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Contribution to climate change

Why could Contributions to Climate change be important?
Working with timeseries in linear models: Using the ramp response

output(t1) = \int_{-\infty}^{t1} \text{pulse}(t1 - t2) \cdot \text{input}(2) \, dt
ramp(t) = \int_{0}^{t} \int_{0}^{t} \text{pulse}(ttt) \, dt \, dt

(for the default models, we know both pulse and ramp explicitly)

For piecewise linear input: \( m = \text{input}(m, dt) \), \( n = \text{output}(n, dt) \)

Response to wedge-input: difference of 2 ramp responses

Response to full input: Sum of \textit{wedge responses}.

define \( \text{Ramp}_{n,k} = \text{ramp}((n - k) \cdot dt) \) \( n > k \)

\[ \Delta_{k,m} = ((k+m-1) - 2(k=m) + (k=m+1)) \quad (m > 0) \]

\[ R_{n,m} = \sum_{k=0}^{N} \text{Ramp}_{n,k} \frac{\Delta_{k,m}}{dt} \quad \text{then} \quad \text{out} = R \cdot \text{in} \quad \text{(Matrix multiplication)} \]

Alternatively: Use the step response

For staircase input:

\[ \text{step}(t) = \int_{0}^{t} \text{pulse}(tt) \, dt \quad \text{define:} \quad \text{Step}_{n,k} = \text{step}((n - m - 0.5) \cdot dt) \quad (n > k) \]

\[ \Delta_{k,m} = (k=m-1) - (k=m) \quad (m > 0) \quad \text{R = step} \triangle \quad \text{then} \quad \text{out} = R \cdot \text{in} \]
"Exact" continuous output:

\[ TOL := 10^{-4.5} \quad \text{outcontinuous}(t) := \left[ C_{puls}(tt) \cdot \text{emiter}(t - tt) \right] \cdot \left( \frac{1}{(1 - tcut) \cdot (1 + tcut)} \right) \cdot \text{em}(t) \text{ cut at } tcut = 250 \pm dt \cdot 0.5 \]

\[ \text{test} := 250 \]

\[ \text{out}(t) := \int_{0}^{t} C_{puls}(tt) \cdot \text{emiter}(t - tt) \cdot \text{dt} \]

Ramp response based output is exact response to linearly interpolated input, which is almost the same as the exact continuous output, provided emissions are cut at end of period. Step response based output have similar performance.

\[ \text{outcontinuous(test)} = 70.029 \quad \text{outC}(\text{test}) = 70.029 \]

\[ \text{outC}(\text{test}) = 69.969 \]

\[ \text{outC}(\text{test}) = 70.065 \]

dt = 5

**Input**

![Input graph]

**Output**

![Output graph]

\[ t := 240, 240.05 \ldots 260 \]
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**Solving models**

**One box:**

\[
\frac{dQ}{dt} = \lambda Q + a x(t) \\
Q(t) = \int_{-\infty}^{t} \text{pulse}(t-t) \cdot x(t') dt' \\
\text{pulse}(\tau) = e^{-\lambda \tau} a
\]

**Many boxes:**

\[
\frac{dQ}{dt} = M \cdot Q + A \cdot X(t) \\
Q \text{ and } X \text{ or } A \text{ vectors, } M \text{ a matrix} \\
Q(t) = \int_{-\infty}^{t} \text{PULSE}(t-t) \cdot X(t') dt' \\
\text{where}
\]

\[
eig = \text{eigenvals}(M) \\
S = \text{eigenvects}(M) \\
M = S \cdot \text{diag}(\text{eig}) \cdot S^{-1} \\
\text{PULSE}(\tau) = \left(S \cdot \text{diag}(\exp(\text{eig} \cdot \tau)) \cdot S^{-1}\right) \cdot A
\]

If we are only interested in one variable \(y = B^T Q\) (could be the sum of \(Q\)), and \(x\) is a number while \(A\) is a vector, \(B^T\)PULSE is a simple sum over exponential terms, which explains while the default models work.

**Solving stepwise:**

\[Q_n = \text{PROP} \cdot Q_{n-1} + \text{STEP} \cdot X_{n-1} + \text{RAM1} \cdot \Delta X_n + \text{R2} \cdot \Delta^2 X_n + \text{R3} \cdot \Delta^3 X_n + \ldots\] (break after RAM1 if \(x\) is piecewise linear)

\[\text{PROP}(dt) = S \cdot \text{diag}(\exp(\text{eig} \cdot dt)) \cdot S^{-1}\]

\[\text{STEP}(dt) = S \cdot \text{diag}\left(\frac{\exp(\text{eig} \cdot dt) - 1}{\text{eig}}\right) \cdot S^{-1} \cdot A\]

\[\text{RAM1}(dt) = S \cdot \text{diag}\left(\frac{\exp(\text{eig} \cdot dt) - 1}{\text{eig} \cdot dt}\right) \cdot S^{-1} \cdot A\]

\[\text{RAM}_n(\tau) = \frac{1}{dt} \int_0^\tau \text{RAM}_{n-1}(\tau) \cdot d\tau\]

Note: any eigenvalues of zero need special treatment.

