R_SV_INV (r,t,p)	The discounted salvage cost of learning investments for an ETL technology that is credited back to the regional objective function (MR_PRICE) for learning technologies remaining past the end of the modeling horizon.

2.8.2.1 R_CCAP(r,t,p)

Description: The cumulative investment in capacity for an ETL technology.

Purpose and Occurrence: This variable tracks the cumulative investment in capacity for an ETL technology which then determines, along with the progress ratio, how much the investment cost is to be adjusted for the learning gains.

This variable is generated for each ETL technology in all time periods beginning from the period that the technology is first available (START). It appears in the cumulative capacity definition constraint (MR_CUINV) that defines it as the initial cumulative capacity (ETL-CUMCAP0) plus investments in new capacity (R_INV) made up to and including the current period. It also appears in the cumulative capacity interpolation constraint (MR_CC). This constraint equates R_CCAP(r,t,p) to the sum over k of the variables R_LAMBD(r,t,p,k) used to represent the cumulative investment in capacity lying in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve.

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent technology capacity.

Bounds: This variable is not directly bounded. It may be indirectly bounded by specifying a bound (IBOND(BD)) on the level of investment in new capacity (R_INV).

2.8.2.2 R_CCOST(r,t,p)

Description: The cumulative cost of investment in capacity for an ETL technology.

Purpose and Occurrence: This variable defines the interpolated cumulative cost of investment in capacity in terms of the continuous variables R_LAMBD and the binary variables R_DELTA, and the internal model parameters ALPH and BETA. ALPH and BETA represent the intercepts on the vertical axis and the slopes, respectively, of the line segments in the piecewise linear approximation of the cumulative cost curve.

This variable is generated for each ETL technology in all time periods beginning from the period that the technology is first available (START). It appears in the cumulative cost interpolation equation (MR_COS) that defines it. It also appears in the equations MR_IC1 and MR_IC2 that define the R_IC variables that represent the portions of the cumulative cost of investment in capacity that are incurred in period t.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Bounds: None.

2.8.2.3 R_DELTA(r,t,p,k)

Description: *Binary* variable (takes the value 0 or 1) used for an ETL technology to indicate in which interval of the piecewise linear approximation of the cumulative cost curve the cumulative investment in capacity (R_CCAP) lies.

Purpose and Occurrence: To indicate which step on the learning curve a technology achieves. A value of 1 for this variable for interval k, and zero values for intervals ≠ k, imply that the cumulative investment in capacity (R_CCAP) lies in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve.

This binary variable, along with the associated continuous variable R_LAMBD, are generated for each ETL technology in all time periods beginning from the period that the technology is first available (START), and for each interval in the piecewise linear approximation. It appears in the constraint MR_DEL, whose purpose is to ensure that, for each ETL technology in each period, it has a value of 1 for exactly one interval k (with zero values for intervals ≠ k); and in the cumulative cost interpolation constraint (MR_COS). It also appears in the pair of constraints MR_LA1 and MR_LA2, whose purpose is to ensure that R_LAMBD, if positive for interval k, is between the two break points on the horizontal axis for interval k in the piecewise linear approximation. (See below under "Purpose and Occurrence" for the variable R_LAMBD.)

Finally, this binary variable appears in two constraints MR_EXPE1 and MR_EXPE2, whose purpose is to reduce the domain of feasibility of the binary variables and thereby improve solution time for the Mixed Integer Program (MIP).

Units: None. This is a binary variable that takes the value 0 or 1.

Bounds: This binary variable is not directly bounded.

2.8.2.4 R_IC(r,t,p)

Description: The portion of the cumulative cost of investment in capacity for an ETL technology (R_CCOST) that is incurred in period t.

Purpose and Occurrence: This variable represents the portion of the cumulative cost of investment in capacity for an ETL technology that is incurred in period t, and so is subject to the same discounting in the objective function (MR_PRICE) that applies to other period t investment costs.

This variable is calculated as the difference between the cumulative costs of investment in capacity for period t and t-1, and is generated for each ETL technology in all time periods beginning from the period that the technology is first available (START). Apart from its appearance in the objective function, this variable appears in the constraints MR_IC1 and MR_IC2 that define it in the first period that the technology is available, and in subsequent periods, respectively. It also appears in the salvage of investments for learning technologies constraint (MR_SV) which calculates the amount to be credited back to the objective function for learning capacity remaining past the modeling horizon (R_SV_INV).

Units: Million 2000 US$, or any other unit in which costs are tracked.

Bounds: None.

2.8.2.5 R_LAMBD(r,t,p,k)

Description: *Continuous* variable used for an ETL technology to represent the portion of cumulative investment in capacity (R_CCAP) that lies in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve.

Purpose and Occurrence: A positive value for this variable for interval k, and zero values for intervals ≠ k, imply that the cumulative investment in capacity (R_CCAP) lies in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve. This continuous variable, along with the associated binary variable R_DELTA, are generated for each ETL technology in all time periods beginning from the period that the technology is first available (START), and for each interval in the piecewise linear approximation.

Since this variable represents the portion of the cumulative investment in capacity (R_CCAP) that lies in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve, the value of R_LAMBD – if positive – is required to be between CCAPK(k-1,p) and CCAP(k,p), where the internal model parameters CCAPK are the break points on the horizontal axis in the piecewise linear approximation of the cumulative cost curve. A zero value for R_LAMBD is also allowed. These requirements on the value of R_LAMBD are imposed via the pair of constraints MR_LA1 and MR_LA2, in which the value for R_LAMBD is subject to lower and upper bounds of CCAPK(k-1,p) * R_DELTA and CCAP(k,p) * R_DELTA respectively, where R_DELTA = R_DELTA(r,t,p,k) is the binary variable associated with R_LAMBD = R_LAMBD(r,t,p,k).

This variable also appears in the cumulative capacity interpolation constraint (MR_CC), and the cumulative cost interpolation constraint (MR_COS).

Units: PJ/a, Gw, or Bvkm/a, or any other unit defined by the analyst to represent technology capacity.

Bounds: The pair of constraints MR_LA1 and MR_LA2 that are discussed above have the effect of either bounding R_LAMBD between CCAPK(k-1,p) and CCAP(k,p), or forcing R_LAMBD to be zero.

2.8.2.6 R_SV_INV(r,t,p)

Description: The discounted salvage cost of learning investments for an ETL technology.

Purpose and Occurrence: This variable credits back to the regional objective function (MR_PRICE) the discounted salvage cost of learning technology investments that remain available past the end of the modeling horizon.

The variable is controlled by the MR_SV constraint which is generated for each learning technology for all periods in which some portion of the investment will remain available past the end of the modeling horizon.

Units: Million 2000 US$, or any other unit in which costs are tracked.

Bounds: None.

2.8.3 Equations

The equations that are used to model the Endogenous Technological Learning option in MARKAL are presented in Table 2-25 below. Since the primary role of the variables and equations used to model ETL is to control the standard MARKAL investment variable (R_INV) and the associated dynamic cost of these investments, ETL is rather self-contained. That is the R_INV variable links the ETL decisions to the rest of the model, and the R_IC investment cost variable determines the associated contribution to the regional objective function (MR_PRICE). Note that the special clustered learning ETL option involves one additional equation (MR_CLU, EQRef - ETLE.13), as compared with the modeling of endogenous technology learning where there are no clusters.

> Reminder: the ETL formulation is activated at run time from the data handling systems (ANSWER and VEDA-FE) which in turn set the $SET ETL 'YES' switch.

Table 2-25. ETL-specific model constraints

EQRef	Constraints (Indexes)	Constraint Description	GAMS Ref
ETLE.1	MR_CC (r,t,p)	The Cumulative Capacity Interpolation constraint for an ETL technology. This constraint defines the cumulative investment in capacity for a technology (R_CCAP) in a period as the sum over all intervals k of the continuous variables R_LAMBD(r,t,p,k) that represent cumulative investment in capacity as lying in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve.	MMEQTEG.ML

EQRef	Constraints (Indexes)	Constraint Description	GAMS Ref
ETLE.2	MR_CLU (r,t,p)	Constraint that is generated only for the special clustered learning ETL option (CLUSTER). For a key learning ETL technology it defines investment in new capacity (R_INV) as the weighted sum of investments in new capacity of the associated clustered technologies.	MMEQUAC.ML
ETLE.3	MR_COS (r,t,p)	The Cumulative Cost Interpolation constraint for an ETL technology. This constraint defines the interpolated cumulative cost of investment in capacity for a technology (R_CCOST) in a period in terms of the binary variables R_DELTA and the continuous variables R_LAMBD, and the internal model parameters ALPH and BETA.	MMEQTEG.ML
ETLE.4	MR_CUINV (r,t,p)	The Cumulative Capacity Definition constraint for an ETL technology. Defines the cumulative investment in capacity for a technology in a period as the initial cumulative capacity (ETL-CUMCAP0) plus the sum of investments in new capacity (R_INV) made up to and including this period.	MMEQTEG.ML
ETLE.5	MR_DEL (r,t,p)	The constraint for an ETL technology that ensures that in each period, there is exactly one interval k for which the binary variable R_DELTA(r,t,p,k) has value 1 (with zero values for intervals ≠ k).	MMEQTEG.ML
ETLE.6	MR_EXPE1 (r,t,p,k)	One of two constraints for an ETL technology to improve MIP solution time by reducing the domain of feasibility of the binary variables R_DELTA.	MMEQTEG.ML
ETLE.7	MR_EXPE2 (r,t,p,k)	Second of two constraints for an ETL technology to improve MIP solution time by reducing the domain of feasibility of the binary variables R_DELTA.	MMEQTEG.ML
ETLE.8	MR_IC1 (r,t,p)	The constraint for an ETL technology that defines the portion of the cumulative cost of investment in capacity (R_IC) that is incurred in period t, where t = START period for the technology.	MMEQTEG.ML
ETLE.9	MR_IC2 (r,t,p)	The constraint for an ETL technology that defines the portion of the cumulative cost of investment in capacity (R_IC) that is incurred in each period t where t > START period for the technology.	MMEQTEG.ML
ETLE.10	MR_LA1 (r,t,p,k)	The constraint for an ETL technology that sets a lower bound on the continuous variable R_LAMBD(r,t,p,k).	MMEQTEG.ML
ETLE.11	MR_LA2 (r,t,p,k)	The constraint for an ETL technology that sets an upper bound on the continuous variable R_LAMBD(r,t,p,k).	MMEQTEG.ML
ETLE.12	MR_SV (r,t,p)	The constraint for an ETL technology that, in periods appropriately close to the model horizon, defines the salvage cost of learning investment variable (R_SV_INV) in terms of the investment cost in said period (R_IC).	MMEQTEG.ML
ETLE.13	MR_SV2 (r,t,p)	The constraint for an ETL technology that, in periods appropriately far from the model horizon, defines the salvage cost of learning investment variable (R_SV_INV) to be zero.	MMEQTEG.ML