Slightly nonlinear Models (Hilda, Wigley-Raper):

\[
\frac{dQ}{dt} = M \cdot Q + A \cdot X(Q, t) \\
\text{solve as above, stepwise, but iterate on } Q \text{ in each step}
\]
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Historical emissions, concentrations and models...can they be made consistent?

Case: LULUCF emissions

![Graph showing historical LULUCF emissions](image)

- Red line: LULUCF implied
- Blue line: LULUCF Houghton et al.
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Historical emissions, concentrations and models...can they be made consistent?

Case: LULUCF emissions

The CO2 concentration

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Historical emissions, concentrations and models...can they be made consistent?

Case: LULUCF emissions

Fertilisation (extra NPP) as function of CO2conc

Comparing implied LULUCF with Houghton estimate

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Historical emissions, concentrations and models... can they be made consistent?

Look on data for each region: region: OECD

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**Contribution to climate change**

**What kind of measure?**

It seems logical, that we want a measure, with the property, for two sources A and B:

\[ T(A + B) = T(A) + T(B) \]

(Here I use T, but it could be any index)

**Total effect partitioned on sources**

This means linearity between sources (Parties) and between gases. We would further like the contributions to sum up to the total effect (then tstart must be -\(\infty\)).

So, what we are after, is Metrics, so that for sources \(p\) and emission of gases \(g\):

\[
T(t2) = \int_{tstart}^{tend} \sum_{(E,g,p)} \text{Metric}_{E,g,p}(t2, t1, E_g) E_{g,p}(t1) \, dt1
\]

We would have to include all sources and all emissions of gases, known to contribute to climate change (including CFC's and aerosols), but not emissions arising from feedback (fertilisation).

Note that I assumed the Metric only to depend on the total emissions of each gas:

\[
E(t)_g = \sum_p E(t)_{g,p}
\]

(But the Metric for gas \(g_1\) could depend on the emissions of gas \(g_2\))

**Linearity**

**No future feedback on contributions from present emissions**

We could further demand, that \(\text{Metric}(t2, t1, E_g)\) should only depend on \(E(t)_{g,p}\) for \(t \leq t1\).

Why? Because otherwise, the contribution from one source at \(t1\) to the effect at \(t2\), would depend on emissions (of own and other Parties) at times after the time of emission at \(t1\), which could not be known at the time \(t1\). So this is a reasonable requirement, if we want the future contribution of Parties to be known (or calculable, assuming good models), at the time of emissions.
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Contribution to climate change

A measure that has the stipulated properties: The timeordered (or timesliced) method of Ian Enting:

\[
\text{Temperature rise from CO2-emissions}
\]

The difference \( dT(t_2, 2000, dt) \) between the temperature-paths at \( t_2 \), with emissions set to 0 in year 2000+\( dt \) and in year 2000 is contributed to the emissions occurring between 2000 and 2000+\( dt \):

\[
\text{Metric} (t_2, 2000) = \frac{dT(t_2, 2000, dt)}{E(2000) \cdot dt}
\]

We can do it for several cases, one by one. The result is independent of the order, as long as we are dealing with "infinitesimal changes" (a small \( dt \)) - and the models (as hopefully the Earth System) is without discontinuities, or thresholds...

Hence we get the desired Measure:

\[
T(t_2) = \sum_{(t_1, t_0, E)} \text{Metric} (t_2, t_1, E) \int_{t_0}^{t_1} \text{...} dt
\]

We can do it for any index (Temperature, sea level rise, damage..) even for non-linear gas or energy models, feedbacks from temperature to carbon etc.