EQRef	Constraints (Indexes)	Constraint Description	GAMS Ref
EQ#27	MR_PRICE (r)	The endogenously calculated cost of investments for learning technologies (R_IC) needs to be discounted and included in the regional objective function (MR_PRICE) in place of the traditional investment calculation using variable R_INV.	MMEQPRIC.ML

2.8.3.1 MR_CC(r,t,p)

Description: The Cumulative Capacity Interpolation constraint for an ETL technology.

Purpose and Occurrence: This constraint defines the cumulative investment in capacity for a technology in a period (R_CCAP) as the sum over all intervals k of the *continuous* variables R_LAMBD(r,t,p,k) that represent cumulative investment in capacity as lying in the k^{th} interval of the piecewise linear approximation of the cumulative cost curve. This constraint links the cumulative capacity investment variable (R_CCAP) to the variables R_LAMBD. In combination with other ETL constraints, it is fundamental to ensuring the validity of the piecewise linear approximation of the cumulative cost curve.

This equation is generated in each time period beginning from the START period for each ETL technology.

Units: Technology capacity units.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:
Primal: The level of this constraint must be zero in a feasible solution.
Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.1}: MR\_CC_{r,t,p}\ \forall p \in teg$$

Cumulative investment in capacity in the current period.

$$R\_CCAP_{r,t,p,k}$$

$$\{=\}$$

Sum over all intervals k (in the piecewise linear approximation of the cumulative cost curve) of the *continuous* variables R_LAMBD in the current period t.

$$\sum_{k} R\_LAMBD_{r,t,p,k}$$

2.8.3.2 MR_CLU(r,t,p)

Description: For a key learning ETL technology it defines investment in new capacity (R_INV) as the weighted sum of investments in new capacity of the attached clustered technologies. The weights used are the numeric values of the CLUSTER parameter.

Purpose and Occurrence: Defines the relationship between investment in new capacity for a key learning ETL technology and investment in new capacity for the associated clustered technologies.

This equation is generated in each time period beginning from the START period for each ETL technology that is a key learning technology, that is, that has associated clustered technologies.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:
Primal: The level of this constraint must be zero in a feasible solution.
Dual variable: The dual variable (DVR_CLU) of this constraint in the MIP solution is of little interest.

Remarks: Activation of the special clustered learning ETL option occurs automatically if data is included for the ETL-CLUSTER parameter.

GAMS Routine: MMEQUAC.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.2}: MR\_CLU_{r,t,p} \ \forall \left[\begin{array}{l} (p \in teg) \wedge (NTCHTEG_{r,p} > 0) \wedge \\ (START_{r,p} \geq t) \end{array} \right]$$

Investment in new capacity (for key learning technology $p \in teg$) in period t.

$$R\_INV_{r,t,p}$$

$$\{=\}$$

The weighted sum of the investments in new capacity in period t of the clustered technologies p' attached to the key learning technology p ∈ teg, and whose START period is less than or equal to t. The weights used are the numeric values of the CLUSTER parameter.

$$\sum_{\substack{p'\$\left(CLUSTER_{r,p,p'}>0\right)\wedge \\ START_{r,p'}\leq t}} (CLUSTER_{r,p,p'} * R\_INV_{r,t,p'})$$

2.8.3.3 MR_COS(r,t,p)

Description: The Cumulative Cost Interpolation constraint for an ETL technology.

Purpose and Occurrence: This constraint defines the interpolated cumulative cost of investment in capacity for a technology in a period (R_CCOST) in terms of the binary variables R_DELTA and the continuous variables R_LAMBD, and the internal model parameters ALPH and BETA, where ALPH and BETA represent the intercepts on the vertical axis and the slopes, respectively, of the line segments in the piecewise linear approximation of the cumulative cost curve. For a more precise definition, see "Mathematical Description" below. In combination with other ETL constraints, it is fundamental to ensuring the validity of the piecewise linear approximation of the cumulative cost curve.

This equation is generated in each time period beginning from the START period for each ETL technology.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9..

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\#ETLE.3:} \; MR\_COS_{r,t,p} \; \forall p \in teg$$

Interpolated cumulative cost of investment in capacity in the current period.

$$R\_CCOST_{r,t,p}$$

$$\{=\}$$

Sum over all intervals k (in the piecewise linear approximation of the cumulative cost curve) of ALPH times the *binary* variable R_DELTA plus BETA times the *continuous* variable R_LAMBD, for the current period t, where ALPH and BETA represent the intercepts on the vertical axis and the slopes, respectively, of the k^{th} interval.

$$\sum_{k} (ALPH_{k,p} * R\_DELTA_{r,t,p,k} + BETA_{k,p} * R\_LAMBD_{r,t,p,k})$$

2.8.3.4 MR_CUINV(r,t,p)

Description: The Cumulative Capacity Definition constraint for an ETL technology.

Purpose and Occurrence: This constraint defines the cumulative investment in capacity for a technology in a period (R_CCAP) as the initial cumulative capacity (CCAP0) plus the sum of investments in new capacity made up to and including this period.

This equation is generated in each time period beginning from the START period for each ETL technology.

Units: Technology capacity units.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:
Primal: The level of this constraint must be zero in a feasible solution.
Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\#ETLE.4:}\ MR\_CUINV_{r,t,p}\ \forall p \in teg$$

Cumulative investment in capacity in the current period.

$$R\_CCAP_{r,t,p}$$

$$\{=\}$$

Cumulative investment in capacity at the start of the learning process.

$$CCAP0_{r,p} +$$

Sum of the investments made since the technology is first available.

$$\sum_{u=START} R\_INV_{r,u,p}$$

2.8.3.5 MR_DEL(r,t,p)

Description: The constraint for an ETL technology that ensures that in each time period, there is exactly one interval k for which the *binary* variable R_DELTA(r,t,p,k) has value 1 (with zero values for intervals ≠ k).

Purpose and Occurrence: To ensure that only one of the *binary* variable R_DELTA(r,t,p,k) has value 1 for each technology. This constraint, in combination with other ETL constraints, is fundamental to ensuring the validity of the piecewise linear approximation of the cumulative cost curve.

This equation is generated in each time period beginning from the START period for each ETL technology.

Units: None.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:
Primal: The level of this constraint must be 1 in a feasible solution.
Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.5}: MR\_DEL_{r,t,p} \ \forall p \in teg$$

Sum over all intervals k (in the piecewise linear approximation of the cumulative cost curve) of the *binary* variables R_DELTA in the current period t.

$$\sum_{k} R\_DELTA_{r,t,p,k}$$

$$\{=\}$$

2.8.3.6 MR_EXPE1(r,t,p,k)

Description: One of two constraints for an ETL technology to improve MIP solution time by reducing the domain of feasibility of the binary variables R_DELTA.

Purpose and Occurrence: To improve MIP solution time this constraint takes advantage of the observation that cumulative investment is increasing with time, thus ensuring that if the cumulative investment in period t lies in segment k, then it will not lie in segments k-1, k-2, …, 1 in period t+1.

This equation is generated for each ETL technology in each time period beginning from the START period and excluding the final period (TLAST), and for each interval k in the piecewise linear approximation of the cumulative cost curve.

Units: None.

Type: *Binding.* The equation is a greater than or equal to (≥) constraint.

Interpretation of the results:

Primal: The level of this constraint must be greater than or equal to zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.6}: MR\_EXPE1_{r,t,p,k} \; \forall (p \in teg) \wedge (t < TLAST)$$

Sum over intervals j ≤ k of binary variables R_DELTA(r,t,p,j), for the k^{th} interval, in period t.

$$\sum_{j \le k} (R\_DELTA_{r,t,p,j})$$

$$\{\ge\}$$

Sum over intervals j ≤ k of binary variables R_DELTA(r,t,p,j), for the k^{th} interval, in period t+1.

$$\sum_{j \le k} (R\_DELTA_{r,t+1,p,j})$$

2.8.3.7 MR_EXPE2(r,t,p,k)

Description: Second of two constraints for an ETL technology to improve MIP solution time by reducing the domain of feasibility of the binary variables R_DELTA. Both constraints rely on the observation that cumulative investment is increasing as time goes on.

Purpose and Occurrence: To improve MIP solution times this constraint is derived from the observation that if cumulative investment in period t lies in segment k, then it must lie in segment k or k+1 or k+2 etc … in period t+1.

This equation is generated for each ETL technology in each time period beginning from the START period and excluding the final period (TLAST), and for each interval k in the piecewise linear approximation of the cumulative cost curve.

Units: None.

Type: *Binding.* The equation is a less than or equal to (≤) constraint.

Interpretation of the results:

Primal: The level of this constraint must be less than or equal to zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.7}: MR\_EXPE2_{r,t,p,k} \; \forall (p \in teg) \wedge (t < TLAST)$$

Sum over intervals j ≥ k of binary variables R_DELTA(r,t,p,j), for the k^{th} interval, in period t.

$$\sum_{j \ge k} (R\_DELTA_{r,t,p,j})$$

$$\{\le\}$$

Sum over intervals j ≥ k of binary variables R_DELTA(r,t,p,j), for the k^{th} interval, in period t+1.

$$\sum_{j \ge k} (R\_DELTA_{r,t+1,p,j})$$

2.8.3.8 MR_IC1(r,t,p)

Description: The constraint for an ETL technology that defines the portion of the cumulative cost of investment in capacity (R_IC) that is incurred in period t, where t = START period for the technology.

Purpose and Occurrence: To determine the variable R_IC which represents the current investment cost incurred in the first period a learning technology is available according to the cumulative investments made in that period. where R_IC then enters the regional objective function (MR_PRICE) subject to the same discounting that applies to other period t investment costs.

This equation is generated in the START period for each ETL technology.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.8}: MR\_IC1_{r,t,p} \ \forall (p \in teg) \wedge (t = START_{r,p})$$

The portion of the cumulative cost of investment in capacity that is incurred in period t, in this case the first period the technology is available.

$$R\_IC_{r,t,p}$$

$$\{=\}$$

The cumulative cost of investment in new capacity in the first period t (t = START).

$$R\_CCOST_{r,t,p} -$$

The initial cumulative cost of investment in new capacity for a learning technology.

$$CCOST0$$

2.8.3.9 MR_IC2(r,t,p)

Description: The constraint for an ETL technology that defines the portion of the cumulative cost of investment in capacity that is incurred in each period t where t > START period for the technology.

Purpose and Occurrence: To determine the variable R_IC which represents the current investment cost incurred in period t according to the cumulative investments made thus far, where R_IC then enters the regional objective function (MR_PRICE) subject to the same discounting that applies to other period t investment costs.

This equation is generated in each time period greater than the START period for each ETL technology.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$EQ\#\ ETLE.9: MR\_IC2_{r,t,p}\ \forall (p \in teg) \wedge (t > START_{r,p})$$

The portion of the cumulative cost of investment in capacity that is incurred in period t.

$$R\_IC_{r,t,p}$$

$$\{=\}$$

The cumulative cost of investment in new capacity as of period t.

$$R\_CCOST_{r,t,p} -$$

The cumulative cost of investment in new capacity as of the previous period t-1.

$$R\_CCOST_{r,t-1,p}$$

2.8.3.10 MR_LA1(r,t,p,k)

Description: The constraint for an ETL technology that sets a lower bound on the continuous variable R_LAMBD(r,t,p,k).

Purpose and Occurrence: To set the lower bound for R_LAMBD(r,t,p,k) to CCAPK(r,k-1,p) * R_DELTA, where CCAPK(r,k-1,p) is the left hand end of the k^{th} interval and R_DELTA = R_DELTA(r,t,p,k) is the binary variable associated with R_LAMBD(r,t,p,k). If binary variable R_DELTA = 1, the effect is to set a lower bound on variable R_LAMBD(r,t,p,k) of CCAPK(r,k-1,p), whereas if R_DELTA = 0 the effect is to set a lower bound of 0.This constraint, in combination with other ETL constraints, is fundamental to ensuring the validity of the piecewise linear approximation of the cumulative cost curve.

This equation is generated in each time period beginning from the START period for each ETL technology, and for each interval k in the piecewise linear approximation of the cumulative cost curve.

Units: Technology capacity units.

Type: *Binding.* The equation is a greater than or equal to (≥) constraint.

Interpretation of the results:

Primal: The level of this constraint must be greater than or equal to zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.10}: MR\_LA1_{r,t,p,k} \; \forall p \in teg$$

Portion of the cumulative investment in capacity that lies in the k[th] interval (of the piecewise linear approximation of the cumulative cost curve), in the current period.

$$R\_LAMBD_{r,t,p,k}$$

$$\{\geq\}$$

Left hand end of the k[th] interval (CCAPK(r,k-1,p)) times binary variable R_DELTA(r,t,p,k), in the current period.

$$CCAPK_{r,k-1,p} * R\_DELTA_{r,t,p,k}$$

2.8.3.11 MR_LA2(r,t,p,k)

Description: The constraint for an ETL technology that sets an upper bound on the continuous variable R_LAMBD(r,t,p,k).

Purpose and Occurrence: To set the upper bound of R_LAMBD(r,t,p,k) to CCAPK(r,k,p) * R_DELTA, where CCAPK(r,k,p) is the right hand end of the k[th] interval and R_DELTA = R_DELTA(r,t,p,k) is the binary variable associated with R_LAMBD(r,t,p,k). If binary variable R_DELTA = 1, the effect is to set an upper bound on variable R_LAMBD(r,t,p,k) of CCAPK(r,k,p), whereas if R_DELTA = 0 the effect is to set an upper bound of 0.This constraint, in combination with other ETL constraints, is fundamental to ensuring the validity of the piecewise linear approximation of the cumulative cost curve.

This equation is generated in each time period beginning from the START period for each ETL technology, and for each interval k in the piecewise linear approximation of the cumulative cost curve.

Units: Technology capacity units.

Type: *Binding.* The equation is a less than or equal to (≤) constraint.

Interpretation of the results:
Primal: The level of this constraint must be less than or equal to zero in a feasible solution.
Dual variable: The dual variable (DVR_LA2) of this constraint in the MIP solution is of little interest.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.11}: MR\_LA2_{r,t,p,k} \ \forall p \in teg$$

Portion of the cumulative investment in capacity that lies in the k^th interval (of the piecewise linear approximation of the cumulative cost curve), in the current period.

$$R\_LAMBD_{r,t,p,k}$$

$$\{\leq\}$$

Right hand end of the k^th interval (CCAPK(r,k,p)) times binary variable R_DELTA(r,t,p,k), in the current period.

$$CCAPK_{r,k,p} * R\_DELTA_{r,t,p,k}$$

2.8.3.12 MR_SV(r,t,p)

Description: The constraint for an ETL technology that, in periods appropriately close to the model horizon, defines the salvage cost of learning investment variable (R_SV_INV) in terms of the variable R_IC. Note that for a technology with lifetime LIFE, investment in new capacity in period t will be available until (t + LIFE – NYRSPER), and hence will still be available at the end of the model horizon (TLAST) if t satisfies:

t + LIFE – NYRSPER > TLAST.

Purpose and Definition of the variable R_SV_INV facilitates taking account of the salvage cost of

Occurrence: learning investments for ETL technologies in the objective function (MR_PRICE).

This equation is generated for each ETL technology in each time period t that satisfies the above condition on t.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks:

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\#ETLE.12:} MR\_SV_{r,t,p} \ \forall(p \in teg) \wedge \begin{pmatrix} t + LIFE - NYRSPER > \\ TLAST \end{pmatrix}$$

Salvage costs of learning investments in period t whose capacity remains active after the modelling horizon.

$$R\_SV\_INV_{r,t,p}$$

$$\{=\}$$

Portion of cumulative investment cost that is incurred in period t (R_IC(r,t,p)) which remains active after the end of the modeling horizon and must be credited back to the objective function (MR_PRICE). The SAL_ETLINVr,t,p multiplier is not actually a parameter, but the expression

$$R\_IC_{r,t,p} * (SAL\_ELTINV_{r,t,p})$$

Where

$$SAL\_ELTINV_{r,t,p} =$$

$$\left[1 - \left(\begin{pmatrix} 1 + DISCOUNT\$not\ DISCRATE_p + \\ DISCRATE_p \end{pmatrix} ** \\ \left(-NYRSPER * \left(ord(t) + LIFE_p - card(t) - 1 \right) \right) \right) \right] /$$

$$\left[\begin{pmatrix} 1 + DISCOUNT\$not\ DISCRATE_p + \\ DISCRATE_p \end{pmatrix} ** \\ \left(NYRSPER * \left(card(t) + 1 - ord(t) \right) \right) \right] /$$

$$\left[1 - \left(\begin{pmatrix} 1 + DISCOUNT\$not\ DISCRATE_p + \\ DISCRATE_p \end{pmatrix} ** \\ \left(-NYRSPER * LIFE_p \right) \right) \right]$$

2.8.3.13 MR_SV2(r,t,p)

Description: The constraint for an ETL technology that, in periods appropriately far from the model horizon, defines the salvage cost of learning investment variable (R_SV_INV) to be zero. Note that for a technology with lifetime LIFE, investment in new capacity in period t will be available until (t + LIFE – NYRSPER), and hence will no longer be available at the end of the model horizon (TLAST) if t satisfies:

t + LIFE – NYRSPER ≤ TLAST.

Purpose and Occurrence: Definition of the variable R_SV_INV facilitates taking account of the salvage cost of learning investments for ETL technologies in the objective function (MR_PRICE).

This equation is generated for each ETL technology in each time period t that satisfies the above condition on t.

Units: Money units, e.g., million 2000 US$, or any other unit in which costs are tracked.

Type: *Binding.* The equation is an equality (=) constraint.

Interpretation of the results:

Primal: The level of this constraint must be zero in a feasible solution.

Dual variable: The dual variables of mixed integer problems have limited usefulness, as discussed in section 1.9.

Remarks: The analyst controls the activation of the ETL formulation at run time by means of the $SET ETL ‘YES’ switch and the inclusion of the required ETL data.

GAMS Routine: MMEQTEG.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# ETLE.13}: MR\_SV2_{r,t,p} \ \forall (p \in teg) \wedge \begin{pmatrix} t + LIFE - NYRSPER \leq \\ TLAST \end{pmatrix}$$

Salvage costs of learning investments in period t for which the capacity will no longer be available after the end of the modelling horizon.

$$R\_SV\_INV_{r,t,p}$$

$$\{=\}$$

2.8.3.14 MR_PRICE(r)
- see EQ# 27 in section 2.5 for a general description without ETL

Description: Regional objective function adjusted to include the endogenously determined investment cost (R_IC) for new investments in learning technologies.