The method mirrors our situation: We can't change the past, but we know that our actions (our emissions) at each instant changes the future.
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**Contribution to climate change**

When we use simple linear models for CO2 and temperature, the metric is very easy to calculate, as follows:

\[
\begin{align*}
\text{Time ordered response:} & \quad E_{ot} := \text{ECO}2^{<\bullet>} \\
\text{Eended}_{k,j} := & \text{i}(k,j,0,1) \cdot \text{Eot}_{k} \\
\text{Cended} := & \text{C}_0 + \text{RC Eended} \\
\text{Delay}_{j,k} := & (j=k-1) + (j=0) (j=0) \\
\text{Cendedmid} := & 0.5 \left( \text{Cended} + \text{Cended Delay} \right)
\end{align*}
\]

\[\text{d}T^{\downarrow} := \text{RT} \left( \left( \text{dFCO2dC}^{\text{Cendedmid}} \text{RC} \right) \right) \text{ECO}2^{<\bullet>}
\]

\[\text{d}S^{\downarrow} := \text{RT} \left( \left( \text{dFCO2dC}^{\text{Cendedmid}} \text{RC} \right) \right) \text{ECO}2^{<\bullet>}
\]

The Metric is thus (for temperature): \[\text{RT} \left( \left( \text{dFCO2dC}^{\text{Cendedmid}} \text{RC} \right) \right)\]

The arrow denote that we shall first calculate the marginal forcing \(\text{dF}/\text{dC}\) for all the scenarios and all the times, and thereafter multiply element by element with the emissions-to-concentration matrix \(\text{RC}\). This is the kind of trick you can do in mathcad, but it is really just integration from \(C=0\) to \(C(t1)\) of \(\text{dF}/\text{dC} \cdot \text{dC}\) in a vertical direction in the \(F \text{ vs } t\) plot, at time \(t1\), between the scenarios (which gives \(F\)), which then gives \(\text{dT}\) by multiplication with the forcing to temperature matrix \(\text{RT}\).

\[
\begin{align*}
\int_{C_0}^{C} \frac{\text{dF}}{\text{dC}} \text{dC} \\
\text{F}(t) := \int_{-\infty}^{t} \text{dFCO2}(C(t)) \cdot \frac{\text{dC}(t)}{\text{dt}} \text{dt} \quad \text{differential method} \\
\text{where } t \text{ is the time where } F(t) \text{ changes}
\end{align*}
\]

\[
\begin{align*}
\text{F}(t) := \int_{-\infty}^{t} \text{dFCO2}(C(t,t)) \cdot \frac{\text{dC}(t,t)}{\text{dt}} \text{dt} \quad \text{timeslice method, where } C(t,t) \text{ is the concentration at time } t \text{ in the scenario, where emissions stop at } t
\end{align*}
\]

\[
\frac{\text{dC}(t,t)}{\text{dt}} = \text{Cmetric}(t,t) \cdot \text{E}(t)
\]

\[
\text{Defines the Cmetric. Then the F metric is}
\]

\[
\text{dFCO2}(C(t,t)) \cdot \text{Cmetric}(t,t) = \text{dFCO2}(C(t,t)) \cdot \text{Cpulse}(t - t)
\]
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Metrics: Contributions to Concentration increases:

- CO2 (ppmv per Pg C)
- CH4 (ppbv per Tg CH4)
- N2O (ppbv per Tg N)

Credits to Gylvan Meira Filho for having pointed out, that its all about establishing a metric!
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Contribution to climate change

Metrics: Contributions to Radiative forcing increases:

CO2 (W/m² per Pg C)

CH4 (W/m² per Tg CH4)

N2O (W/m² per Tg N)

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Contribution to climate change

Metrics: Contributions to temperature increases:

- CO2 (deg K per Pg C)
- CH4 (deg K per Tg CH4)
- N2O (deg K per Tg N)

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Contribution to climate change

Metrics: Contributions to ocean expansion increases:

- CO2 (m per Pg C)
- CH4 (m per Tg CH4)
- N2O (m per Tg N)

role of historical emissions

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**Contribution to climate change**

**Marginal attribution:**

$$
\frac{d\mathbf{T}^{\phi}}{dt} = \text{RT} \cdot \text{diag}(dF_{\text{co2}} dC(CO2\text{tot})) \cdot \text{RC-ECO2}^{\phi}\leftarrow dT_{1p}(t) = N(t)
$$

**Time rate of change of forcing:**

$$
\frac{d\mathbf{T}^{\phi}}{dt} = \text{RT} \cdot \text{INT} \cdot \text{diag}(dF_{\text{co2}} dC(CO2\text{mod})) \cdot \text{DOT} \cdot \text{RC-ECO2}^{\phi}\leftarrow dT_{2p}(t) =
$$

$$
\int_{-\infty}^{t} \left[ \frac{\delta\mathbf{T}(t_1)}{\delta F(t_2)} \cdot \frac{dF}{dC} \left( C(t_2) \right) \int_{-\infty}^{t_1} \left[ \frac{\delta\mathbf{C}(t_2)}{\delta E(t_3)} \cdot \frac{dE(t_3)}{dt} \right] dt_3 \right] dt_2
$$