Purpose and Occurrence: The objective function is changed (for learning technologies only) by replacing the traditional calculation of discounted cost of investments in new capacity with that of the endogenously determined value. This equation is generated for each region where the learning investment costs occur in each time period beginning from the START period for each ETL technology.

GAMS Routine: MMEQPRIC.ML

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 27}: MR\_PRICE_{r}$$

All the basic objective function terms.

• • •

The calculated investments costs associated with the ETL technologies, adjusted for any salvage.

$$+ \sum_{t,p \in teg} \left[PRI\_DISC_{r,t,p} * \left(R\_IC_{r,t,p} - R\_SV\_INV_{r,t,p} \right) \right]$$

2.9 Modeling of Uncertainty using Stochastic Programming

As discussed in section 1.11 MARKAL has a facility to permit the modeling of uncertainty with respect to assumptions related to future events (e.g., emission limits, demand levels reflecting economic activity) and technology assumptions (e.g., investment cost of new technologies).

In order to model stochastic optimization in MARKAL the standard model formulation (equation and variables) is augmented with an additional index, SOW corresponding to the State of the World. A SOW is established for each future state for which a probability (PROB) is provided by the analyst. The sum of these probabilities must equal 1. Some of the input data differ for different SOW's. Besides the data describing the values to be taken by the model components for each SOW, the only other data required is the *time of resolution* of the uncertainty (START_STG2). The resolution period determines the point in time when the event tree (see section 1.11, Figure 1-12) fans out, and the number of SOW members determines how many branches there are in the event tree. Note that as currently implemented in standard MARKAL only 2-stage stochastic programming is allowed, that is there is only one resolution time (another, not generally available formulation of MARKAL supports general multi-stage stochastic programming where more than one resolution date is permitted, see section 1.11).

A single PROB is provided by the analyst for each SOW, and this probability is applied to all input data.

The stochastic model constructed on the SOW and PROB data is a single linear program (LP) that can be substantially larger that the basic non-stochastic model it is based on. This is because while for the periods before the resolution period (the so called hedging periods) there is only a single set of model equations and variables, from the resolution period onwards there are equations and variables for each SOW.

Note that the stochastic formulation of MARKAL may not be used in conjunction with the multi-region or MACRO/MICRO versions of the model.

Note that the stochastic version of MARKAL may NOT be used in conjunction with the MARKAL LAG/LED energy and material flow options, owing to the inter-temporal nature of these options and the complications involved in handling them.

2.9.1 Sets, Switches and Parameters

As with all other aspects of MARKAL the user describes the stochastic nature of the energy system by means of a Set and the Parameters described in this section (there are no switches employed in the stochastics formulation). Table 2-26 and Table 2-27 below describe the User Input Parameters, and the Matrix Coefficient and Internal Model Sets and Parameters, respectively, that are associated with the stochastic programming option. The core parameters that represent the stochastic data are closely related to their standard MARKAL counterparts. So in the Related Parameters section of the table below, these (stochastic) input parameters are provided; and the user is referred to Table 2-6 for full details of the non-stochastic equivalent parameter.

Table 2-26. Definition of stochastic user input data

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
LAMBDA			• Scalar • [≥=0]; default 0.	•	• Risk aversion indicator.
PROB (sow)		*SOW*	• Fraction • [0-1]; no default	• Provided for each state-of-the-world to be monitored • The sum of all the PROB entries MUST = 1.	The probability to be assigned to values provided for each SOW . • The total system cost for each SOW path is multiplied by PROB in the overall stochastic objective function (MS_Exp_Cst).
S_ARAF *(p,sow,t)*	*S_T_ARAF*	ARAF	• Fraction. • [0-1]; no default.	• Provided if a hydro plant has an annual limit on the reservoir for each SOW.	Annual reservoir availability for each SOW. • See ARAF entry in Table 2-6 for details.
S_BOUND(BD) *(p,bd,sow,t)*	*S_TCHBOUND*	BOUND(BD) TCH_BND	• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Provided if capacity of a technology is to be explicitly limited for each SOW.	Limit on capacity of a technology in a period for each SOW. • See BOUND(BD) entry in Table 2-6 for details.
S_BOUND(BD)O *(p,bd,sow,t)*	*S_TCH_BNDO*	BOUND(BD)O TCH_BNDO	• Units of activity (e.g., GWh, PJ). • [open]; no default.	• Provided if activity of a technology is to be explicitly limited for each SOW.	Limit on total annual activity of a technology in a period. for each SOW • See BOUND(BD)O entry in Table 2-6 for details.
S_BOUND(BD)Or *(s,bd,sow,t)*	*S_SEPBOUND*	BOUND(BD)Or SEP_BND	• Units of activity (e.g., GWh, PJ). • [open]; no	• Provided if activity of a resource supply	Limit on total activity of a resource supply option in a period for each SOW. • See BOUND(BD)Or entry in Table 2-6 for details.

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
			default.	option is to be explicitly limited for each SOW.	
S_COST *(s,sow,t)*	*S_SEPCOST*	COST	• Base year monetary units per commodity unit (e.g., 2000M$/PJ). • [open]; no default.	• Provided whenever a cost is incurred to supply a resource for each SOW.	Annual cost of a resource supply option for each SOW. • See COST entry in Table 2-6 for details.
S_DEMAND *(d,sow,t)*	*S_DEM*	DEMAND DM_DEM	• Units of end-use demand. • [open]; no default.	• Required for each end-use demand that is to be handled stochastically.	Annual demand for an energy service for each SOW. • See DEMAND entry in Table 2-6 for details.
S_ENV_MAXEM *(v,sow,t)* S_ENV_BND(BD) *(v,sow,bd,t)*	*S_EM_BND* *S_EM_FIX*	EM_BOUND EM_FIX	• Tons or thousand tons. • [open]; no default.	• When there is to be an annual limit imposed on emissions for each SOW.	Upper or fixed limit on emissions in a period for each SOW. • See ENV_BOUND(BD) entry in Table 2-6 for details.
S_ENV_CUM *(v,sow)*		ENV_CUM	• Tons or thousand tons. • [open]; no default.	• When there is to be a cumulative limit imposed on emissions for each SOW.	Upper cumulative limit on emissions over the entire modeling horizon for each SOW. • See ENV_CUMMAX entry in Table 2-6 for details.
S_IBOND(BD) *(p,bd,sow,t)*	*S_TCH_IBND*	IBOND(BD) TCH_IBND	• Units of capacity (e.g., GW, PJa). • [open]; no default.	• Provided if new investment in a technology is to be explicitly limited for each SOW.	Limit on new investment in a technology in a period for each SOW. • See IBOND(BD) entry in Table 2-6 for details.
S_SRAF(Z)	*S_T_SRAF*	SRAF(Z)	• Fraction.	• Provided if a	Seasonal reservoir availability for each SOW.

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
(p,z,sow,t)			• [0-1]; no default.	hydro plant has a seasonal limit on the reservoir for each SOW.	• See SRAF(Z) entry in Table 2-6 for details.
SOW[83]	ALLSOW	*PROB* REALSOW SON	• Set • no default.	• Derived from the entries in PROB as the list of states-of-the-world. • Members must be named SW1-SW9. If more that nine SOW are required then the set ALLSOW must be provided explicitly.	This set corresponds to the entries in the probability table (PROB) thereby establishing the list of set members that control how many different states are to be handled, what stochastic data instances can be provided, and how many instances of the stochastic equations and variables are to be handled.
START_STG2			• Year. • no default.	• Provided to indicate when stochastic phase of the model begins.	The period (year) corresponding to the resolution of uncertainty, and thus the last period of the hedging phase and the point from which the event tree fans out for each of the SOW.

[83] SOW is actually not provided explicitly as input data, though the various values for the states-of-the-world must be established by the user in the data handling system as part of establishing the PROB table and related stochastic data.

Table 2-27. Stochastic option-specific matrix coefficients and internal model parameters

Matrix Controls & Coefficients (indexes)	**Type**	**Description & Calculations**
PATH (t,sow,sow)	S	A mapping set that indicates for each period which SOW index to use. For all SOWs this set has SW1 for the first index for all periods prior to the resolution of uncertainty (START_STG2), thereafter for each SOW both indexes are the corresponding SOW. This thus indicates whether equations and variables are to be generated for only SWI or each SOW in a period.
TP_SOW (t,sow)	S	Indicator of periods for each SOW. Only the first SOW, SW1, has an entry for the hedging periods, that is the periods before the resolution of uncertainty (START_STG2). For SWI and all other SOWs there are entries only for the periods after START_STG2.
TPN<tech_grp> (t,sow,p/s)	S	Mapping sets for each technology group (e.g., TCH, CON, DMD, PRC) that indicates from which period a technology or resource option is available (START) to each SOW.
TPNN<tech_grp> (t,sow,sow,p/s)	S	Mapping sets for each technology group (e.g., TCH, CON, DMD, PRC) that indicates for each SOW which SOW applies, that is SW1 during the hedging period or the actual SOW during the event tree periods.

2.9.2 Variables

As noted earlier, the variables that are used to model the stochastic programming version of MARKAL are the same variables that make up the deterministic MARKAL model, with two minor adjustments. The main difference is that the variables require another index corresponding to the state-of-the-world, SOW. To standardize the handling of this index it is always introduced after the period index, thus it is usually the second index (or the first if there is no period index) in the variable. To accommodate this requirement each standard model variable name is adjusted by adding a prefix, S_, to the variable name. So for example the capacity variable, CAP(t,p) becomes S_CAP(t,sow,p). During matrix generation the appropriate SOW index value is then entered into S_CAP according to the set PATH and the period being worked on.

As there is thus essentially no redefinition of the variables for the stochastic formulation, other than the control of the instances of the variable according to the control sets PATH and TPN/TPNN<tech_grp>, the user is referred to section 2.4 for details on the variables of the model. Below in Table 2-28 the variables strictly involved with the stochastic version are listed, however as it is rather straightforward the page of variable details in section 2.4 is not repeated here for the stochastic variables.

Table 2-28. Variables

VAR Ref	Variable (Indexes)	Variable Description
S.1	Exp_ObjZ	The variable equal to the expected sum of the total discounted cost associated with each SOW.
S.2	Multi_Obj	The variable equal to multi-objective function, i.e. the objective function if risk is accounted for. See the constraints section for a definition of the new objective function
S.3	Upside_Dev (sow)	The upside deviation between the total system cost for each SOW, and the expected system cost.

2.9.3 Equations

As noted earlier, and as is the case with the variables, the equations that are used to model stochastics are the same equations that make up the non-stochastic MARKAL model with two minor adjustments. The main difference is that the equations require another index corresponding to the state-of-the-world, SOW. To standardize the handling of this index it is always introduced after the period index, thus it is usually the second index (or the first if there is no period index) in the equation. To accommodate this requirement each standard model equation name is adjusted by adding a prefix, MS_, to the equation name. So for example the capacity transfer equation, EQ_CPT(t,p) becomes MS_CPT(t,sow,p). During matrix generation the appropriate SOW index value is then entered into MS_CPT according to the set PATH and the period being worked on.

As there is thus essentially no redefinition of the equations for the stochastic formulation, other than the objective function (below) and the control over the generation of the appropriate equations and variables according to PATH and TPN/TPNN<tech_grp> control sets mentioned above, the user is referred to section 2.5 for details on the core equations of the model. Below in Table 2-29 the few equations directly related to only stochastic version are listed, and then elaborated in some more detail, first for the standard stochastic objective function and then for the risk aversion formulation.

Table 2-29. New constraints of stochastic MARKAL

EQRef	Constraints (Indexes)	Constraint Description	GAMS Ref
MS.1	MS_Exp_Cst	The expected total system cost, taking into consideration the probability of each event path, to be minimized as the objective function if risk aversion is assumed equal to 0.	MMEQUA.MS
MS.2	MS_UpsideDev (sow)	The Upside deviation between the total system cost for sow and the expected system cost.	MMEQUA.MS
MS.2	MS_MLT_OBJ (sow)	The multi-objective function, i.e. the objective function if risk is accounted for, by adding to the expected cost a risk term obtained by multiplying a risk intensity LAMBDA by the upside standard deviation. The upside standard deviation is equal to the square root of the probability weighted sum of the squared upside deviations for every state of nature,.	MMEQUA.MS

2.9.3.1 MS_Exp_Cst - Expected total discounted system cost, the stochastic objective function.

Description: The expected total system cost over the entire horizon, using the probability of each event.

Purpose and The objective function of the stochastic programming MARKAL when risk aversion is assumed to be 0.
Occurrence: This equation is generated once, as the objective function.

GAMS Routine: MMEQUA.MS

MATHEMATICAL DESCRIPTION

$$\text{EQ\#S.1}: MS\_Exp\_Cst$$

The objective function RHS variable to be minimized, if risk aversion is assumed to be 0.

$$Exp\_ObjZ$$

$$\{=\}$$

The total expected system cost..

$$\sum_{sow}\left(PROB_{sow} * S\_ObjZ_{sow}\right)$$

2.9.3.2 MS_Deviatn(sow) - Determination of the deviation between the total system cost and the expected system cost.

Description: The deviation between the total system cost for the expected system cost is equal to the upside deviation minus the downside deviation.

Purpose and Occurrence: To determine the up/downside deviations.

This equation is generated for each SOW.

GAMS Routine: MMEQUA.MS

MATHEMATICAL DESCRIPTION

$$\text{EQ\# S.2}: \boldsymbol{MS\_UpsideDev(sow)}$$

The upside deviation variable

$$\boldsymbol{Upside\_Dev_{sow}}$$

$$\{=\}$$

The positive part of the difference between total discounted system costs associated with a given path and the expected total discounted cost.

$$\left[\boldsymbol{S\_ObjZ_{sow} - Exp\_ObjZ}\right]^{+}$$

2.9.3.3 MS_MLT_OBJ - The objective function if risk is accounted for.

Description: The stochastic multi-objective function, the objective if risk is accounted for, by attaching a weight lambda to the upside variance.

Purpose and To minimize the total expected dis-utility over the entire horizon, i.e. the expected cost plus the LAMBDA-weighted expected deviation

Occurrence: This equation is generated once, as the objective function.

GAMS Routine: MMEQUA.MS

MATHEMATICAL DESCRIPTION

$$\text{EQ\#S.3}: MS\_MLT\_OBJ$$

The objective function RHS variable to be minimized.

$$Multi\_Obj$$

$$\{=\}$$

The expected total system costs associated with all paths.

$$Exp\_ObjZ +$$

The upside standard deviation is equal to the square root of the squared upside deviations for every state of nature, multiplied according to their probabilities.

$$\boldsymbol{Lambda * sqrt\left(\sum_{sow}\left(PROB_{sow} * (Upside\_Dev_{sow})^2\right)\right)}$$

2.10 The MARKAL-MICRO version of the elastic demand mechanism

As discussed in Section 1.6.3, MARKAL-MICRO is an alternate partial equilibrium formulation of MARKAL that allows endogenous, price sensitive useful energy demand, jointly with the price sensitive supply of energy. It accomplishes this by means of minimizing the total surplus for the entire energy system via non-linear, convex programming.

The standard version of MARKAL includes a similar feature but it uses a step-wise linear approximation of the demand curve that permits computing the equilibrium via Linear Programming. One difference between the two versions is that MARKAL-MICRO accepts cross-price elasticities, whereas Standard MARKAL accepts only own-price elasticities. Cross-price elasticities allow inter-demand substitutions, as for example between passenger car transport and mass transit.

The easiest way to specify demand functions for MARKAL-MICRO is to calibrate them to the reference exogenous demands of MARKAL. A manual procedure has been implemented[84]. To calibrate the demand function without substitution, the inputs for the calibration procedure are:

1) the exogenous demand for the different demand categories and their associated shadow prices from a reference MARKAL run, and

2) a price elasticity for each demand category.

The procedure, written in the SHFTDD.GMS file, computes the different parameters for the demand functions, i.e. the constant parameter B and the shift-parameter γ, assuming a demand function: $\log \boldsymbol{Q_t} = \boldsymbol{B} + \gamma_t - \varepsilon \log \boldsymbol{P_t}$. These parameters are then used in the MARKAL-MICRO run.

When substitution possibilities are to be modeled, a Constant Elasticity of Substitution function can be implemented. The calibration is again based on the demand and corresponding shadow prices from a MARKAL run, with the substitution elasticity between the demand categories as a given input. It is a two-step procedure:

- 1st step: computation of the share parameters δ_i for the sub-categories and the prices of the aggregate demands through a small simultaneous equation model.
- 2nd step: calibration of the demand function for the aggregate demand categories and for the demand categories without substitution.

Both calibration procedures are written in GAMS, which must be setup and run in a Command Prompt (DOS) window. See the SHFTDD.BAT/CMD[85] command file for more information on running the utilities. The SHFTDD.DD file produced provides the basic inputs for running MICRO which should then be entered into ANSWER or VEDA-FE as the MI-parameters.

Note that owing to the non-linear programming (NLP) nature of the modified objective function, MARKAL-MICRO may not be activated with the other Standard MARKAL features, including multi-region. This is the major reason why the step-wise linearization approach was taken for Standard MARKAL.

[84] Both MACRO and MICRO calibration procedures require running of a utility “manually” in Command Prompt (DOS) window, rather than by means of the ANSWER or VEDA-FE shell.
[85] The BAT version of SHFTDD should be used when working with GAMS20.5 or earlier, the CMD version when using new versions of GAMS.

2.10.1 Sets, Switches and Parameters

Like all other aspects of MARKAL the user describes MARKAL-MICRO by means of a Switch, and the Parameters described in this section. Table 2-30 and Table 2-31 below describe the User Input data, and the Matrix Coefficient and Internal Model Parameters, respectively, that are associated with MARKAL-MICRO.

Table 2-30. Definition of MARKAL-MICRO user data

Input Data (Indexes)	Alias/Internal Name	Related Parameters	Units/Range & Defaults	Instance (Required/Omit/ Special Conditions)	Description
MARKAL-MICRO			• Switch. • [MARKAL-MICRO]; default not run.	• Provided when running MICRO (see next column).	Unlike the more recent additions to Standard MARKAL, MARKAL-MICRO is activated through the data handling system (ANSWER or VEDA-FE) by passing the 'MARKAL-MICRO' switch to the main GAMS routines by the <case>.GEN file[86].
MI-ACOEF *(d)*	*PARDD* *(d,'acoef')*	*CCONS(d)* *ELP(d)*	• Monetary unit/ commodity unit. • [0-1]; no default.	• Provided if a demand is to be subject to price elasticity.	Constant for the demand function. • Generated by the SHFTDD.GMS calibration routine.
MI-ELASP *(d)*	PARDD *(d,'elasp')*	*ELP(d)*	• Fraction. • [0-1]; no default.	• Provided if a demand is to be subject to price elasticity.	Price elasticity of each demand category (DM).
MI-SHFTDD (d,t)	SHFTDD *(d)*	*SHFTPAR(d,t)*	• Number. • [0-any]; no default.	• Provided if a demand is to be subject to price elasticity.	Demand shifting parameter. • Generated by the SHFTDD.GMS calibration routine.
MM-DMTOL	DMTOL	*DMMIN(d)*	• Fraction. • [0-1]; no default.	• Provided to determine the lower bound for MICRO demands.	Fraction by which MICRO demands may be lowered.