**Time ordered attribution:**

$$
\frac{d\mathbf{T}^{\phi}}{dt} = \text{RT} \cdot (dF_{\text{co2}} dC(C\text{ended}mid) \cdot \text{RC}) \cdot \text{ECO2}^{\phi}\leftarrow dT_{3p}(t) =
$$

$$
\int_{-\infty}^{t} \left[ \frac{\delta\mathbf{T}(t_1)}{\delta F(t_2)} \cdot \frac{dF}{dC} \left( \Psi(E,t_2,t_3) \right) \int_{-\infty}^{t_1} \left[ \frac{\delta\mathbf{E}(t_2,t_3)}{\delta E(t_4)} \cdot \frac{dE(t_4)}{dt} \right] dt_3 \right] dt_2 dt_3
$$

Here $\Psi[E,t_2,t_3]$ is a functional, giving the concentration at $t_2$ following from total emissions $E(t)$ up to $t=t_3$, and zero thereafter, while

$$
\frac{\delta\Psi}{\delta E}(E,t_2,t_3) = \frac{(\Psi(E,t_2,t_3 + dt_3) - \Psi(E,t_2,t_3)}{E(t_2) \cdot dt_3}
$$

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Contribution to climate change

Temperature rise:

- $dT_{k,1}$
- $dT_{k,2}$
- $dT_{k,3}
- dT_{k,4}$

$1500 + k$
The overall difference between the methods, seems to be small, in the this case (the A2 scenario). We would like to look into other scenarios to see whether this holds in all cases (especially the more interesting scenarios leading to stabilization of the greenhouse gas concentrations.

$$\max \left( \frac{dT1 - dT3}{dT3 + 10^{-20}} \right) = 0.018$$

Here we take max over all the 4 parties and all times.

The maximum difference seems to be of the order of 2%

$$\max \left( \frac{dT2 - dT3}{dT3 + 10^{-20}} \right) = 0.017$$

$$\max \left( \frac{dS1 - dS3}{dS3 + 10^{-20}} \right) = 0.011$$

$$\max \left( \frac{dS2 - dS3}{dS3 + 10^{-20}} \right) = 0.01$$

With bug corrected, this is .010, .015, .010, .010
Metrics: Contributions to temperature increases (deg K per Tg N):

Values, going right along an emission year on the vertical axis, gives the contribution from a pulse in the emission year. It is seen, that the "Time rate of change metric" iso-lines bends to the right. This means we can be responsible for ever, even though the pulse has vanished (N2O and CH4), whereas the responsibility in the time ordered metric fades out.
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Contribution to climate change

Comparing 3 metrics: Temperature rise for CO2 emissions: \((10^{-3} \text{ deg C per Pg C/yr})\)

**Note:** Calculated for A2 scenario, and with default gas and energy models

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.

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Comparing 3 metrics: Sea level rise from CO2 emissions: (m per Pg C/yr)

**Note:** Calculated for A2 scenario, and with default gas and energy models.

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.
Comparing 3 metrics: Temperature rise for N2O emissions: (deg C per Tg N/yr)

Note: Calculated for A2 scenario, and with default gas and energy models

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.
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Contribution to climate change

Comparing 3 metrics: Sea level rise from N2O emissions: (m per Tg N/yr)

Note: Calculated for A2 scenario, and with default gas and energy models.

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.

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Comparing 3 metrics: Temperature rise for CH4 emissions: (deg C per Tg CH4/yr)

Note: Calculated for A2 scenario, and with default gas and energy models

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations. The fact decay of a CH4 pulse makes it hard to see the detailed differences in the metrics.
Comparing 3 metrics: Sea level rise from CH4 emissions: (m per Tg CH4/yr)

Note: Calculated for A2 scenario, and with default gas and energy models

Values, going right along an emission year on the vertical axis, gives the contribution per unit emitted in that year to the index in the attribution year on the horizontal axis. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.
Values, going up from a given attribution year on the horizontal axis, gives the contribution per unit emitted in the year given by the vertical axis. Red areas shows where the normalised marginal metric hits harder than the time slice metric. For CO2 and N2O this is the last about 50 years before the attribution year, whereas earlier emissions are hit less hard. In that sense the normalised marginal method favours early emitters, compared to the time slice method. Note: we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.

Calculated for the A2 scenario with 3 gases and the default gas and energy models
Values, going up from a given attribution year on the horizontal axis, gives the contribution per unit emitted in the year given by the vertical axis. Red areas shows where the normalised marginal metric hits harder than the time slice metric. For CO2 and N2O this is the last about 50 years before the attribution year, whereas earlier emissions are hit less hard (except from emissions earlier than about 100 to 150 years before the attribution). In that sense the normalised marginal favours early emitters compared to the time slice method. Note we make reservations on the issue, whether the numerical accuracy is sufficient in these calculations.

Calculated for the A2 scenario with 3 gases and the default gas and energy models