[86] As noted above, MARKAL-MICRO may not be activated for a multi-region run, and so like any other single-region MARKAL run is performed in one job step and only requires a <case>.GEN file.

Table 2-31. MARKAL-MICRO matrix coefficients and internal model parameters

Matrix Controls & Coefficients (indexes)	Type	Description & Calculations
CCONS *(d)*	I	Component of the consumer surplus objective coefficient - $$CCONS_d = \exp(MI{-}ACOEF_d / MI{-}ELASP_d) * (1 / ELP_d)$$
DMMIN *(d)*	I	Minimum level to which a demand may be lowered.
ELP *(d)*	I	Component of the consumer surplus objective coefficient - $$ELP_d = [1 - (1 / MI{-}ELASP_d)]$$
SHFTPAR *(d,t)*	I	Component of the consumer surplus objective coefficient - $$SHFTPAR_d = \exp(SHFTDD_d / MI{-}ELASP_d) * NYRSPER * (ord(t) - 1)$$

2.10.2 Variables

MARKAL-MICRO only needs two additional variables to Standard MARKAL, one to link the demands (DDD) into the objective function, and the other the objective function variable to be minimized (PARTOBJZ). Both are presented in Table 2-32 below.

Table 2-32. MARKAL-MICRO variables

VAR Ref	Variable (Indexes)	Variable Description
MI.1	DDD (t,d)	Endogenously determined demand level.
MI.2	PartObjZ	The variable equal to the consumer/produce surplus that is to be minimized.

2.10.3 Equations

Similarly to the situation with the variables, MARKAL-MICRO only needs two morel equations than Standard MARKAL, one to handle the endogenous demands (EQ_DD) that are linked into the objective function, and the other for the alternate objective function (EQ_CSPRICE). Both are presented in the following sections.

2.10.3.1 EQ_CSPRICE

Description: The total consumer/producer surplus to be minimized as the objective function of the model.

Purpose and Occurrence: To minimize total system cost allowing for elastic demands, by doing the same to the consumer/producer surplus.

This equation is generated once, as the objective function.

GAMS Routine: MMEQUA.MI

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 27}: EQ\_CSPRICE$$

The total system cost excluding consumer surplus terms, as calculated for Standard MARKAL.

$$ObjZ -$$

Discounted Consumer Surplus.

$$\sum_{t,d}\left[PRI\_DF_t * \begin{pmatrix} \left(CCONS_d * SHFTPAR_{d,t} * DDD_{t,d} ** ELP_d\right) - \\ \left(CCONS_d * SHFTPAR_{d,t} * DMMIN_d ** ELP_d\right) \end{pmatrix} \right]$$

$$\{=\}$$

RHS variable to be minimized corresponding to the consumer/producer surplus.

PartObjZ

2.10.3.2 EQ_DD

Description: Demand coupling equation that associates the flexible demand variable with the level of output from each of the demand devices.

Purpose and Occurrence: To determine the level of each of the elastic demands. This equation is generated once, as the objective function.

GAMS Routine: MMEQUA.MI

MATHEMATICAL DESCRIPTION

$$\text{EQ\# 27}: E\text{Q\_DD}_{t,d}$$

Output of each demand device that services this demand.

$$\sum_{p\$DMD\_DMS_{p,d}} \left(DEM\_CAP_{t,p,d} * CAP_{t,p}\right)$$

$$\{=\}$$

$$DDD_{t,d}$$

2.11 GAMS Modeling Environment for MARKAL[87]

MARKAL is written in a modular fashion employing the General Algebraic Modeling System (GAMS)[88]. GAMS is a 2-pass compile and execute system that first builds the current code from the individual routines according to "control switches" ($SET environment variables) and parameters (%1, %2, …, %n) passed at invocation time. Once fully resolved this code is then executed. The overall organization of the MARKAL model execution environment is also presented in this section.

2.11.1 MARKAL Source Code[89]

The MARKAL modeling environment is composed of a series of components that consist of:

- ANSWER or VEDA-FE for assembling and managing input data and scenarios;
- "DOS" command scripts that manage system files and invoke the appropriate GAMS routines;
- GAMS command directive files that set model variants and identify input files;
- input data extracted from ANSWER or VEDA-FE (the <scenario>_<region>.DD/DDS files);
- the actual GAMS source code;
- the solver(s), called from the SOLVE.* GAMS routines;
- output files, including the run listing file, the quality control and other log files, and the results files passed to ANSWER or VEDA-BE; and
- ANSWER or VEDA-BE for processing the model results.

Figure 2-10 below, depicts the interaction of the various components of the model execution environment (ANSWER/VEDA-FE and VEDA-BE are not explicitly shown or discussed here: see the appropriate system manuals for each component).

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[87] Owing to major changes in the internals of GAMS beginning with Version 20.6 (released March 25, 2002 as Rev 132) MARKAL has two execution "threads" or paths. The command (CMD) mode is the path that must be used with all GAMS systems 20.6 or higher. The batch (BAT) mode is the path that is backward compatible with the older versions of GAMS. Note that for single-region models, running with either path works with all versions of GAMS. Having said this, some very old GAMS systems may encounter $301 compile errors owing to long source code lines. A user should either update their GAMS license or edit the flagged long lines.

[88] Anthony Brooke, David Kendrick, Alexander Meeraus and Ramesh Raman, GAMS - A User's Guide, December 1998.

[89] The MARKAL model source code is available from the ETSAP Primary Systems Coordinator. Contact Gary Goldstein, Hggoldstein@irgltd.comH, for more information.

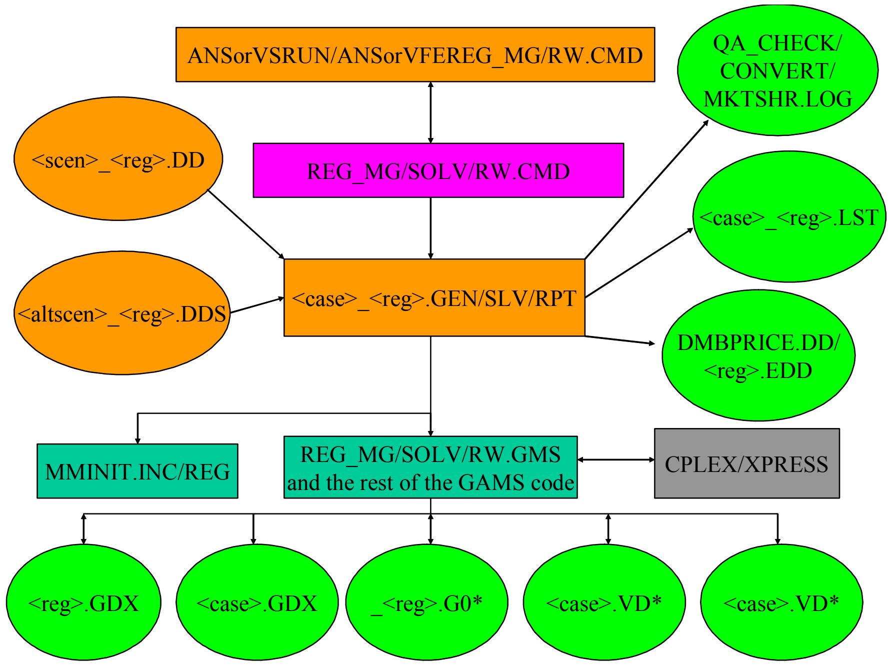

Figure 2-10. MARKAL execution environment[90,91]

Table 2-33. Components of the MARKAL execution environment

Component	Nature of Component	Purpose
*.ANS	ANSWER-specific GAMS Code	GAMS routines that do pre/post-processing for ANSWER.
*.BAT/CMD	Command Scripts	DOS (BAT) and Windows (CMD) command scripts that oversee the running of MARKAL in a Command Prompt window.
*.GMS	General GAMS Code	GAMS routines used for interpolation, output formatting and such.
*.GP	MARKAL-Goal Programming GAMS Code	The modules that are specifically tied to the MARKAL goal programming formulation.
*.INC	MARKAL Core GAMS Code	The modules that make-up the core part of MARKAL, and so are involved in all variants of the model.
*.LIV	MARKAL Lumpy Investment GAMS Code	The modules that are specifically tied to the MARKAL lumpy investment formulation.

[90] The orange boxes are dynamically produced by ANSWER/VEDA-FE at job submission time, the violet boxes are the main "DOS" command script files that oversee the job run, blue boxes are MARKAL GAMS source code, the gray box is the solvers, and the green boxes are output files generated from the system.
[91] In earlier versions of ANSWER a '+' was used as a separator between <scen>/<altscen>/<case> and <reg>.*, but as of ANSWERv5.3.2 the underscore ("_") has been adopted as it is with VEDA-FE.

Component	Nature of Component	Purpose
*.MED	MARKAL Elastic Demand (PE) GAMS Code	The modules that are specifically tied to the MARKAL elastic demand (partial equilibrium) formulation.
*.MEV	MARKAL Environmental Damage GAMS Code	The modules that are specifically tied to the MARKAL environmental damage formulation.
*.MI	MARKAL-MICRO GAMS Code	The modules that are specifically tied to MARKAL-MICRO.
*.ML	MARKAL-Endogenous Technology Learning GAMS Code	The modules that are specifically tied to the MARKAL endogenous technology learning (ETL) formulation.
*.MM	MARKAL-MACRO GAMS Code	The modules that are specifically tied to MARKAL-MACRO.
*.MRK	MARKAL-specific GAMS Code	The modules that are specifically tied to MARKAL only (e.g., model declaration and solve).
*.MS	MARKAL-Stochastics GAMS Code	The modules that are specifically tied to the MARKAL stochastic programming formulation.
*.MTS/MKT	SAGE Time-stepped and Market Share GAMS Code	The modules that control the time-stepped execution of SAGE and the market share and inter-period technology learning algorithms.
*.VFE/VBE	VEDA-specific GAMS Code	GAMS routines that do pre/post-processing for VEDA-FE/BE.
<altscen>_<reg>. DDS	Alternate Data for each Region	Augmenting information to be applied to the reference data generated by ANSWER or VEDA-FE for each region, plus <altscen>_TRADE if appropriate.
<case>_<reg>.GEN <case>.SLV <case>_<reg>.RPT	GAMS Command Directives Files	The GAMS command directive files identify the DD/DDS files associated with each region, set the desired model variant switches (discussed below), and call the main matrix generation, solve and report writer drivers (REG_MG.REG, REG_SOLV.REG and REG_RW.REG). [Note that only <case>.SLV exists, that is there is not one for each region.]
<case>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data and model results from the optimization. Generated during the solve phase and accessed by the report writer.
<case>.VD <case>.VDE <case>.VDS	VEDA Results	The VEDA data, SAGE RES elements and set membership information passed by the report writer for subsequent processing.

Component	Nature of Component	Purpose
<reg>.GDX	GAMS GDX	The GAMS dynamic data exchange file that contains all the data declarations for each region. Generated during matrix generation and accessed during the solve and report writing phases.
<reg>.EDD/ DMBPRICE.DD	Elastic Demand Reference Data	The files (multi/single-region), generated during a reference run, containing the prices of demands and annual/total system costs used to "seed" the flexible demand mechanism employed by MARKAL when run as a partial equalibrium formulation. [There are also associated *.IMP files formatted for direct importing of this information into ANSWER.]
<scen>_<reg>.DD	Reference Data for each Region	The full RES data specification generated by ANSWER or VEDA-FE for each region, plus <scen>_TRADE.DD.
<case>_<reg>.LST	GAMS Output Listing	GAMS writes a execution log (including error message, if any) to the LST file for each region (and TRADE). The <case>_TRADE.LST file also contains the complete solution listing.
_<reg>.G0*	GAMS Save	The GAMS save files contain the model declaration for each region. Generated during the matrix generation and accessed by the report writer.
ANSRUN.CMD or VSRUN.CMD ANSREG_MG.CMD or VFEREG_MG.CMD ANSREG_RW.CMD or VFEREG_RW.CMD	"DOS" Command Script	ANSRUN.CMD or VSRUN.CMD are command scripts generated by ANSWER or VEDA-FE respectively with the job name and pathing information (to the source code) for the current run. ANSREG_MG.CMD and ANSREG_RW.CMD or VFEREG_MG.CMD and VFEREG_RW.CMD are the associated script files with the list of regions for which the matrix generator and report writer are to be run to guide those job steps. [Note that for those running older versions of GAMS, prior to 20.6, *.BAT files replace their *.CMD counterparts when running under ANSWER, though VEDA-FE only generates *.CMD files.]
CPLEX.OPT XPRESS. OPT	Solver options file	The solver options file used to specify options to the solver that is used in the model solve phase.
MMINI*.*	GAMS Initialization Routines	The modules that declare all "empty" GAMS data structures appearing in the model code. This ensures that no execution errors occur for models that do not use certain features in a particular run.
QA_CHECK.LOG CONVERT.LOG MKTSHR.LOG	MARKAL LOG files	During execution two LOG files are written (with a third LOG file written for a SAGE run). QA_CHECK.LOG reports on the correctness and consistency of the underlying data describing each regional energy system. CONVERT.LOG reports on the actions taken to adjust the lifetimes of technologies for which the technical lifetime (LIFE) is not a multiple of the number of years per period (NYRSPER). For a SAGE run, MKTSHR.LOG give a trace of the actions taken between periods to apply the Market Share algorithm.

Component	Nature of Component	Purpose
RMARKAL.CMD or SAGE.CMD REG_MG.CMD or SAGE_MG.CMD REG_SOLV.CMD or SAGE_SOL.CMD REG_RW.CMD or SAGE_RW.CMD	"DOS" Command Script	Main drivers overseeing overall execution and the execution of the matrix generator, solve and report writer steps. Command scripts ANSRUN.CMD or VSRUN.CMD generated by ANSWER or VEDA-FE respectively, see above, call RMARKAL.CMD that oversees overall execution. (In the case of SAGE, it is SAGE.CMD that oversees overall execution.) RMARKAL.CMD in turn calls REG_MG.CMD, REG_SOLV.CMD and REG_RW.CMD to carry out the matrix generation, solve and report writing steps. Other CMD routines are called from these for file management etc as needed. [Also, CURVER.CMD echoes the current version of the source code to the screen during the run.]
REG_MG.CMD or SAGE_MG.CMD REG_SOLV.CMD or SAGE_SOL.CMD REG_RW.CMD or SAGE_RW.CMD	GAMS Main MARKAL Component Drivers	The main matrix generation, solve and report writer drivers called from the <case>_<reg>.GEN, <case>.SLV and <case>_<reg>.RPT directives files. [Note that for those running older versions of GAMS, prior to 20.6, *.BAT files replace their *.CMD counterparts when running under ANSWER, though VEDA-FE only generates *.CMD files.]

As noted previously, MARKAL has been developed based on the GAMS modeling framework. GAMS dynamic substitution for environment variables is employed to fully establish the appropriate executable code. This is possible as GAMS is a two-pass system, first fully resolving the source code (substituting the string associated with the %env_variable% environment variables), and then executing the resulting code.

Below in figure 2-11 is a small example showing a sample GAMS control directives file (generated dynamically by ANSWER or VEDA-FE according to the variant switches needed), followed by both the original GAMS source code and the associated fully resolved executable code.

```
* ----- Identify the "shell" and run name
$TITLE <ANSWER or VEDA-FE>: CASE DUTEST
$SET SHELL '<ANSWER or VEDA-FE>';
* ----- GAMS options. Set LIMROW=x, LIMCOL=y to activate an equation listing
OPTION LIMROW=0, LIMCOL=0, SOLPRINT=ON, SYSOUT=OFF, ITERLIM=2000000;
OPTION RESLIM=50000, PROFILE=0, SOLVEOPT=REPLACE;
* ----- Uncomment if switching from the default CPLEX solver to XPRESS
*OPTION LP=XPRESS;
* ----- Turn off the listing of the data and source code
$OFFLISTING
* ----- Permit multiple instances of declarations
$ONMULTI
* ----- %MODVER% a GAMS environment variable indicating if BAT/CMD execution,  passed on call line.
* ----- It is used to ensure backward compatibility between older and current versions of the model.
* ----- Identify the source code version (note: Title header printed in LST file comes from MMINIT.INC)
$IF NOT '%MODVER%' == 'CMD' $LOG   *** Running MARKAL Standard ***
$IF         '%MODVER%' == 'CMD' $LOG   *** Running MARKAL Version 5+ ***
* ----- Include the files containing "empty" declaration of all sets and parameters used in the code
$IF    '%MODVER%' == 'CMD' $INCLUDE MMINIT.INC
$IF    '%MODVER%' == 'CMD' $INCLUDE MMINIT.REG
$IF    '%MODVER%' == 'CMD' $INCLUDE MMINIT.<ANS or VFE>
$IF NOT '%MODVER%' == 'CMD' $CLEAR G_TRADE TRD_FROM TRD_COST  BI_TRDENT
      BI_TRDCST BI_TRDELC BI_TRDCSTE REG_XMONY REG_XCVT REG_XMACRO

* get the full set declarations from the matrix generation steps for each region, and complete initializations
$INCLUDE REG_FULL.SET
$INCLUDE MMINIANS.REG

* name the case
SET SCENCASE / 'DUTEST'  'BASE scenario run for Demo only to 2020' /;

* include the TRADE .DD and .DDS files
$IF NOT '%MODVER%' == 'CMD' $INCLUDE BASE+TRADE.DD
$IF         '%MODVER%' == 'CMD' $SET TRDFILE $INCLUDE BASE+TRADE.DD

* include appropriate $SETs according to model variant, including activating multi-region and elastic demand
$SET STOCHRPT 'NO'
$SET RUN_MREG 'YES'
$SET MARKALED 'YES'

*  Do the ANSSER adjustments after loading the regional GDX files
$IF NOT '%MODVER%' == 'CMD' $INCLUDE <ANS or VEDA>2GAMS.REG

* activate SAGE Market Share algorithm (INV/INVPCT/ACTPCT(for DMD-only))
*$SET SAGE 'YES'
*$SET MKTSHR 'ACTPCT'

* Solve
* - adjust GMARKAL or MARKAL-MACRO for MARKAL(ED) vs MACRO by ANSWER or VEDA-FE
$BATINCLUDE REG_SOLV.REG GMARKAL SOLVE
```

Figure 2-11. DUTEST.SLV solve command directive file for MARKAL run with elastic demands activated

```
***************************************************************************
*
*   MMEQUA.REG has the actual equation declarations and specifications for a multi-region
*       run. Most of the details regarding the actual value of the coefficients are handled in
*         MMCOEF.INC.
*
***************************************************************************

*==================================================================*
*  Declare the individual equations and provided informative text to be
*   printed in the *.LST files for the user
*==================================================================*
 EQUATIONS

 ...

 MR_DEM(*, *, *)            demand relation (=G=)
 ...

 ;

*==================================================================*
*  Generation of individual equations, passing the multi-region specs                              *
*==================================================================*

 ...

*==================================================================*
*  Useful Energy Demands                                       *
*==================================================================*
$BATINCLUDE MMEQDEM.INC MR '' '_R' 'R_' 'REG,' '(REG)' 'TP' DM

 ...
```

Figure 2-12. MMEQUA.REG equation declaration and call to MMEQDEM.INC for the actual inclusion of the demand equation

```
*=====================================================================*
*  MMEQDEM.INC Demand Equation
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
*    %4 - variable/coef prefix => 'DM' or 'S_' or 'R_'
*    %5 - REGional indicator => '' or '' or 'REG,'
*    %6 - regional scaling => '' or '' or '(REG)'
*    %7 - loop control set => 'TP' or 'TP_SOW(TP,SOW)' or 'TP'
*    %8 - DM or S indicator for table names => 'DM' or 'S' or 'DM'
*=====================================================================*
* only generate if demand provided
%1_DEM(%5%7, DM)$(TS(TP) AND (%8_DEM%3(%5DM, %2TP) GT 0)) .. MMSCALE%3%6 *
(  SUM(DMD$DMD_DMS%3(%5DMD, DM),
          DEM_CAP%3(%5TP, DMD, DM) * %4CAP(%5TP, %2DMD) )

* if requested include elastic demands
$ IF '%MARKALED%' == 'YES' $BATINCLUDE MMEQDEM.MED %1 '%2' '%3' '%4' '%5'

)

  =G=

* Set the RHS to the demand level
  MMSCALE%3%6 * %8_DEM%3(%5DM, %2TP);

************************************************************************
*   MMEQUADM.MED has the EQ_DEM part of the Elastic Demands
*    %1 - equation name prefix 'EQ' or 'MS' or 'MR'
*    %2 - SOW indicator => '' or 'SOW,' or ''
*    %3 - coef qualifier => '' or '' or '_R'
*    %4 - variable/coef prefix => 'DM' or 'S_' or 'R_'
*    %5 - REGional indicator => '' or '' or 'REG,'
************************************************************************

* Growth in Demand
  - SUM(DMSTEPS%3(%5DM,'UP',JSTEP),
%4ELAST(%5TP,%2DM,'UP',JSTEP)$(DM_ELAST%3(%5DM,'UP',TP) NE 0) )

* Reduction in Demand
  + SUM(DMSTEPS%3(%5DM,'LO',JSTEP),
%4ELAST(%5TP,%2DM,'LO',JSTEP)$(DM_ELAST%3(%5DM,'LO',TP) NE 0) )
```

Figure 2-13. MMEQDEM.INC and MMEQDEM.MED prior to substitution of

```
*----- GAMS call for the source code module
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.INC

*----- All environment variable parameters substituted in MMEQDEM.INC
MR_DEM(REG,TP, DM)$(TS(TP) AND (DM_DEM_R(REG,DM, TP) GT 0)) .. MMSCALE_R(REG) *
* demand device production of useful demand services
(   SUM(DMD$DMD_DMS_R(REG,DMD
          DEM_CAP_R(REG,TP, DMD, DM) * R_CAP(REG,TP, DMD))

*----- Since %MARKALED% == 'YES' call is made to add the elastic demand parameters
*** BATINCLUDE C:\ANSWERV5\GAMS_SRCPRDVD\MMEQDEM.MED

* growth in demand
  - SUM(DMSTEPS_R(REG,DM,'UP',JSTEP),
          R_ELAST(REG,TP,DM,'UP',JSTEP)$(DM_ELAST_R(REG,DM,'UP',TP) NE 0) )

*  reduction in demand
  + SUM(DMSTEPS_R(REG,DM,'LO',JSTEP),
          R_ELAST(REG,TP,DM,'LO',JSTEP)$(DM_ELAST_R(REG,DM,'LO',TP) NE 0) )

)

  =G=

* Set the RHS to the demand level
MMSCALE_R(REG) * DM_DEM_R(REG,DM, TP);
```

Figure 2-14. MMEQDEM.INC, including MMEQDEM.MED, after substitution of GAMS environment variables ready for execution

2.11.2 MARKAL Initialization Routines

In order to allow MARKAL the flexibility to include only the desired model options, thereby omitting others, all possible GAMS sets and parameters are declared in "MMINIT" routines. Each and every set and parameter referenced anywhere in the MARKAL code is initialized (to null) in one of these routines by naming it and defining its dimensionality (number of indexes) and scope (set controlling the domain)[92].

In Table 2-34 below the individual MMINIT routines are identified and their purpose elaborated.

[92] For the multi-region sets and parameters employed in the solve step the domains have been set to the universal ('*'), rather than the specific known sets, owing to the approach taken of buffering the individual regional information into the multi-region structures.

Table 2-34. MARKAL initialization routines

Routine	Steps Used[93]	Purpose
MMINIT.INC	MG/SOL	Declare all the single region sets and parameters.
MMINIT.REG/ MMINIANS.REG	MG/SOL	Declare all the multi-region sets and parameters.
MMINT.ANS/VFE	MG/SOL/RW	Declare the rest of the single region sets and parameters.
VEDA2GAMS.VFE/ REG	MG/SOL	Preprocess VEDA generated sets and parameters that need to be mapped to their GAMS equivalents, and initialize selected parameters.

2.11.3 MARKAL Matrix Generator Source Code Overview

For each region the MARKAL matrix generator is invoked to generate all the internal parameters governing the intersections to be created in the final matrix. The resulting GAMS Data Exchange file (<region>.GDX) contains all such information, along with the original input data, for each region. This information, along with the trade specification, is then picked up by the solve step and assembled into the full multi-region matrix. The _<region>.G0* restart files are also passed to the report writer to re-establish the individual regional data needed for each report.

Each regional matrix generation step is controlled by the <case>_<region>.GEN GAMS command directive file generated by ANSWER or VEDA-FE. An overview of the calls and actions is given below.

- <case>_<region>.GEN
  - * initialize
    - MMINIT.INC
    - MMINIT.GP
    - MMINIT.REG
    - MMINIT.ANS/VFE
  - * obtain the regional data
    - <scenario>_<region>.DD
    - <altscenario>_<region>.DDS
  - * activate any model run switches
    - $SET Run switches
  - * complete the data setup
    - ANSorVEDA2GAMS.ANS/VFE
    - REG/SAGE_MG.GMS
      - Perform QC checks
      - Derive the single region model coefficients

[93] MG = Matrix Generation, SOL = Solve, RW = Report Writer

        - Move the region specific set and parameter information into their multi-region counterparts
- Files produced for each region
    - <case>_<region>.LST[94] with execution trace along with any error messages
    - QA_CHECK/CONVERT.LOG with quality control and any fractional LIFE adjustments
    - REG_FULL.SET, the incrementally built list of master set elements (e.g., ENT, TCH, DM) with values for all regions
    - <region>.GDX for the solve step
    - _<region>.G0* restart files for the report writer

2.11.4 MARKAL Solve Source Code Overview

The MARKAL solve source code is invoked to generate the final matrix and solve the model. The GAMS Data Exchange files (<region>.GDX) produced by the regional matrix generation step are loaded and mapped to their regional counterpart. For SAGE a loop over each time period first sets lower bounds on market share technologies, solves, adjusts market shares if necessary and re-solves (writing to the MKTSHR.LOG), then augments the investment costs for any learning technologies before going on to the next period.

Before the actual solve step is invoked a pre-step is done to assemble REG_FULL.GDX from REG_FULL.SET. The solve step is controlled by the <case>.SLV GAMS command directive file generated by ANSWER or VEDA-FE. An overview of the calls and actions is given below.

- <case>.SLV
  * initialize
    - MMINIT.INC
    - MMINIT.REG
    - MMINIT.ANS/VFE
    - $INCLUDE REG_FULL.SET to get the full list of all master sets assembled
    - MMINIANS.REG
  * activate any model run switches
    - $SET Run switches
  * prepare and solve the model
    - REG/SAGE_SOLV.GMS
  * assemble the multi-region model
        - Perform QC checks on trade
        - Get each <region>.GDX file, assembling the full matrix

[94] Note that the <case>_<region>.LST is generated first for each region in the matrix generation step, but later overwritten during the report writer step. If an error occurs during an regional matrix generation step the user must abort the run (via Ctrl-C in the DOS box) and examine the associated LST file using any text editor.

        - If SAGE establish a loop for solving each period

    * solve the model

        - Solve
            - If SAGE
                - apply market share adjustments and resolve
                - Determine any SETL related adjustments to the investment cost of learning technologies for the next period
        - Check on solver result status

- Files produced
    - <case>.LST with execution trace along with any error messages, as well as the solution dump and trade status
    - QA_CHECK/MKTSHR.LOG with trade and the actions taken by the market share algorithm
    - <case>.GDX for the report writer

2.11.5 MARKAL Report Writer Source Code Overview

For each region the MARKAL report writer is invoked to produce the ANSWER/VEDA-BE "dump" of model results. The individual <region>._G0* restart files and full GAMS Data Exchange file (<case>.GDX) containing the multi-region solution is processed to produce the region specific information. The resulting files can then be imported into ANSWER/VEDA-BE for subsequent analysis.

Each regional report writer step is controlled by the <case>_<region>.RPT GAMS command directive file generated by ANSWER or VEDA-FE. An overview of the calls and actions is given below.

- <case>_<region>.RPT

    * initialize and obtain model results

    - MMINIT.ANS/VFE
    - Load the <region>._G0* restart and <case>.GDX solution files

    * activate any model run switches

    - $SET Run switches
    - REG/SAGE_RPT.GMS

    * prepare the result information for each region

        - Buffer information from the multi-region solution for this region
        - Write the ANSWER or VEDA-BE solution information

- Files produced for each region
    - <case>_<region>.LST with execution trace along with any error messages[95]
    - <case>.ANT or VDE/VDS/VD ANSWER or VEDA exchange files. T

[95] Note that the <case>_<region>.LST is generated first for each region in the matrix generation step, but later overwritten during the report writer step. If an error occurs during an regional matrix generation step the user must abort the run (via Ctrl-C in the DOS box) and examine the associated LST file using any text editor.